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Bifurcation in Transport Barrier Formation

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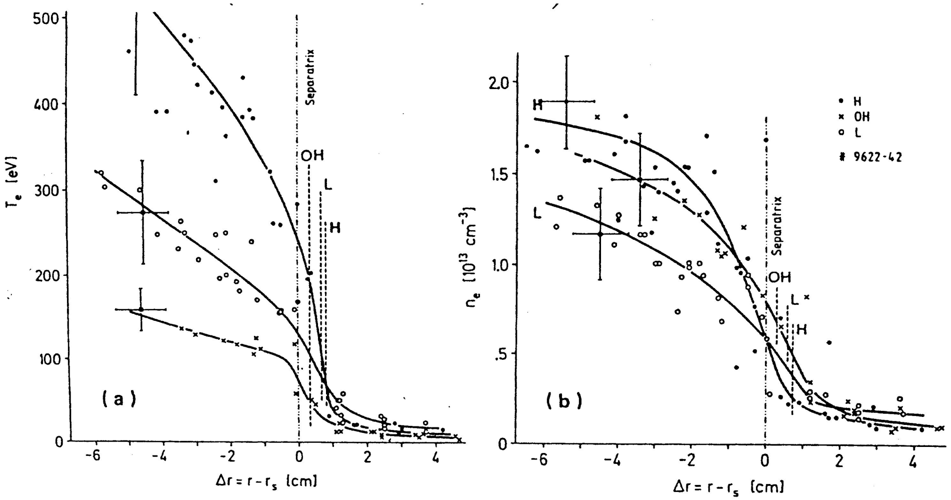
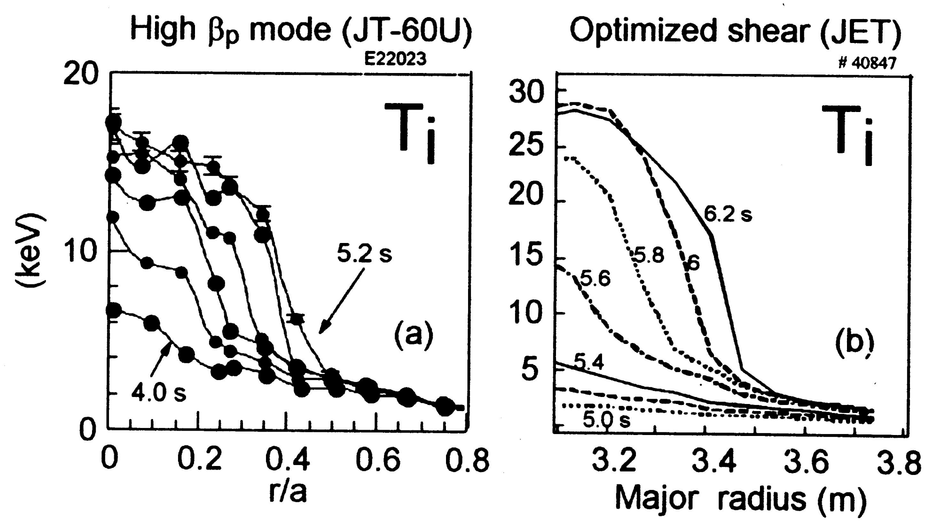
- **Transport Barriers in Tokamaks**
- **Transport Models and Simulation Results**
- **Possibility of Bifurcation**
- **Summary**

Improved Confinement and Transport Barrier

- **Edge Transport Barrier**: H mode (ASDEX, 1982)

