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Analysis of Alfvén Eigenmodes Driven by Energetic Ions in Toroidal Plasmas with Weak or Negative Magnetic Shear

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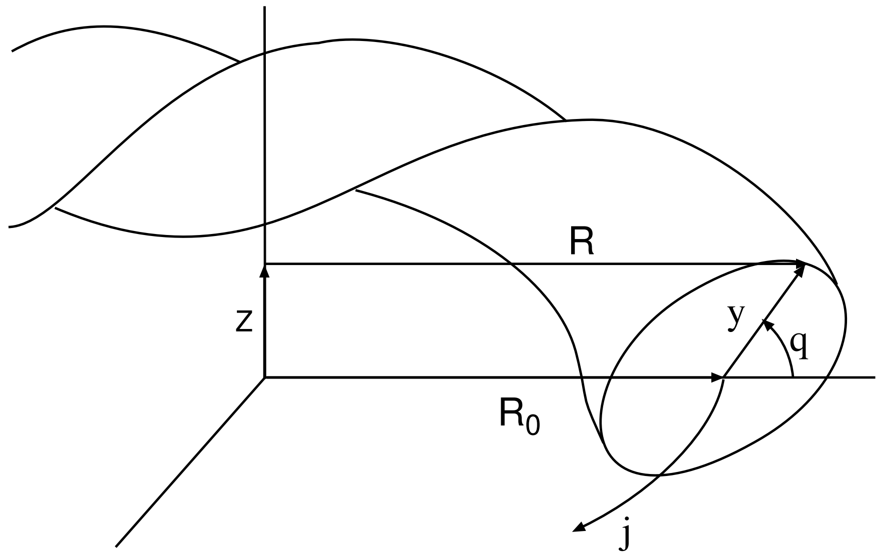
Linear Stability Analysis of Alfvén Eigenmode

- **MHD Analysis** (Ideal, Resistive)
- **MHD including Kinetic Effect** (perturbative)
  - Eigen function from MHD analysis, Growth rate including kinetic effects
- **Kinetic Analysis** (Electron thermal motion, Ion gyromotion, Drift motion)
  - PENN code (Jaun, Alfvén Lab)
  - TASK/WM (Fukuyama)
- **Ballooning Expansion** (High n mode)
  - HINST (Gorelenkov, Cheng)
  - 2D-WKB (Vlad, Chen, Zonka)
- **3D Full Wave Code: TASK/WM**
  - Magnetic surface coordinates from MHD Equilibrium Analysis
  - Boundary value problem of Maxwell's equation. Dielectric tensor)
  - Fourier mode expansion in poloidal and toroidal direction, FDM in radius)
  - Looking for complex eigen frequency which maximize the integral of wave field.

Magnetic Flux Coordinates

- Flux Coordinates (Non-Orthogonal)
  - Minor radius direction:
    Poloidal magnetic flux ψ
  - Poloidal direction: θ
  - Toroidal direction: φ



- Co-variant expression of $\boldsymbol{E}$

$$\boldsymbol{E} = E_1\boldsymbol{e}^1 + E_2\boldsymbol{e}^2 + E_3\boldsymbol{e}^3$$

where contra-variant basis

$$\boldsymbol{e}^1 = \boldsymbol{\nabla}\psi, \qquad \boldsymbol{e}^2 = \boldsymbol{\nabla}\theta, \qquad \boldsymbol{e}^3 = \boldsymbol{\nabla}\varphi$$

- J : Jacobian $\quad J = \dfrac{1}{\boldsymbol{e}^1 \cdot \boldsymbol{e}^2 \times \boldsymbol{e}^3} = \dfrac{1}{\boldsymbol{\nabla}\psi \cdot \boldsymbol{\nabla}\theta \times \boldsymbol{\nabla}\varphi}$

- g : Metric tensor $\quad g_{ij} = \boldsymbol{e}_i \cdot \boldsymbol{e}_j$, where co-variant basis $\boldsymbol{e}_i \equiv \partial\boldsymbol{r}/\partial x_i$

Wave Equation

- Maxwell's equation for stationary wave electric field $\boldsymbol{E}$

  (angular frequency ω, light velocity c)

$$\nabla \times \nabla \times \boldsymbol{E} = \frac{\omega^2}{c^2} \overleftrightarrow{\epsilon} \cdot \boldsymbol{E} + \mathrm{i}\,\omega\mu_0 \boldsymbol{j}_{\mathrm{ext}}$$

  - $\overleftrightarrow{\epsilon}$: Dielectric Tensor

    [Effects of finite temperature (Cyclotron damping, Landau damping)

  - $\boldsymbol{j}_{\mathrm{ext}}$: Antenna Current

- Wave Equation in Non-Orthogonal Coordinates (radial components)

$$\begin{aligned}(\nabla \times \nabla \times \boldsymbol{E})^1 \;=\; & \frac{1}{J}\left[\frac{\partial}{\partial x^2}\left\{\frac{g_{31}}{J}\left(\frac{\partial E_3}{\partial x^2} - \frac{\partial E_2}{\partial x^3}\right) + \frac{g_{32}}{J}\left(\frac{\partial E_1}{\partial x^3} - \frac{\partial E_3}{\partial x^1}\right) + \frac{g_{33}}{J}\left(\frac{\partial E_2}{\partial x^1} - \frac{\partial E_1}{\partial x^2}\right)\right\}\right. \\ & \left. - \frac{\partial}{\partial x^3}\left\{\frac{g_{21}}{J}\left(\frac{\partial E_3}{\partial x^2} - \frac{\partial E_2}{\partial x^3}\right) + \frac{g_{22}}{J}\left(\frac{\partial E_1}{\partial x^3} - \frac{\partial E_3}{\partial x^1}\right) + \frac{g_{23}}{J}\left(\frac{\partial E_2}{\partial x^1} - \frac{\partial E_1}{\partial x^2}\right)\right\}\right]\end{aligned}$$

