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Analysis of ICRF Waves and TAE in Helical Devices

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- Full Wave Analysis in Toroidal Plasmas
- Formulation of 3D Full Wave Analysis
- Numerical Analysis (Cold, Hot)
- Summary

Full Wave Analysis in Toroidal Devices

- **Motivations**
  - RF Heating and Current Drive (Fast wave, Alfvén wave)
  - Low Frequency Instabilities (Alfvén eigenmodes)
  - Diagnostics (Ion cyclotron emission)
- **Features**
  - Configuration: 2D (Axi-symmetric, Linear-helical), 3D
  - Plasma model: Cold, Hot (No FLR), FLR (Fast wave, Differential, Integral)
  - Numerical method: Finite difference, Finite element, Mode expansion
- **Various Codes**

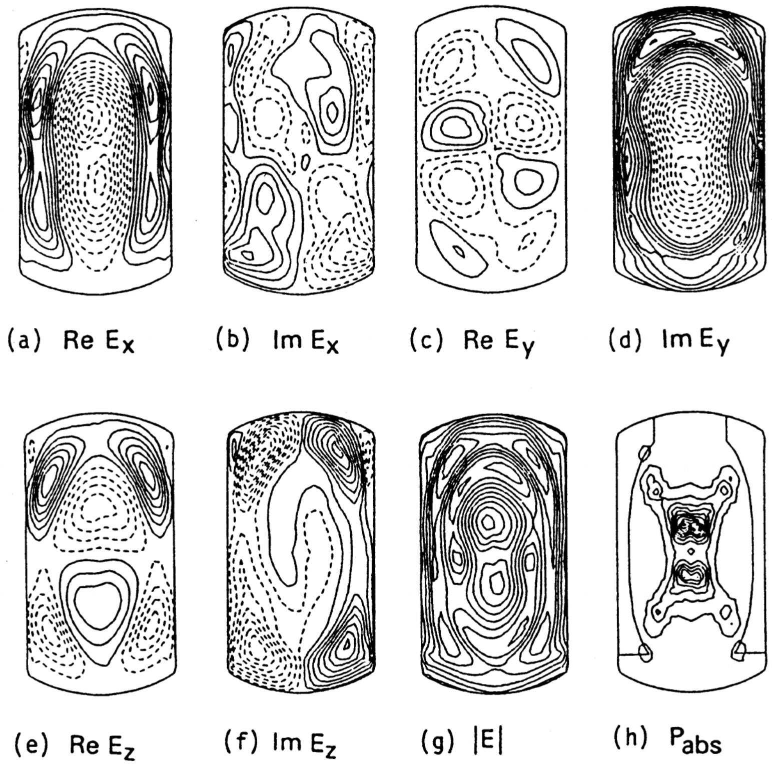
  ALCYON (Cadarache), LION (Lausanne/JET), PENN (Stockholm), PICES (Oak Ridge), TASK/WM (Ours), TORIC (Garching), Bonoli (MIT)

Comparison with Our Previous Analyses

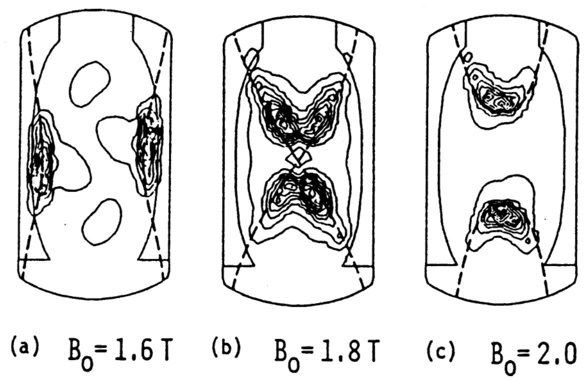
- **Tokamak Plasma** i*A. Fukuyama et al., Comp. Phys. Rep.* **4***(1986) 137*j
  - Configuration: Toroidal symmetry (2D)
  - Plasma: Cold, Hot, FLR (fast wave, differential)
  - Numerical: FDM, FEM, Mode expansion + FDM
- **Linear Helical Plasma** i*A. Fukuyama et al., Nucl. Fusion* **26***(1986) 151*j
  - Configuration: Helical symmetry (2D)
  - Plasma: Cold, Hot (no FLR)
  - Numerical: FEM
- **Toroidal Helical Plasma** i`TASK/WM version 3`)
  - Configuration: No symmetry (Magnetic flux coordinates)
  - Plasma: Cold, Hot, FLR (fast wave)
  - Numerical: Mode Expansion + FDM

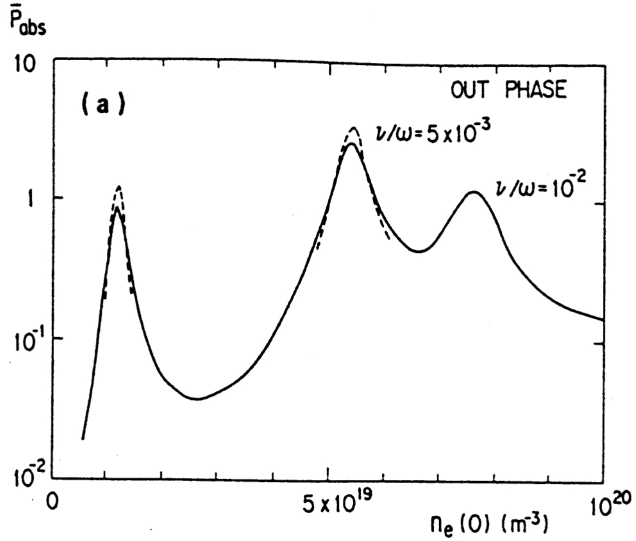
ICRF Waves in Linear Helical Plasmas

Wave Pattern



Power Absorption





Wave Equation

- Maxwell's equation for stationary wave electric field $\boldsymbol{E}$ (angular frequency ω, light velocity cj

$$\boldsymbol{\nabla} \times \boldsymbol{\nabla} \times \boldsymbol{E} = \frac{\omega^2}{c^2} \overleftrightarrow{\epsilon} \cdot \boldsymbol{E} + \mathrm{i}\,\omega\mu_0 \boldsymbol{j}_{\mathrm{ext}}$$