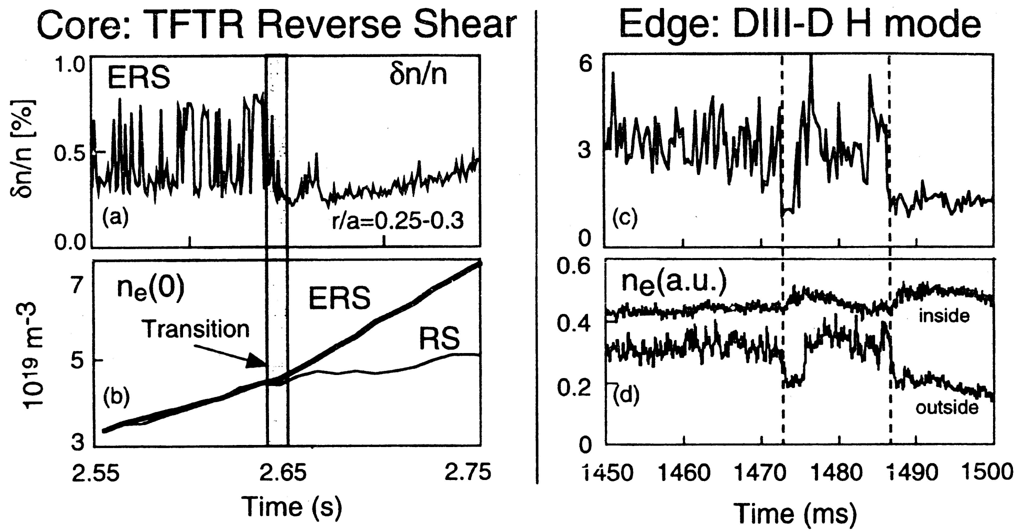


- **Internal Transport Barrier**: High β_p mode, Negative magnetic shear

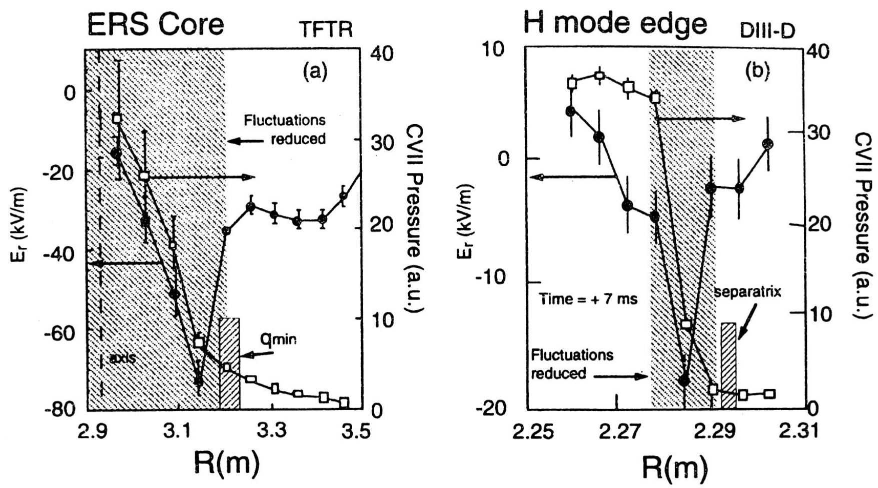


Suppression of Turbulence and Generation of E_r

- **Fluctuation Amplitude**