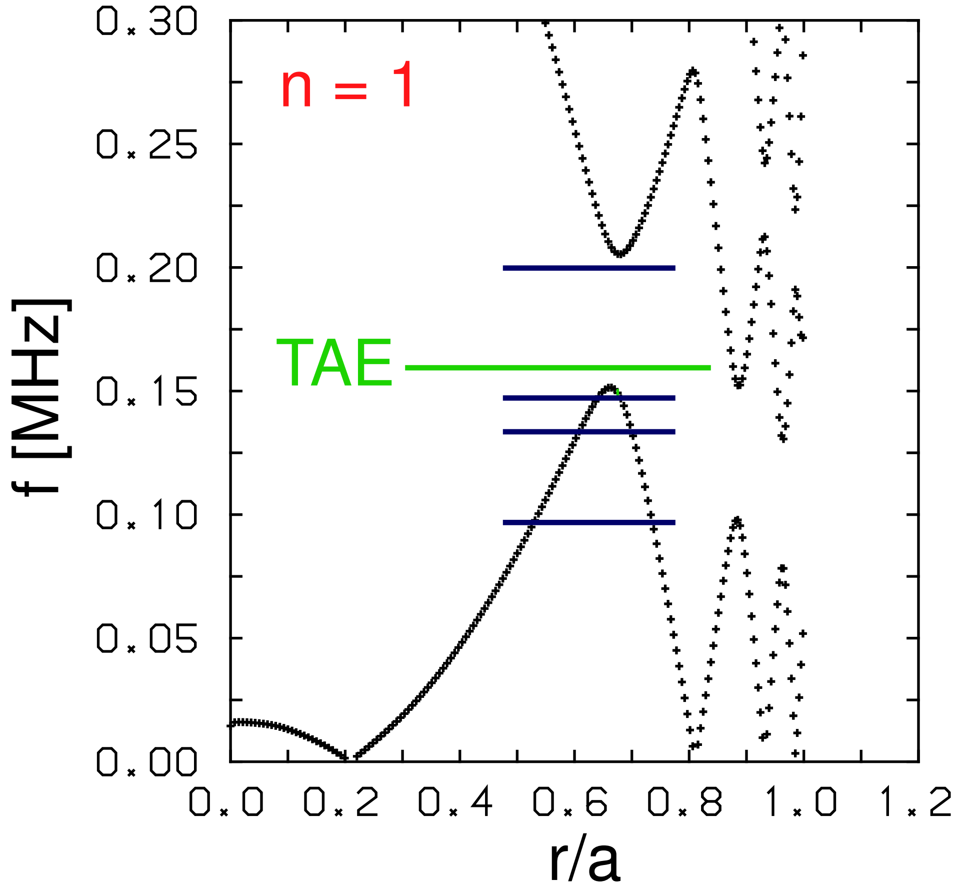
  - $(x^1, x^2, x^3) = (\psi, \theta, \varphi)$
  - Similar expression for poloidal and toroidal components

Example of AE in JT-60U

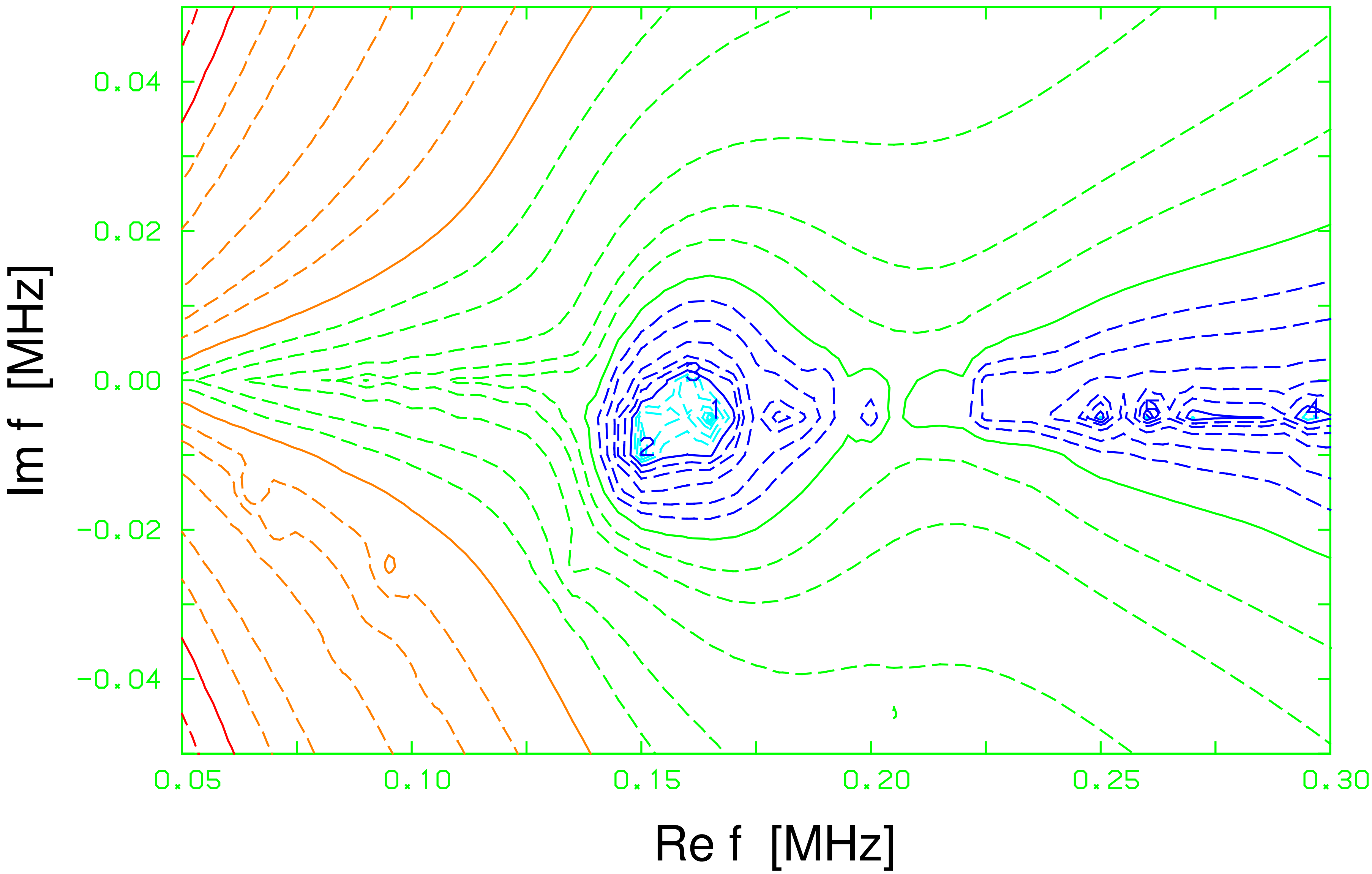
Parameters

R	3.5016 m
a	0.9837 m
κ	.2810
δ	0.3098
b/a	1.1
B_0	3.3119 T
I_p	1.6945 MA
$n_\mathrm{e}(0)$	$0.2356\ 10^{20}\mathrm{m}^{-3}$
$n_\mathrm{e}(a)$	$0.05\ 10^{20}\mathrm{m}^{-3}$
$T_\mathrm{e}(0)$	4.1 keV
$T_\mathrm{e}(a)$	0.8 keV
$T_\mathrm{D}(0)$	3.7 keV
$T_\mathrm{D}(a)$	0.4 keV