  - $\overleftrightarrow{\epsilon}$: Dielectric Tensor
    [Effects of finite temperature (Cyclotron damping, Landau damping)
  - $\boldsymbol{j}_{\mathrm{ext}}$: Antenna Current

- Wave Equation in Non-Orthogonal Coordinates (radial components)

$$\begin{aligned}(\boldsymbol{\nabla} \times \boldsymbol{\nabla} \times \boldsymbol{E})^1 \;=\; & \frac{1}{J}\left[\frac{\partial}{\partial x^2}\left\{\frac{g_{31}}{J}\left(\frac{\partial E_3}{\partial x^2} - \frac{\partial E_2}{\partial x^3}\right) + \frac{g_{32}}{J}\left(\frac{\partial E_1}{\partial x^3} - \frac{\partial E_3}{\partial x^1}\right) + \frac{g_{33}}{J}\left(\frac{\partial E_2}{\partial x^1} - \frac{\partial E_1}{\partial x^2}\right)\right\}\right. \\ & \left. - \frac{\partial}{\partial x^3}\left\{\frac{g_{21}}{J}\left(\frac{\partial E_3}{\partial x^2} - \frac{\partial E_2}{\partial x^3}\right) + \frac{g_{22}}{J}\left(\frac{\partial E_1}{\partial x^3} - \frac{\partial E_3}{\partial x^1}\right) + \frac{g_{23}}{J}\left(\frac{\partial E_2}{\partial x^1} - \frac{\partial E_1}{\partial x^2}\right)\right\}\right]\end{aligned}$$

  - $(x^1, x^2, x^3) = (\psi, \theta, \varphi)$
  - Similar expression for poloidal and toroidal components

Fourier Mode Expansion

- Fourier Expansion in Poloidal and Toroidal Directions
- Spatial variation of wave electric field, medium and the L.H.S. of Maxwell's equation

$$E(\psi,\theta,\varphi) = \sum_{mn} E_{mn}(\psi) e^{\mathrm{i}(m\theta+n\varphi)}$$

$$G(\psi,\theta,\varphi) = \sum_{lk} G_{lk}(\psi) e^{\mathrm{i}(l\theta+kN_p\varphi)}$$

$$J(\boldsymbol{\nabla}\times\boldsymbol{\nabla}\times\boldsymbol{E}) = G(\psi,\theta,\varphi)E(\psi,\theta,\varphi) = \sum_{m'n'}[J(\boldsymbol{\nabla}\times\boldsymbol{\nabla}\times\boldsymbol{E})]_{m'n'} e^{\mathrm{i}(m'\theta+n'\varphi)}$$

- Coupling between various modes (N_h : Rotation number of helical coil in φ)

Mode Number	Toroidal Direcition	Poloidal Direction
Wave electric field $\boldsymbol{E}$	n	m
Medium G	kN_h	l
$J(\boldsymbol{\nabla}\times\boldsymbol{\nabla}\times\boldsymbol{E})$	n'	m'
Relations	$n' = n + kN_\mathrm{h}$	$m' = m + l$

Parallel Wave Number

- Plasma dielectric tensor $\overleftrightarrow{\epsilon}(\psi, \theta, \varphi, k_\parallel^{m''n''})$ depends on $k_\parallel^{m'',n''}$ through the plasma dispersion function $Z[(\omega - N\omega_{\rm cs})/k_\parallel^{m''n''} v_{\rm Ts}]$

$$k_\parallel^{m'',n''} = -i\hat{\boldsymbol{e}}_h\cdot\boldsymbol{\nabla} = -i\hat{\boldsymbol{e}}_h\cdot(\boldsymbol{\nabla}\theta\frac{\partial}{\partial\theta} + \boldsymbol{\nabla}\varphi\frac{\partial}{\partial\varphi}) = -i\hat{\boldsymbol{e}}_h\cdot(\boldsymbol{e}^2\frac{\partial}{\partial\theta} + \boldsymbol{e}^3\frac{\partial}{\partial\varphi}) = m''\frac{B^\theta}{|B|} + n''\frac{B^\varphi}{|B|}$$

- Fourier components of $\boldsymbol{E}$ in the local normalized orthogonal coordinates

$$\begin{array}{ccccccccccc}
(J \overleftrightarrow{\epsilon} \cdot \boldsymbol{E})^i & = & J \overleftrightarrow{g}^{-1} & \cdot & \overleftrightarrow{\mu} & \cdot & \overleftrightarrow{\epsilon}_{sbh} & \cdot & \overleftrightarrow{\mu}^{-1} & \cdot & \boldsymbol{E}_i \\
m' & & \ell_3 & & & & \ell_2 & & \ell_1 & & m \\
n' & & k_3 & & & & k_2 & & k_1 & & n
\end{array}$$