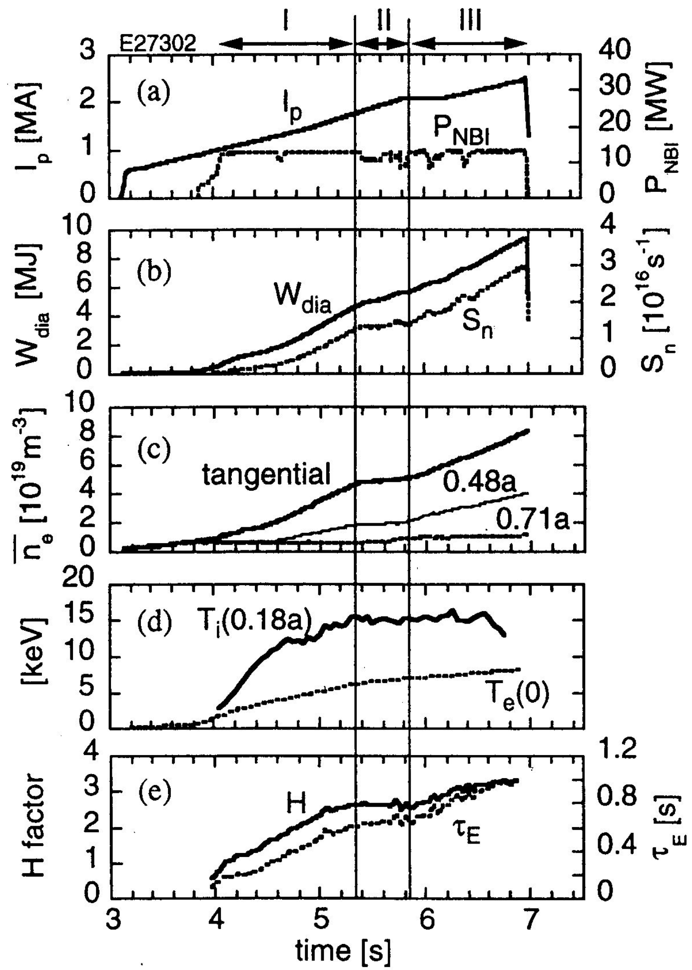


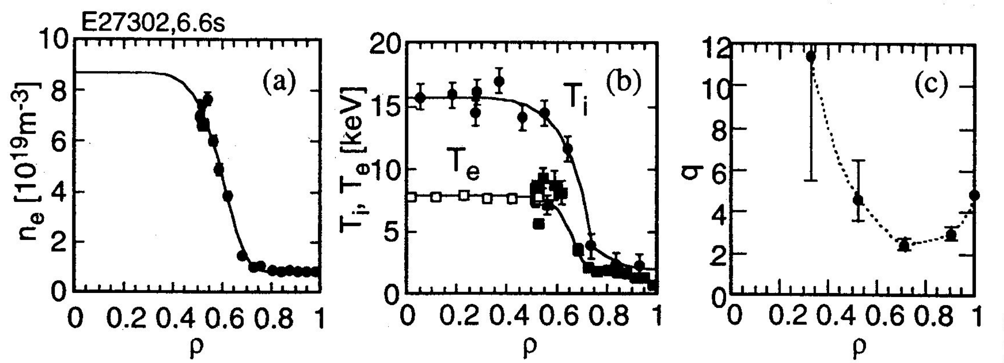
- **Radial Electric Field**



Negative Magnetic Shear Configuration

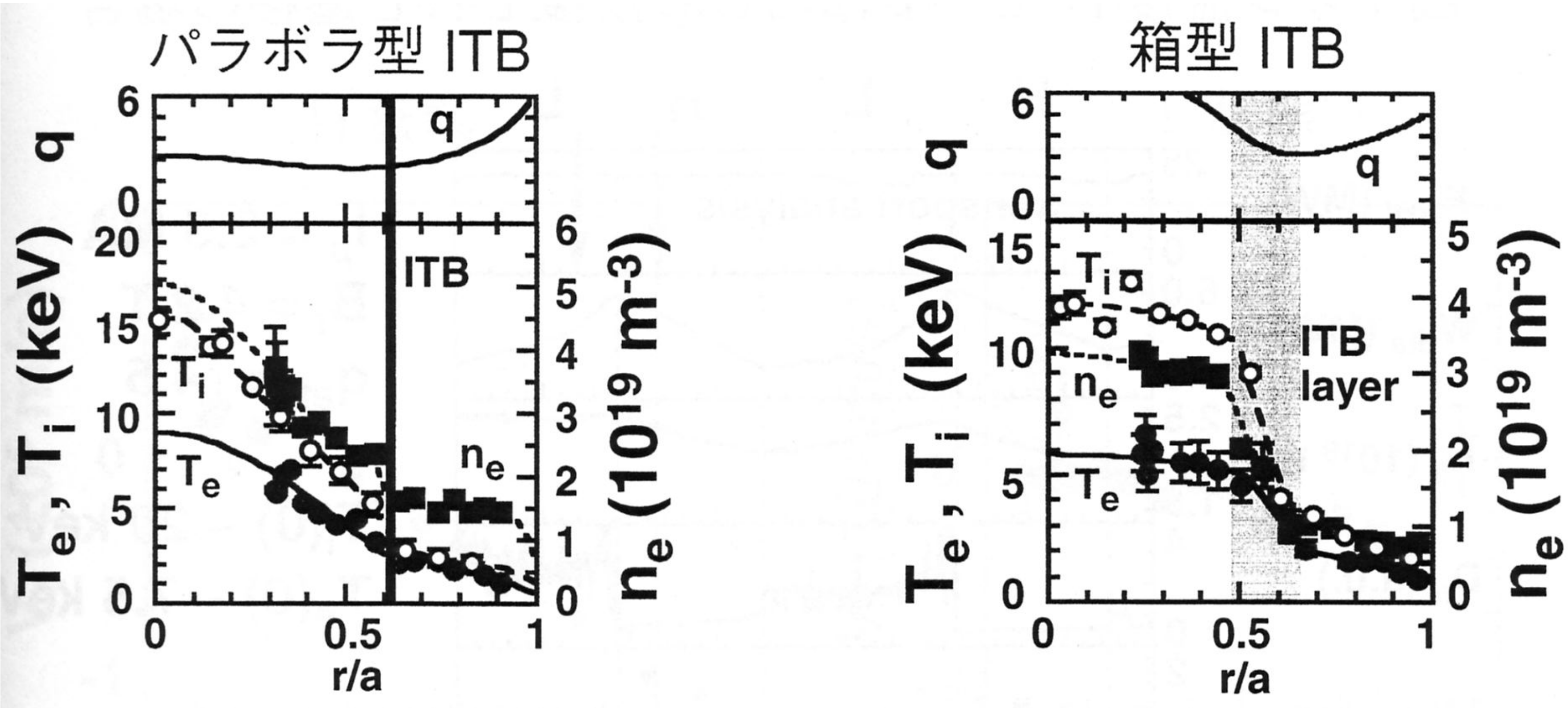
- **ITB Formation by Current Ramp Up** (JT-60U)