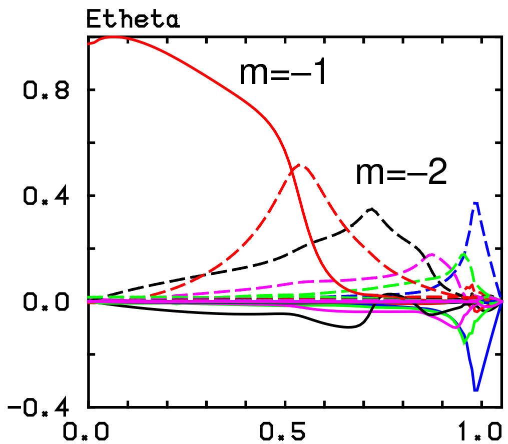
Radial profile of Alfvén resonance frequency



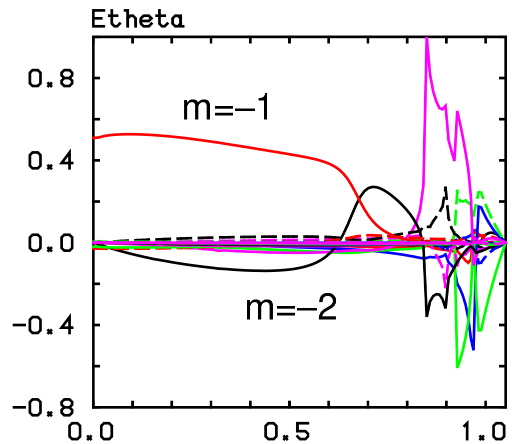
Complex Eigen Frequency of Alfvén Eigenmode



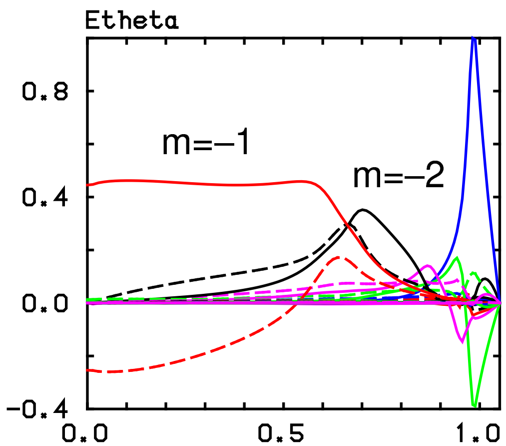
Radial Mode Structure of Alfvén Eigenmode ($n = 1$)



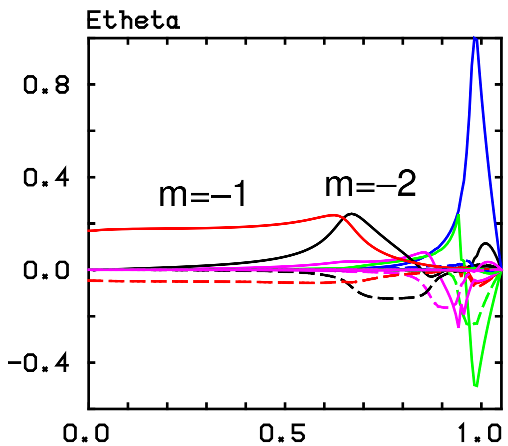
$f_{\mathrm{r}} = 97.0\,\mathrm{kHz}$
$f_{\mathrm{i}} = -23.6\,\mathrm{kHz}$



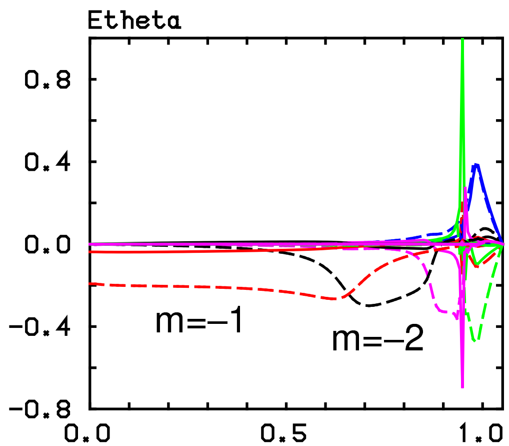
$f_{\mathrm{r}} = 199.4\,\mathrm{kHz}$
$f_{\mathrm{i}} = -3.37\,\mathrm{kHz}$