therefore

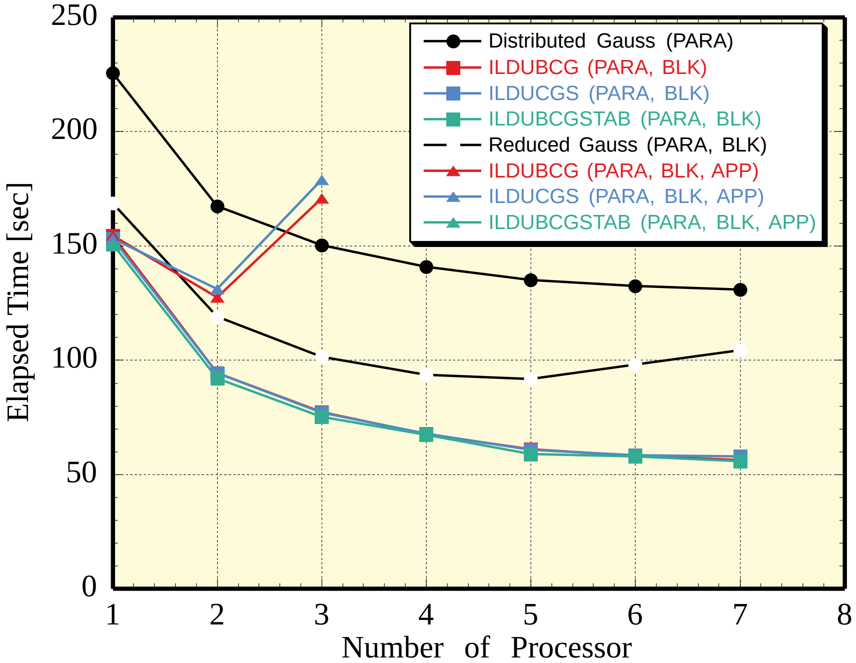
$$m'' = m + \ell_1 + \frac{1}{2}\ell_2 \qquad n'' = n + k_1 + \frac{1}{2}k_2$$

$$m' = m + \ell_1 + \ell_2 + \ell_2 \qquad n' = n + k_1 + k_2 + k_3$$

Parallel Processing by PC Cluster

- **Distributed memory system**iFortran77 + MPI libraryj
- **Dependence of elapsed time on the number of processors**

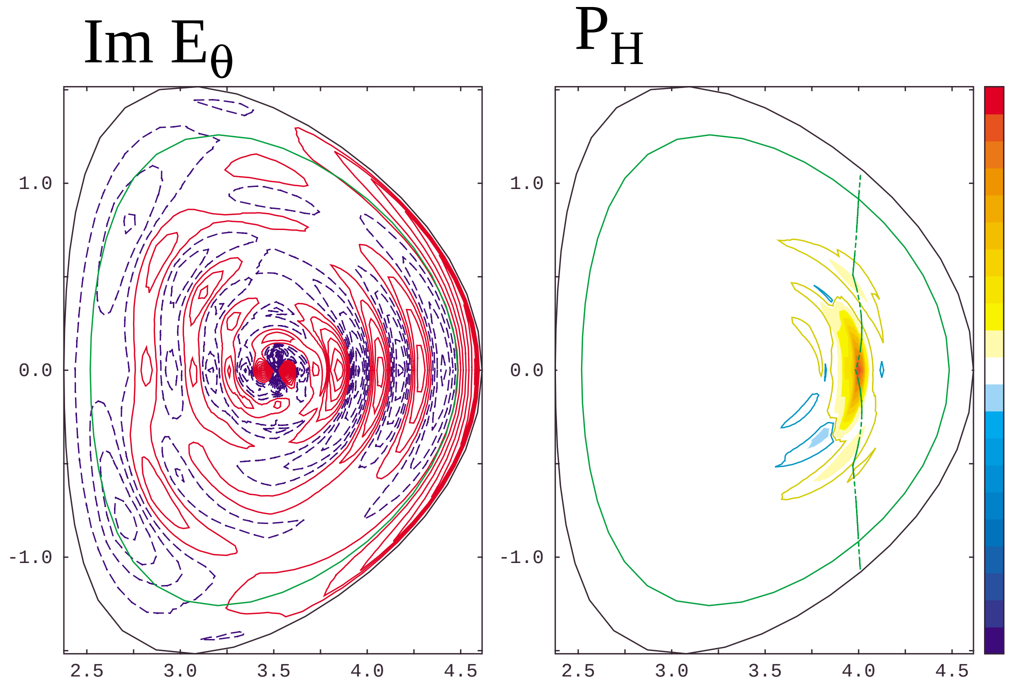
  Radial mesh $N_{Rmax} = 50$, Poloidal mode $N_{\theta max} = 8$, Toroidal mode $N_{\phi max} = 4$