Shape of Internal Transport Barrier

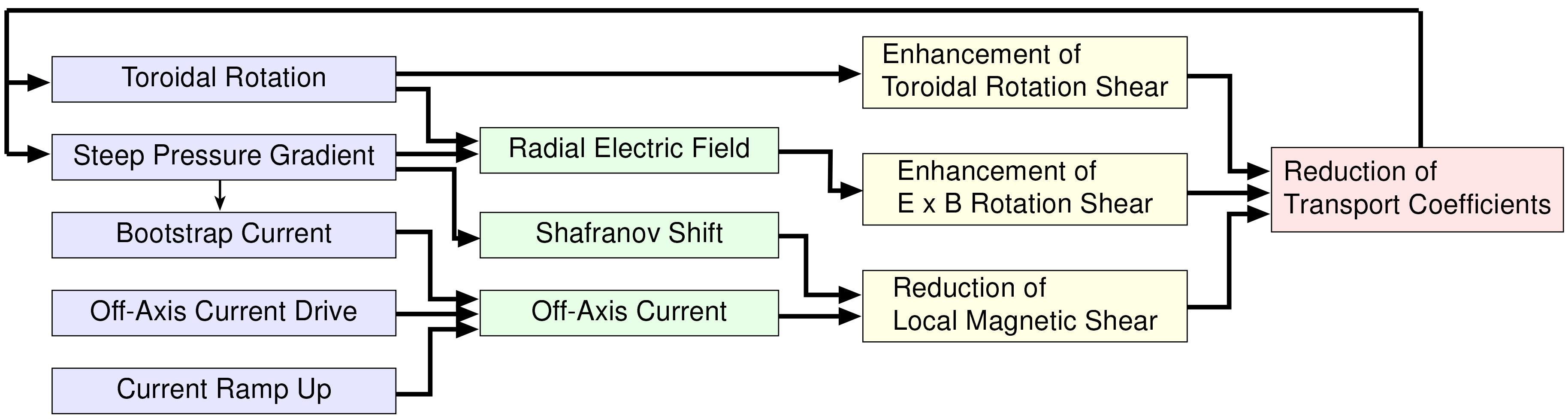
- **ITB**: High β_p mode and Negative magnetic shear (JT-60U)



Physical Mechanism of ITB

- Reduction of Transport Coefficients Due to
  - Weak or negative magnetic shear
  - Large shift of magnetic axis
  - Large rotation velocity shear

- Positive feedback loop to enhance pressure gradient



Modeling of Transport Barrier Formation

- **Mechanism of Turbulence Suppression**
- $E \times B$ **rotation shear**
  - E_r **generation through radial force balance**

$$E_r = -u_\theta B_\phi + u_\phi B_\theta + \frac{1}{e_s}\frac{\mathrm{d}}{\mathrm{d}r}P$$