$f_{\mathrm{r}} = 136.4\,\mathrm{kHz}$
$f_{\mathrm{i}} = -24.1\,\mathrm{kHz}$



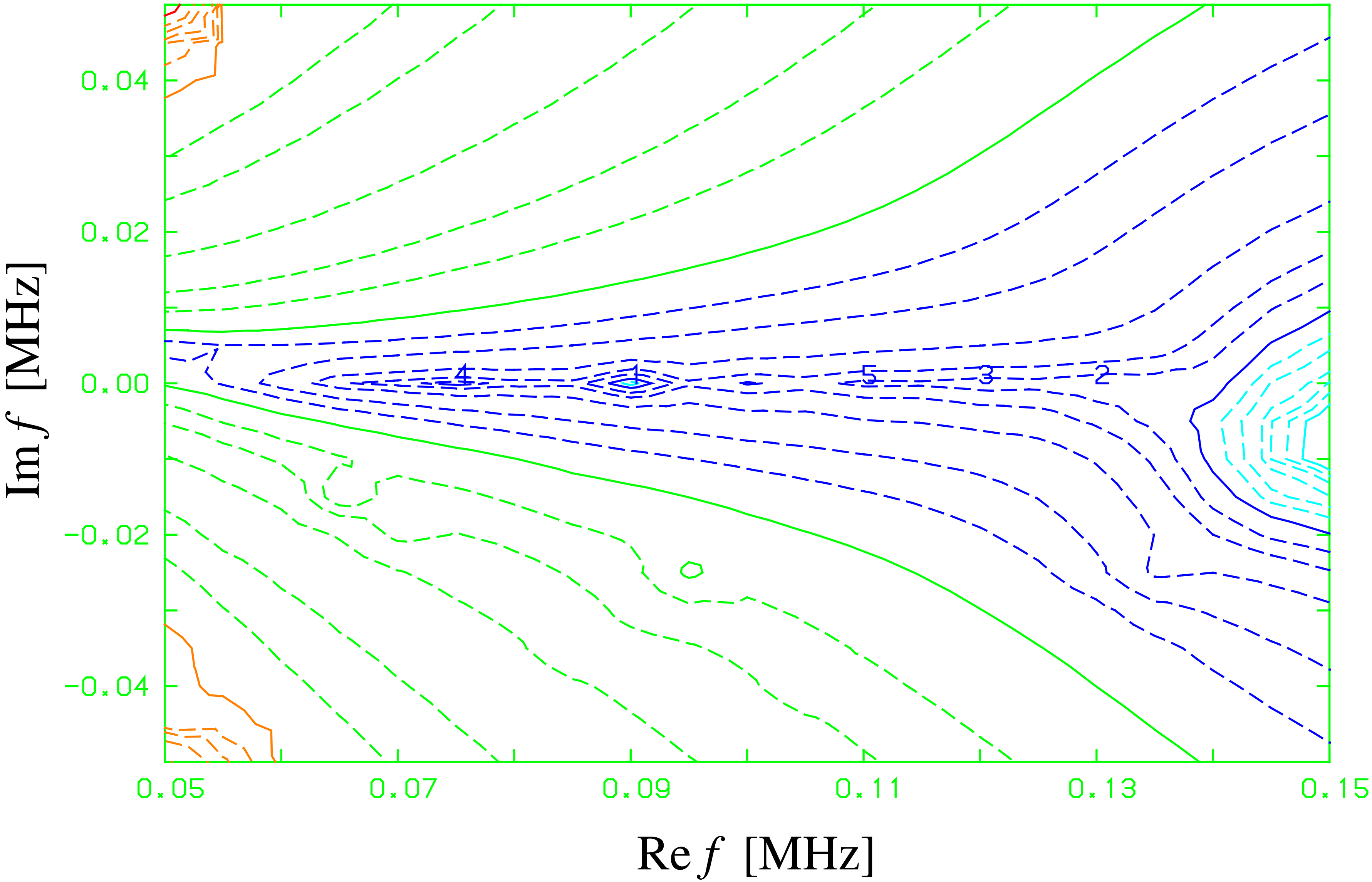
$f_{\mathrm{r}} = 149.8\,\mathrm{kHz}$
$f_{\mathrm{i}} = -8.12\,\mathrm{kHz}$



$f_{\mathrm{r}} = 164.3\,\mathrm{kHz}$
$f_{\mathrm{i}} = -5.08\,\mathrm{kHz}$

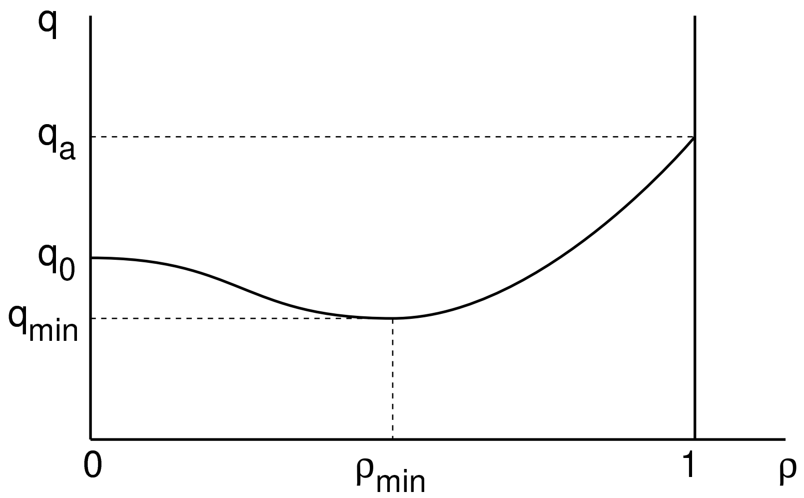
Mode Structure with Energetic Particle

- $n_{F0} = 10^{17}\,\mathrm{m}^{-3}$, $T_B = 500\,\mathrm{keV}$, $L_{nB} = 0.5\,\mathrm{m}$