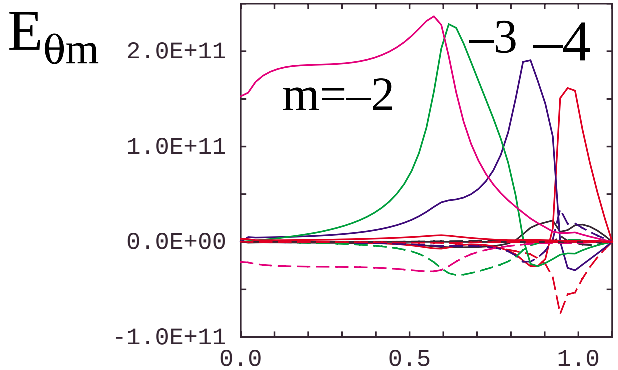
Analysis of ICRF Waves in Tokamaks

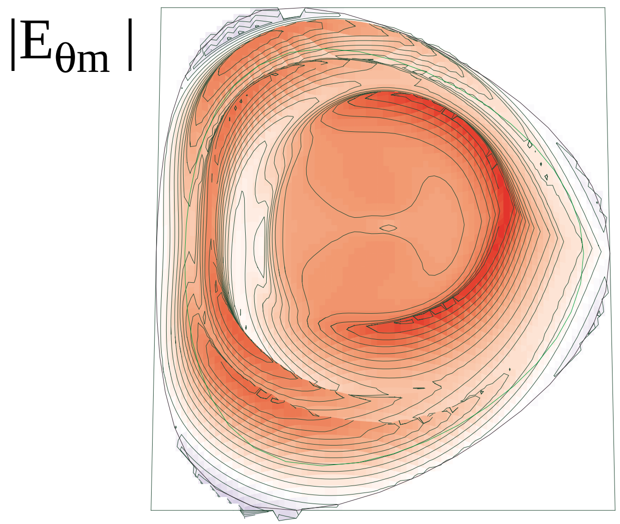
JT-60 plasma parameters

- ICRF Minority Ion Heating



- Toroidal Alfvén Eigenmode

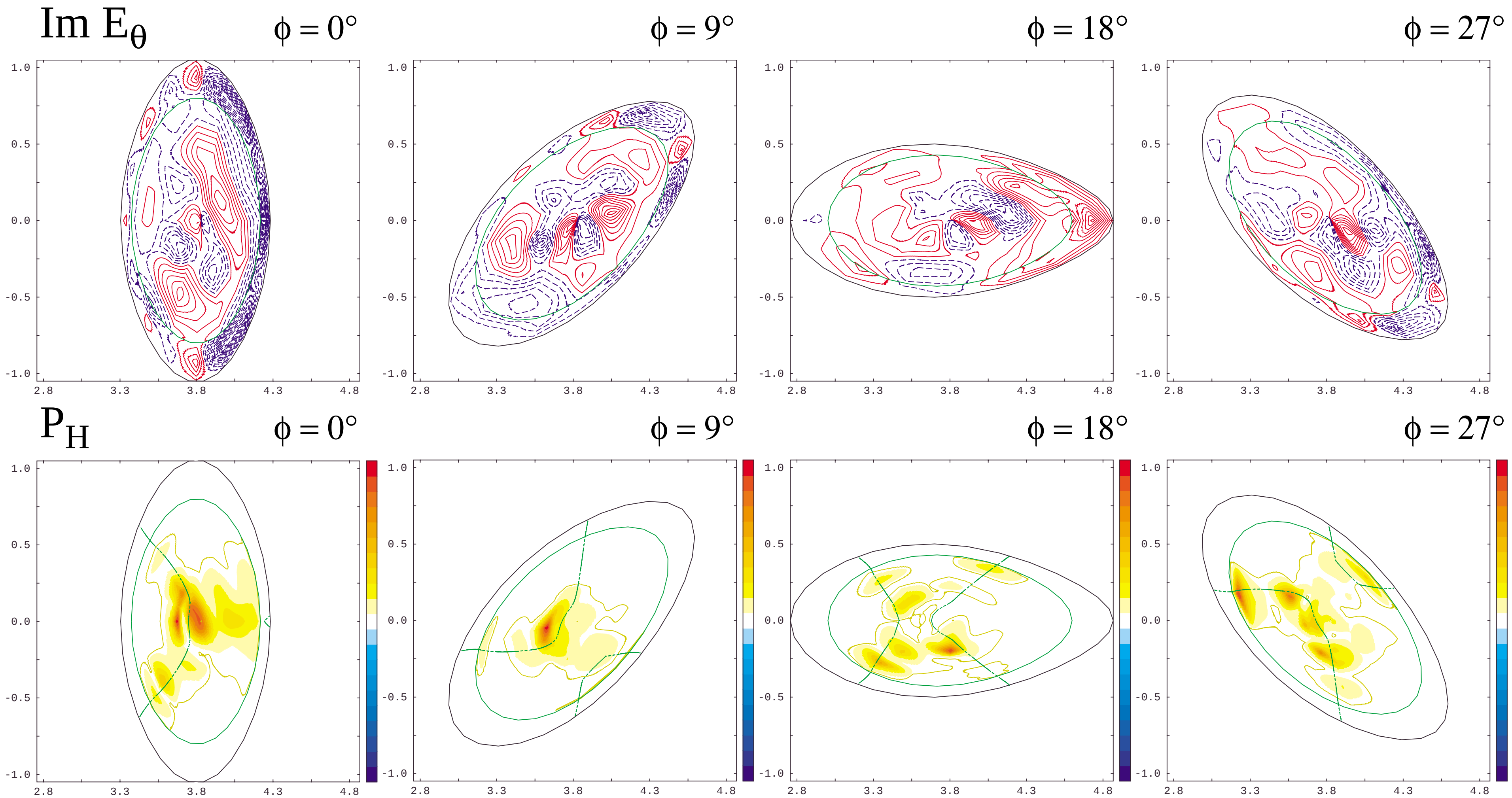




Typical Poloidal Profile (Cold Plasma Model)

LHD $(B_0 = 3.75\,\mathrm{T},\ R_0 = 3.9\,\mathrm{m})$ *Experiment:* $B_0 = 2.75\,\mathrm{T} \rightarrow f = 36.7\,\mathrm{MHz}$