  - $E \times B$ **shearing rate (Hahm and Burrel)**

$$\omega_E = \frac{RB_\theta}{B}\frac{\mathrm{d}}{\mathrm{d}r}\left(\frac{E_r}{RB_\theta}\right)\frac{k_\theta}{k_r}$$

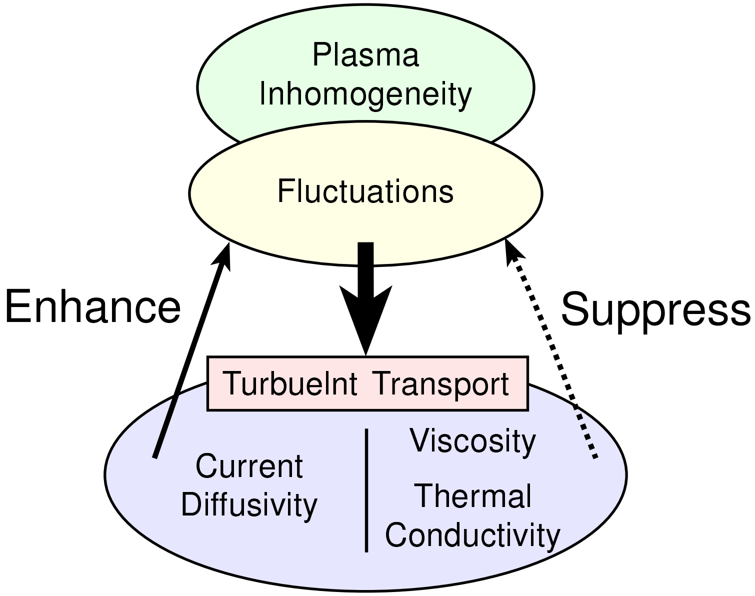
  - **Criteria for suppression**

$$\omega_E > \gamma_{\mathrm{Lin}}$$

- **Magnetic shear** s **and normalized pressure gradient** α
  - **Marginal stability of self-sustained turbulence (CDBM model)**
  - **Thermal diffusivity** χ **as a function of** s **and** α
- **Positive feedback of pressure gradient increase**
  - **In both models,** χ **depends on the pressure gradient.**
  - **In some case, not always, increase of pressure gradient reduces** χ**.**

CDBM Transport Model

- **Self-Sustained Turbulence Theory**
  - **Turbulence sustained by the enhancement of transport coefficients due to the turbulence itself**

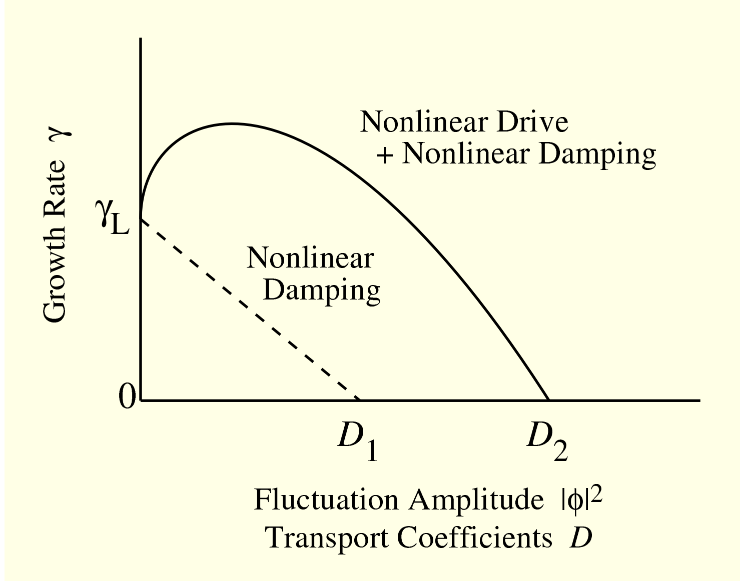


- **Current-Diffusive Ballooning Mode**
  - **Ballooning mode: MHD mode localized in bad curvature region**