Analysis of TAE in Reversed Shear Configuration

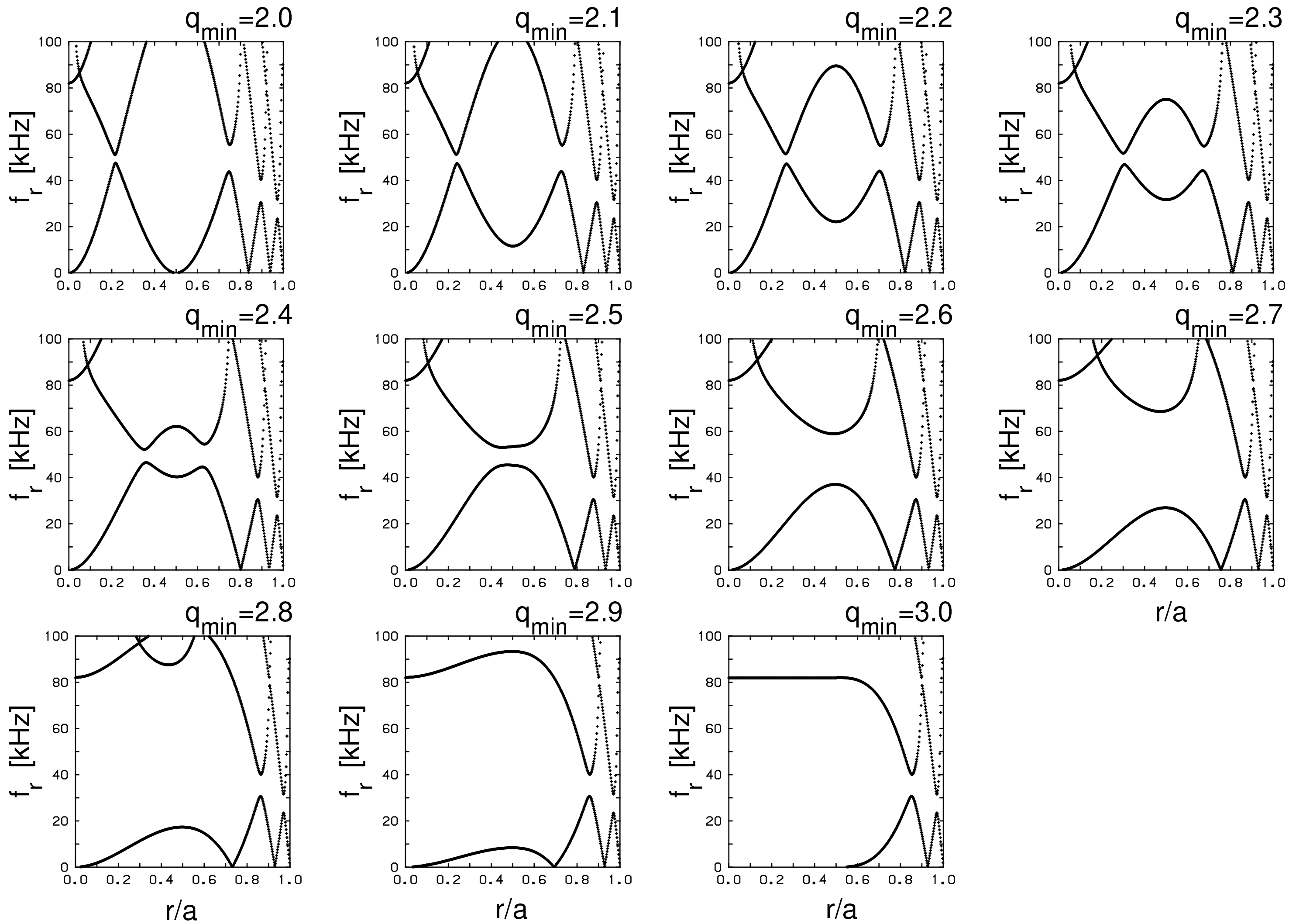
Assumed q profile



Plasma Parameters

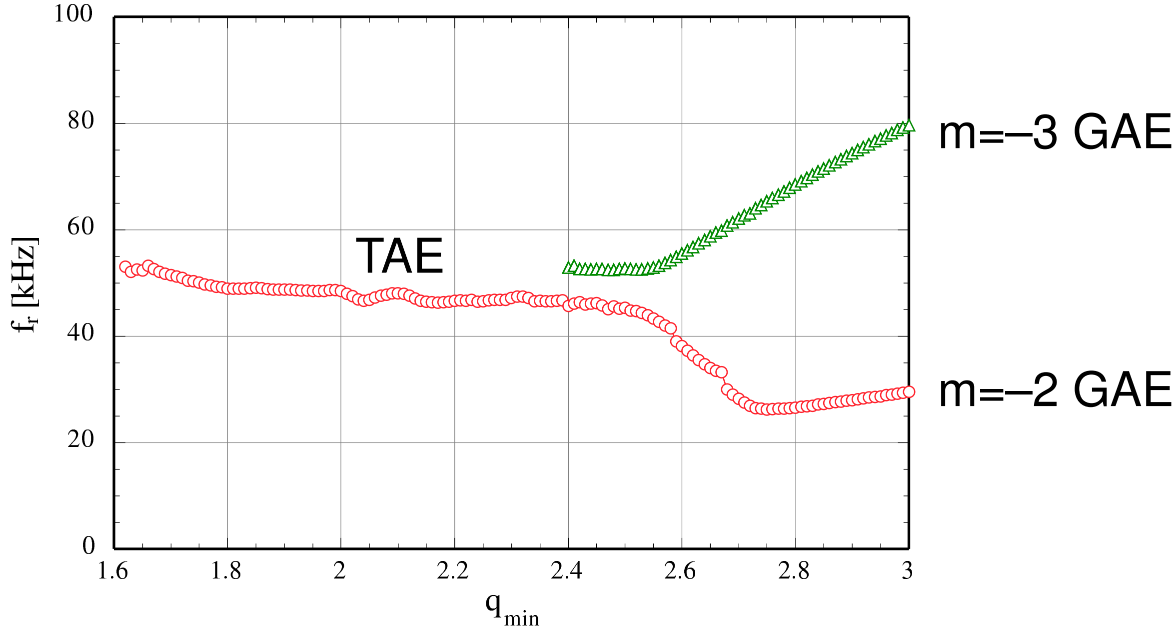
Major Radius	R_0	$3\,\mathrm{m}$
Minor Radius	a	$1\,\mathrm{m}$
Wall Radius	b	$1.2\,\mathrm{m}$
Toroidal Magnetic Field	B_0	$3\,\mathrm{T}$
Center Electron Density	$n_e(0)$	$10^{20}\,\mathrm{m}^{-3}$
Edge Electron Density	$n_e(a)$	$10^{20}\,\mathrm{m}^{-3}$
Central Temperature	$T(0)$	$3\,\mathrm{keV}$
Edge Temperature	$T(a)$	$3\,\mathrm{keV}$
Ion Species		Deuterium
Central Safety Factor	$q(0)$	3
Edge Safety Factor	$q(a)$	5
Toroidal Mode Number	n	1
q-Minimum Radius	ρ_{min}	0.5