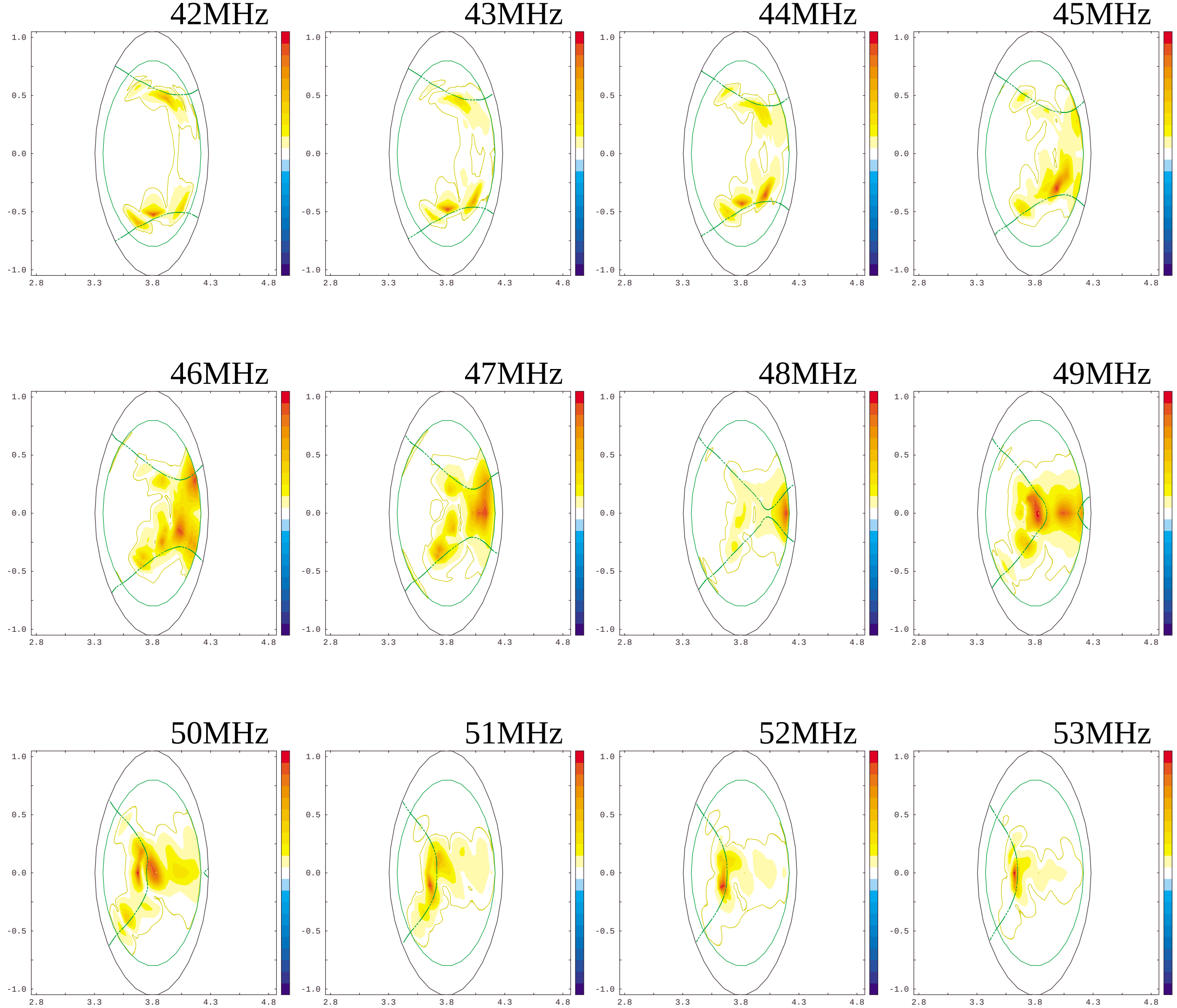
$f = 50\,\mathrm{MHz},\ n_{\mathrm{e}0} = 10^{19}\,\mathrm{m}^{-3},\ n_{\mathrm{H}}/n_{\mathrm{e}} = 0.05,\ N_{r\max} = 100,\ N_{\theta\max} = 16,\ N_{\phi\max} = 4$

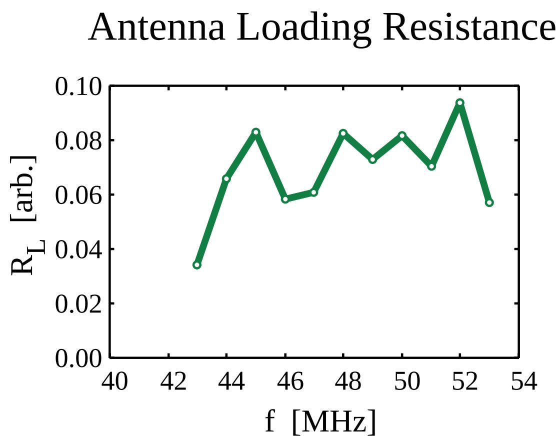


Frequency Dependence (Cold Plasma Model)

LHD ($B_0 = 3.75\,\mathrm{T}$, $R_0 = 3.9\,\mathrm{m}$) $n_{\mathrm{e}0} = 10^{19}\,\mathrm{m}^{-3}$, $n_{\mathrm{H}}/n_{\mathrm{e}} = 0.05$,

P_{abs}

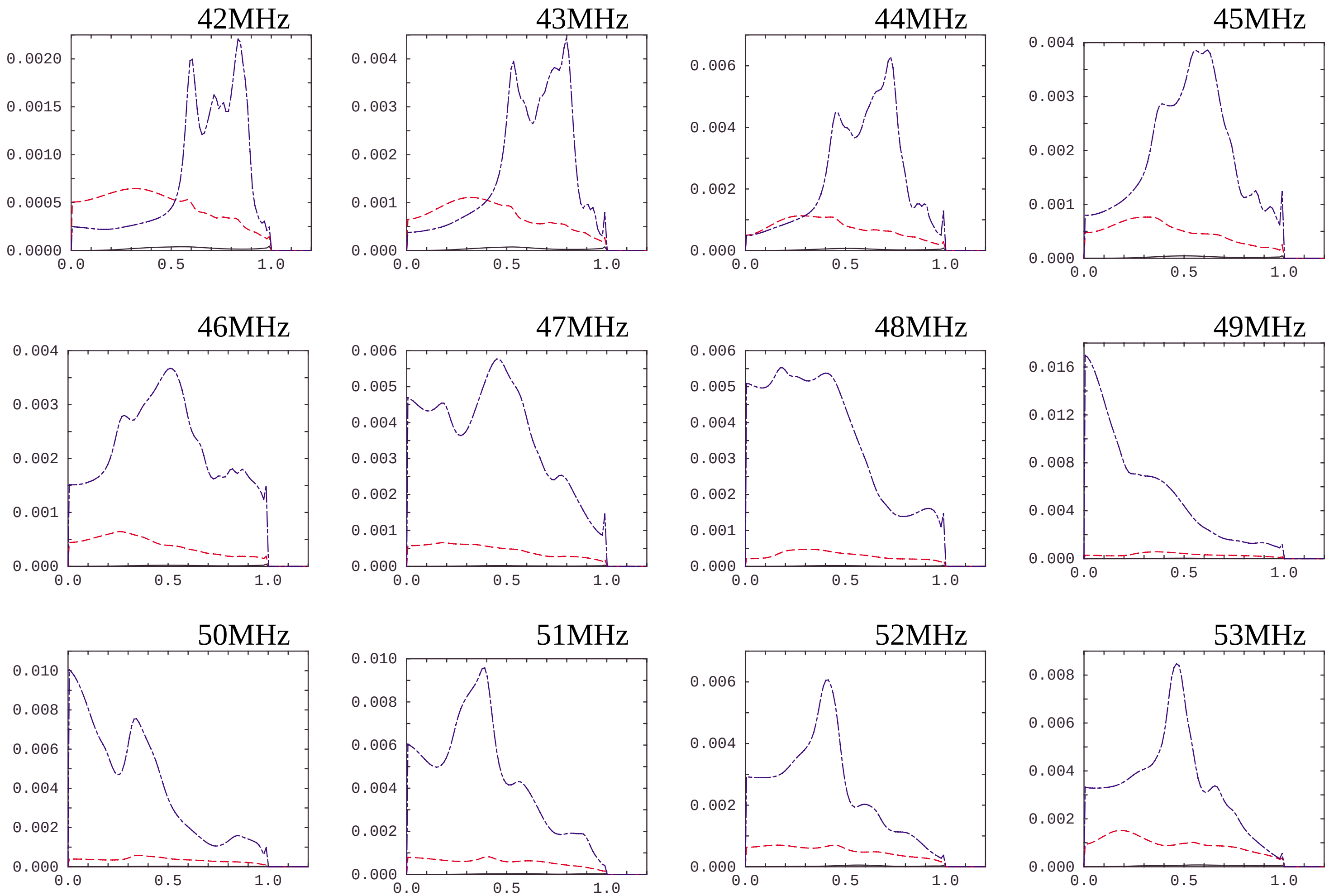




Frequency Dependence (Cold Plasma Model)

LHD $(B_0 = 3.75\,\mathrm{T},\ R_0 = 3.9\,\mathrm{m})$ $\qquad n_{e0} = 10^{19}\,\mathrm{m}^{-3},\ n_\mathrm{H}/n_\mathrm{e} = 0.05,$