$q_{\min}$ Dependence of Alfvén Frequency Profile

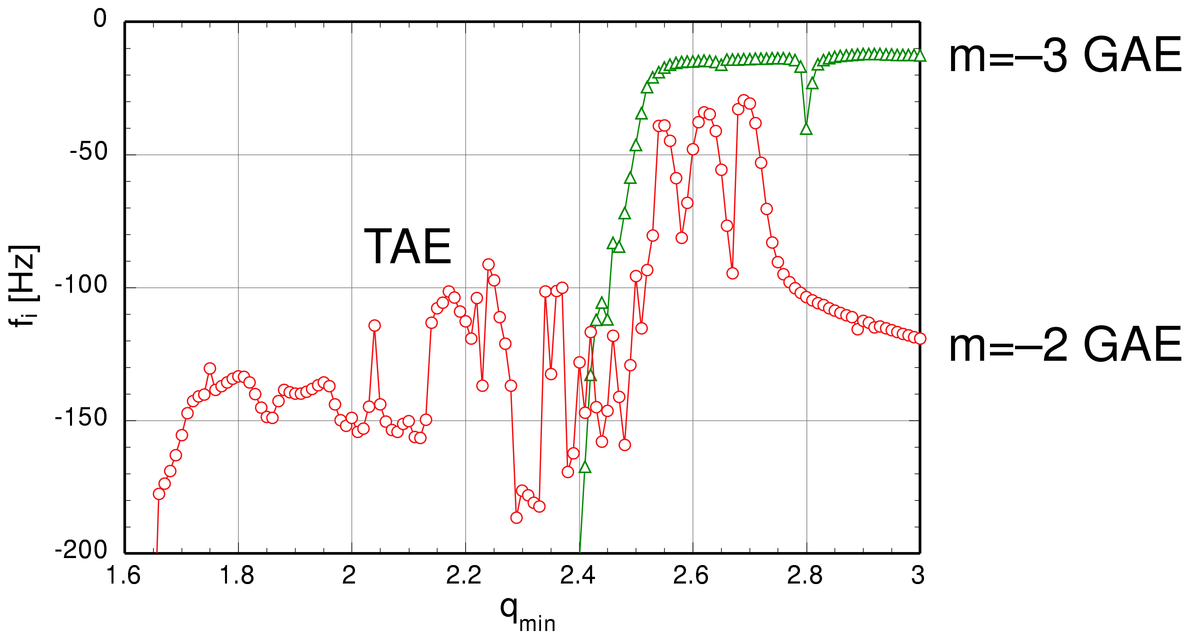


$q_{\min}$ Dependence of Eigen Frequency and Damping Rate

Eigen Frequency



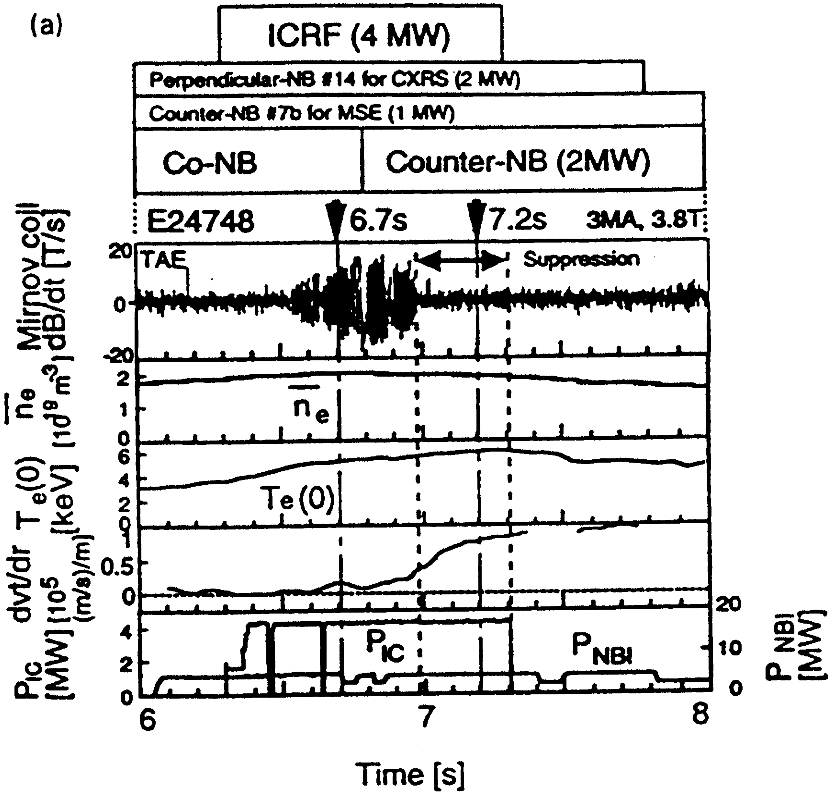
−Damping Rate



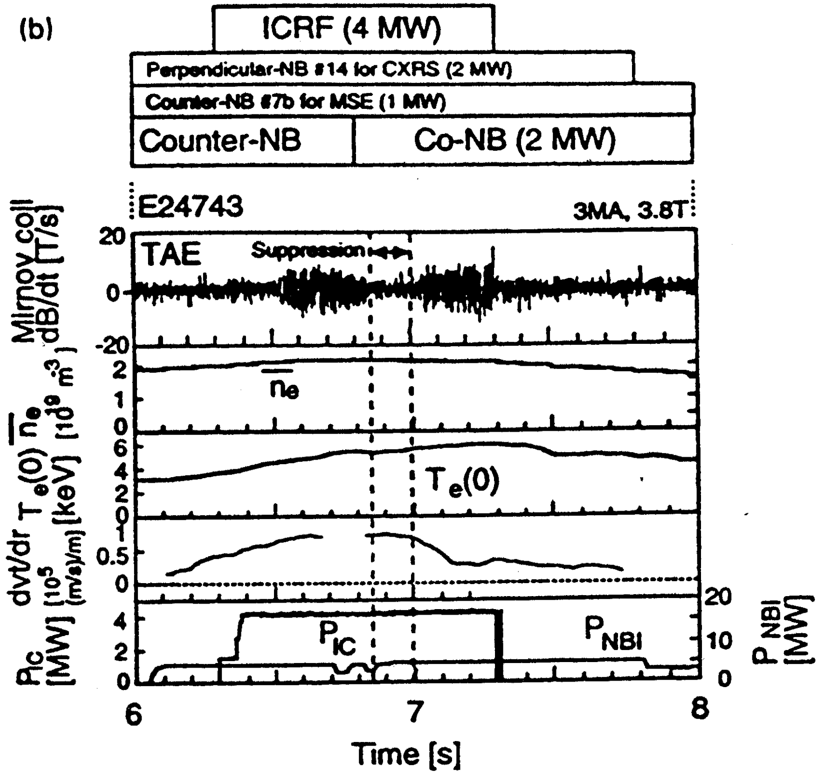
Experimental Results on JT-60U

M. Saigusa et al., Nucl. Fusion 37 (1997) 1559.

Co-NBI $\longrightarrow$ Counter-NBI



Counter-NBI $\longrightarrow$ Co-NBI



Dispersion Relation including Toroidal Rotation

$$\left[k_{||m}^2 - \frac{(\omega - k_{||m}u)^2}{{v_\mathrm{A}}^2}\right]\left[k_{||m+1}^2 - \frac{(\omega - k_{||m+1}u)^2}{{v_\mathrm{A}}^2}\right] - \epsilon^2\frac{(\omega - k_{||m}u)^2(\omega - k_{||m+1}u)^2}{{v_\mathrm{A}}^4} = 0$$

- Parallel wave number $k_{||m} = \dfrac{1}{R}\left(n + \dfrac{m}{q}\right)$
- Alfvén resonance condition without toroidal effect

$$\omega^2 = {k_{||m}}^2(u \pm v_\mathrm{A})^2, \qquad \omega^2 = {k_{||m+1}}^2(u \pm v_\mathrm{A})^2$$

- Condition for frequency gap

$$k_{||m}\left(u - v_\mathrm{A}\right) = k_{||m+1}(u + v_\mathrm{A}\Big)$$

- Safety factor : q

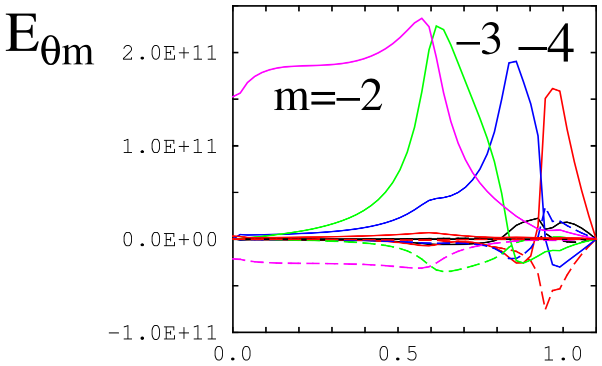
$$q = -\frac{m + 1/2}{n} - \frac{1}{2n}\frac{u}{v_\mathrm{A}}$$