$P_\mathrm{abs}(\rho)$

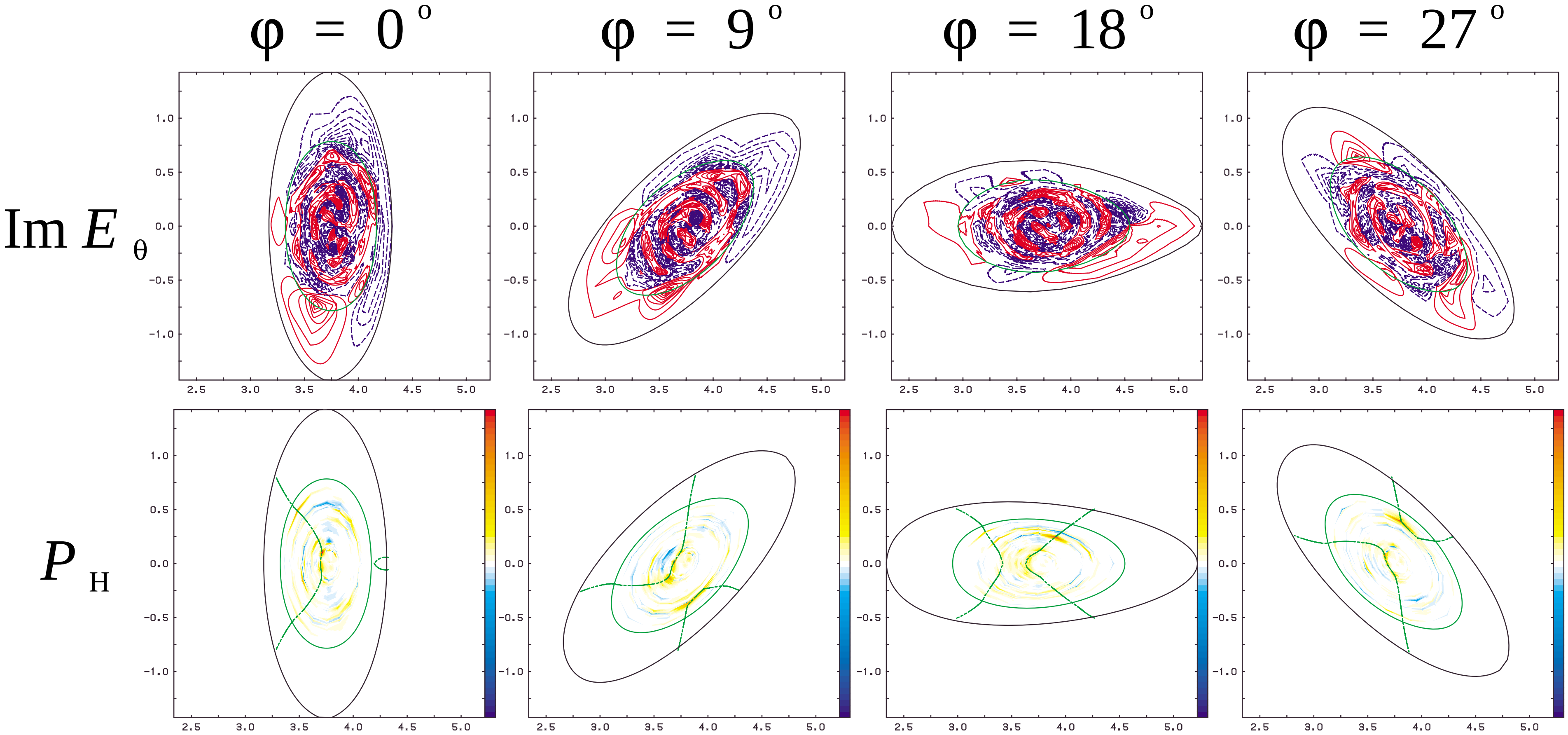


Typical Poloidal Profile

- **LHD** $(B_0 = 3.75\,\mathrm{T},\ R_0 = 3.9\mathrm{m})$

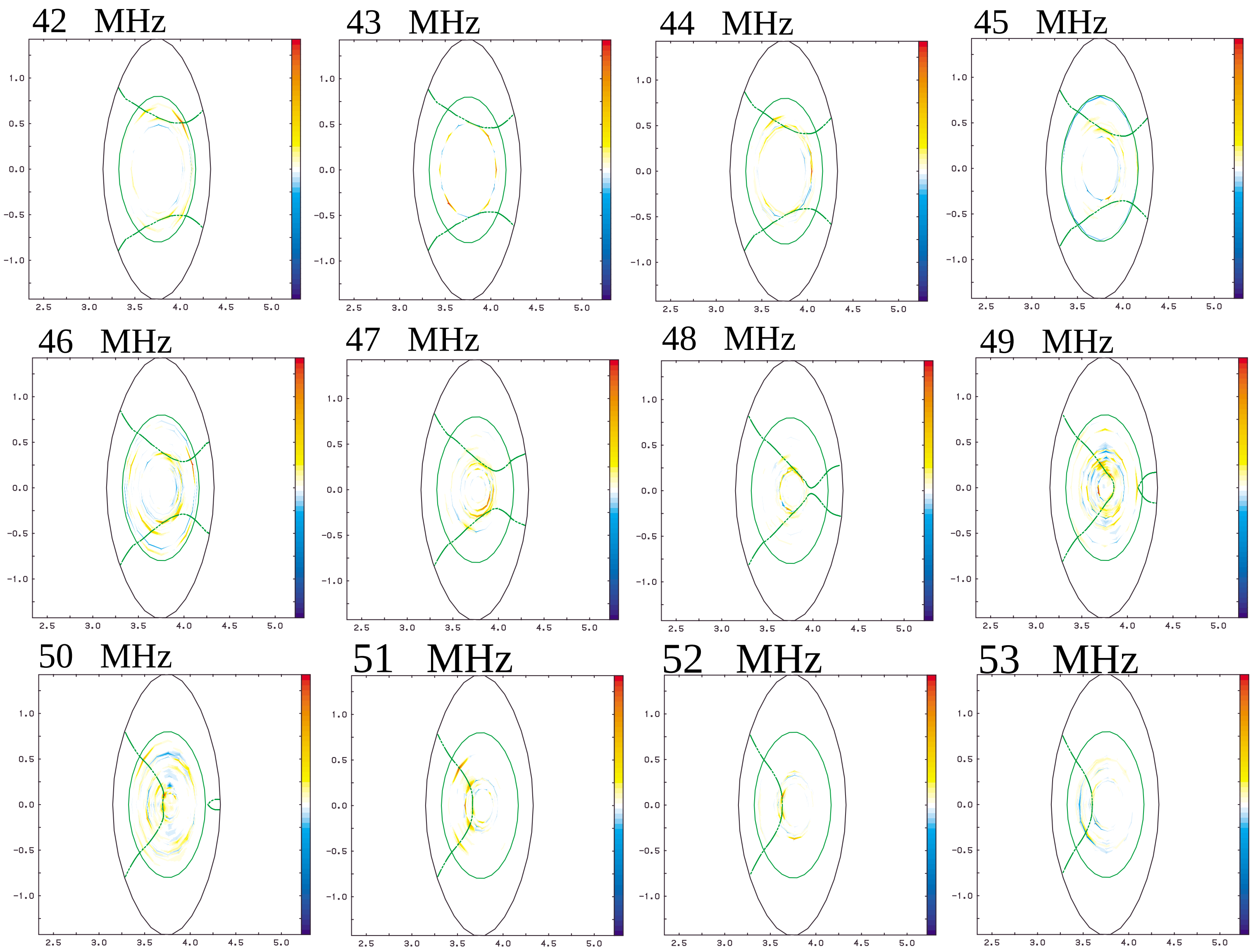
$$f = 50\,\mathrm{MHz},\ n_{e0} = 5 \times 10^{19}\,\mathrm{m}^{-3},\ n_{\mathrm{H}}/n_{\mathrm{e}} = 0.05$$

$$N_{\mathrm{rmax}} = 100,\ N_{\theta\mathrm{max}} = 16,\ N_{\phi\mathrm{max}} = 4$$



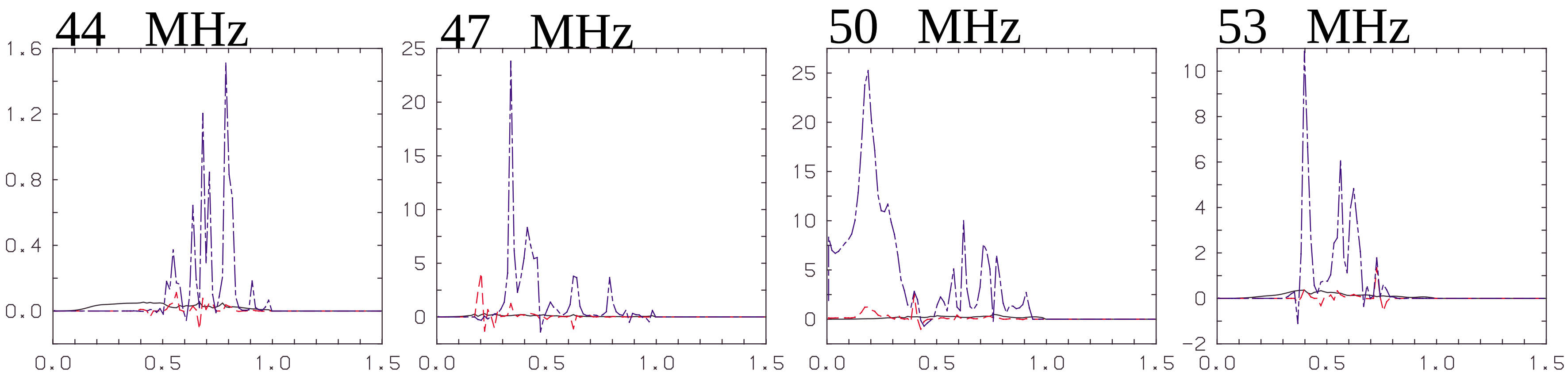
Frequency Dependence of Power Deposition Profile

- **LHD** ($B_0 = 3.75\,\mathrm{T}$, $R_0 = 3.9\mathrm{m}$), $n_{e0} = 5 \times 10^{19}\,\mathrm{m}^{-3}$, $n_\mathrm{H}/n_\mathrm{e} = 0.05$



Radial Profile of P_{abs}

- **LHD** $(B_0 = 3.75\,\mathrm{T},\ R_0 = 3.9\mathrm{m})$, $n_{\mathrm{e0}} = 5 \times 10^{19}\,\mathrm{m}^{-3}$, $n_{\mathrm{H}}/n_{\mathrm{e}} = 0.05$



Alfvén Eigenmodes in Toroidal Helical Plasmas

- **GAE**: single n, single m

  On axis or reversed shear

- **TAE**: single n, multi m

  Poloidal mode coupling $(m, m + \ell)$

$$k_\parallel = \frac{\ell}{2qR}, \qquad q = -\frac{m + \ll /2}{n}$$

- **HAE**: multi n, multi m

  Toloidal mode coupling $(n, n + N_\mathrm{h}k)$

$$k_\parallel = \frac{\ell + N_\mathrm{h}kq}{2qR}, \qquad q = -\frac{m + \ll /2}{n + N_\mathrm{h}k/2}$$

How to find an eigenmode

Maximize the wave amplitude

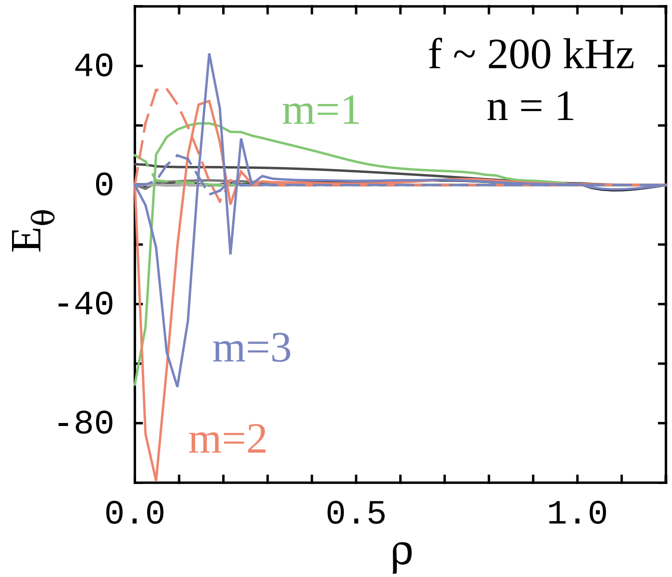
for fixed amplitude excitation

by changing the complex wave frequency

Example of GAE in LHD

Localized near the axis

where $q \sim 2$



Summary

- In order to analyze ICRF waves in toroidal helical devices, the full wave code TASK/WM was extended to three-dimensional configuration.
- Minority ion heating in LHD was studied with cold plasma approximation. The toroidal effect shifts the cyclotron resonance surface and therefore power deposition profile. Radial deposition profile is broader than those in tokamaks.
- Analysis of Alfvén eigenmodes in toroidal helical plasma is available. An example of core-localized mode was found.
- **Remaining Issues**
  - Kinetic analysis in toroidal helical plasmas
  - Finite gyro radius effect as a differential operator
  - Systematic analysis of Alfvén eigenmode in helical devices