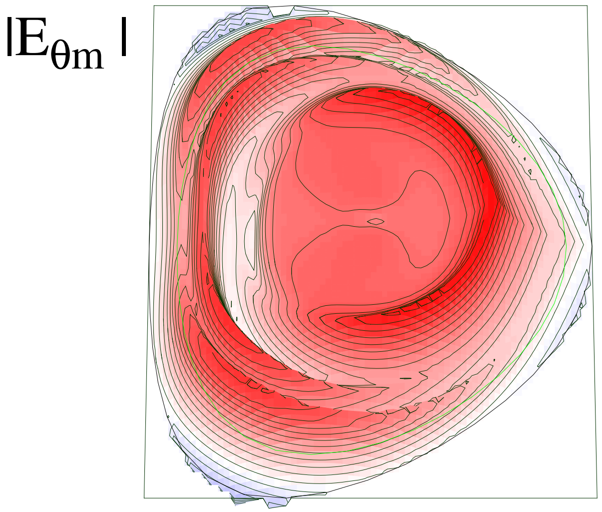
- Eigen frequency ; ω

$$\omega = \frac{v_\mathrm{A}}{2qR}(1 - \frac{u^2}{{v_\mathrm{A}}^2})$$

Effect of Rotation on $n = 1$ mode

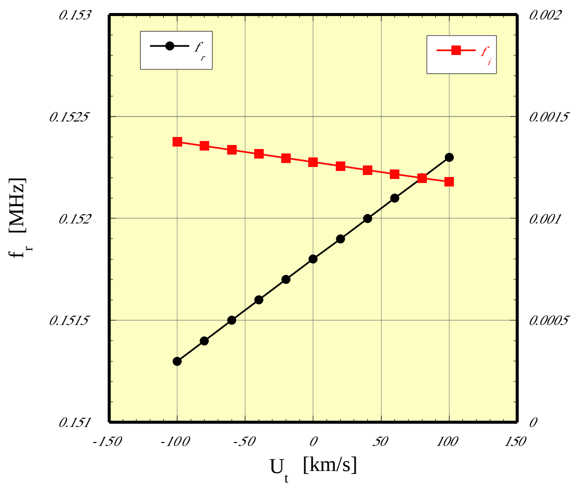
Example of $n = 1$ Eigenmode for JT-60U parameters



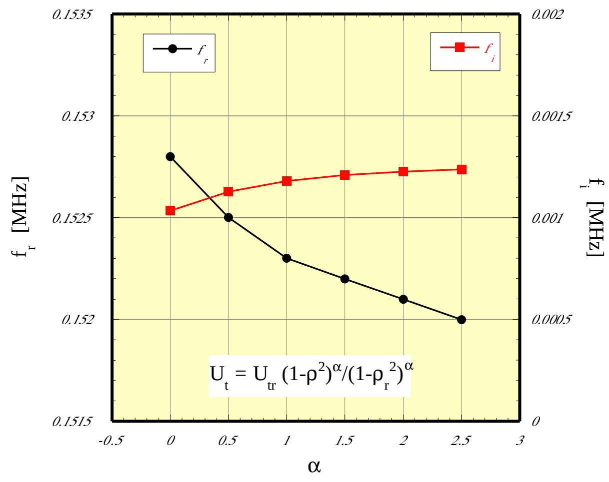


Dependence of eigen frequency and damping rate on

Rotation velocity

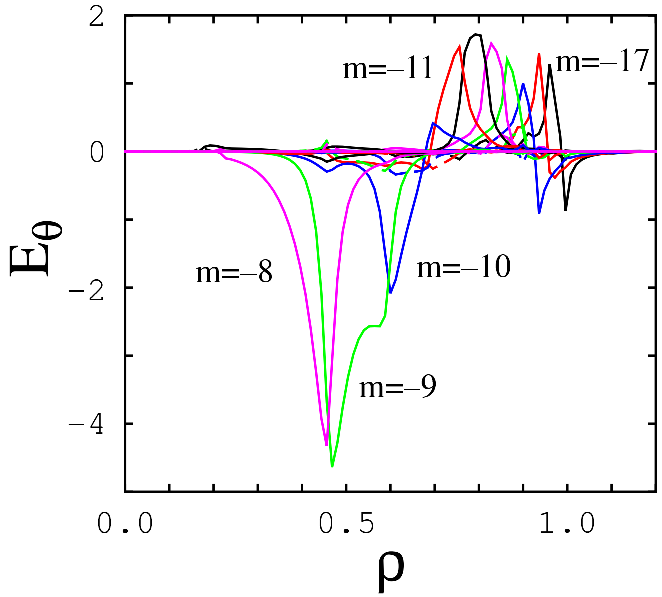


Velocity gradient

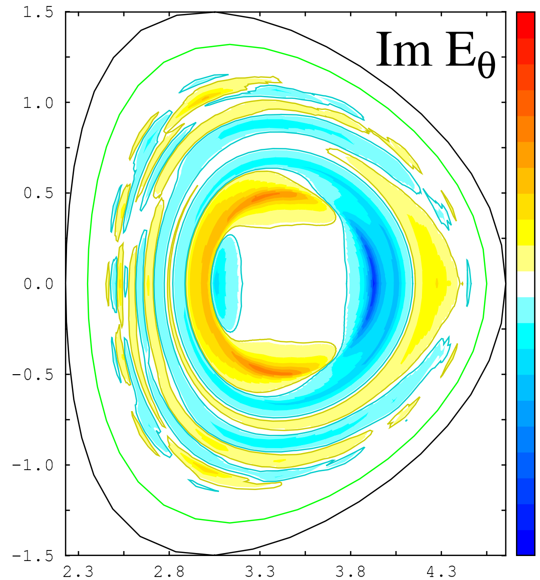


Effect of Rotation on $n = 7$ mode

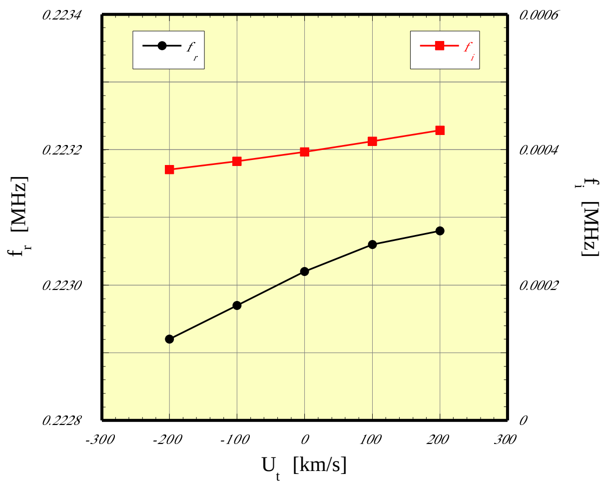
- $n = 7$, $m = -17 \sim -3$, $f = 223\,\mathrm{kHz}$



Shape of eigen function agree with Nova-K



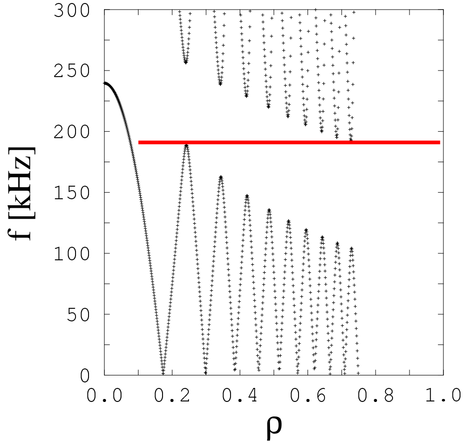
- Rotation velocity dependence: Stabilizing for co rotation (Contradict with exp.)



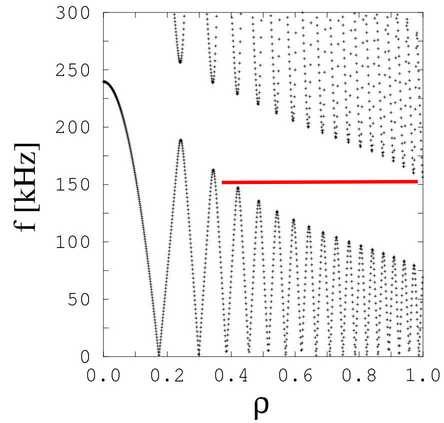
Influence of poloidal mode range : $n = 7$ mode

- Radial structure of Alfvén Continuum

$m = -17 \sim -3$

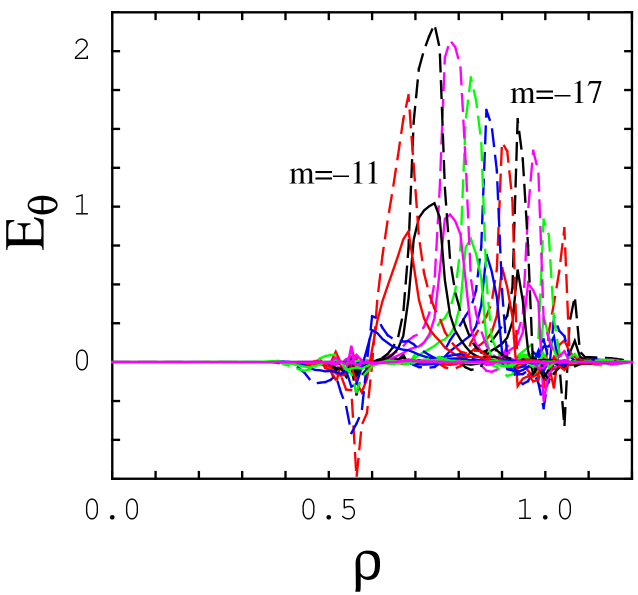


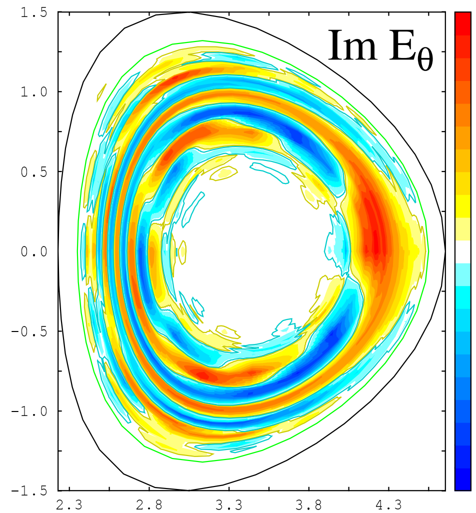
$m = -21 \sim -7$

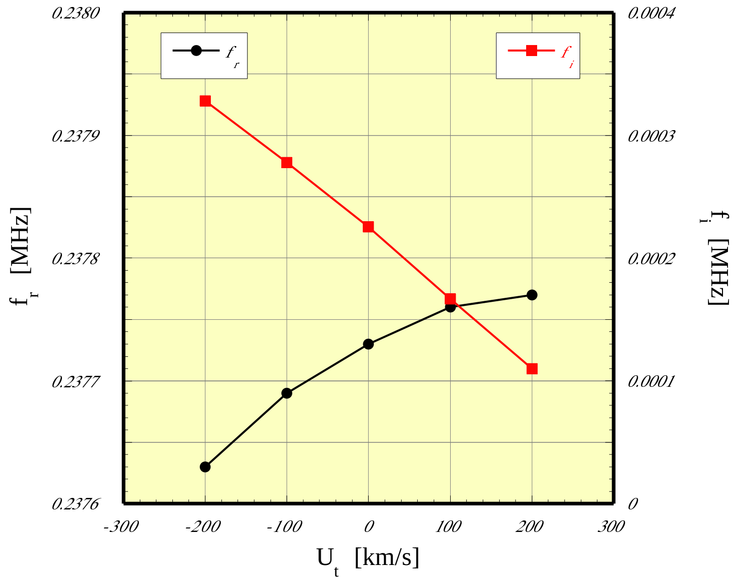


- $n = 7,\ m = -21 \sim -7,\ f = 238\,\mathrm{kHz}$

  Destabilizing for co-rotation (agree with exp.)







Summary

- **We studied the linear stability of Alfvén eigenmode including the effect of kinetic Alfvén waves using the 3D full wave code, TASK/WM.**
- **Negative magnetic configuration supports GAE with single dominant poloidal mode number.**
- **The toroidal rotation changes the TAE eigen frequency mainly through the change of gap position and q value.**
- **Destabilization by co-rotation agrees with experimental observation in JT-60U, though the stability is sensitive to the Alfvén resonance near the plasma surface.**
- **Future work**
  - **Analysis of destabilization by energetic ions**
  - **Analysis of low-frequency modes with trapped particle effects**