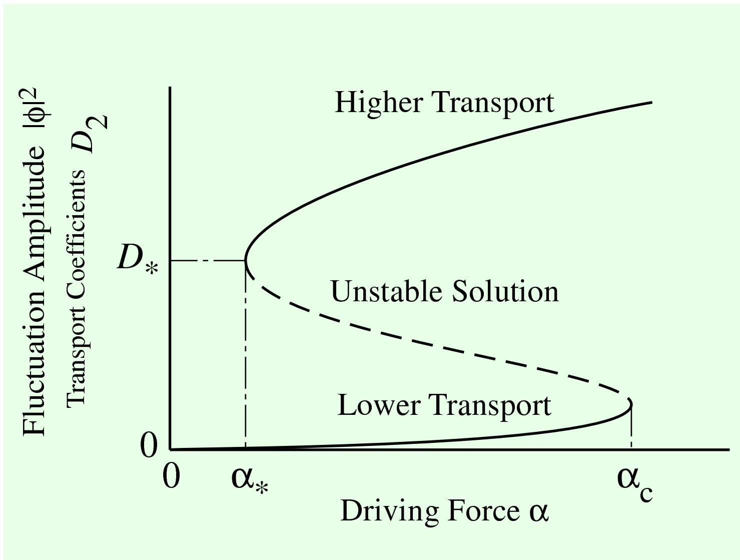
  — **Ideal ballooning mode (second stability)**
  — **Resistive ballooning mode (plasma near edge)**
  — **Current-diffusive ballooning mode (core plasma)**

Self-Sustained Turbulence

- **Conventional Turbulence**
  - Growth of linearly unstable mode
  - Saturation due to NL damping
  - Transport coefficients proportional to the linear growth rate



- **Self-Sustained Turbulence**
  (K. Itoh et al., 1992)
  - Weakly unstable in linear stage
  - NL dissipation destabilizes the mode.
  - Saturation due to the balance between NL drive and NL damping
  - Supported by turbulence simulation (M. Yagi et al., 1995)



Basic Equations of CDBM

- **Reduced MHD Equation** **(One fluid model including transport)**

  Equation of motion $$\frac{\partial}{\partial t}\frac{n_0 m_\mathrm{i}}{B_0}\nabla_\perp^2\phi = B_0\nabla_\parallel j_\parallel + \nabla p \times \nabla\frac{2r\cos\theta}{R_0}\cdot\hat{z} + \mu\frac{n_0 m_\mathrm{i}}{B_0}\nabla_\perp^4\phi$$

  Ohm's law $$\frac{\partial}{\partial t}A = -\nabla_\parallel\phi - \eta j_\parallel + \lambda\nabla_\perp^2 j_\parallel \quad \textbf{where} \quad j_\parallel = -\frac{1}{\mu_0}\nabla_\perp^2 A$$

  Energy equation $$\frac{\partial}{\partial t}p + \frac{1}{B_0}\nabla\phi\times\nabla p_0\cdot\hat{z} = \chi\nabla^2 p$$

- **Transport Coefficients**

  μ : **Ion viscosity**
  η : **Resistivity**
  λ : **Current diffusivity**
  χ : **Thermal diffusivity**

  μ, λ, χ: **Classical (collisional) + Turbulent (amplitude-dependent)**
  η: **Classical (collisional)**

- **Calculate growth rate for given μ, λ, χ (given amplitude)**

Ballooning Mode

- **Ballooning Mode Equation** **(normalized, m: poloidal mode number)**

$$\gamma f^2 \phi = \frac{\partial}{\partial \xi} \frac{f^2}{\gamma + \eta m^2 f^2 + \lambda m^4 f^4} \frac{\partial \phi}{\partial \xi}$$

$$- \mu m^2 f^4 \phi + \frac{\alpha}{\gamma + \chi m^2 f^2} \left[\kappa + \cos \xi + (s\xi - \alpha \sin \xi) \sin \xi \right] \phi$$

- **Magnetic shear** $s \equiv \dfrac{r}{q} \dfrac{dq}{dr}$
- **Normalized pressure gradient** $\alpha \equiv -q^2 R \dfrac{d\beta}{dr}$
- **Magnetic curvature** $\kappa \equiv -\dfrac{r}{R} \left(1 - \dfrac{1}{q^2} \right)$

$$q = rB_0 / RB_\theta, \quad \beta = 2\mu_0 p_0 / B_0^2, \quad f^2 = 1 + (s\xi - \alpha \sin \xi)^2$$

Current diffusivity	λm^4	**Destab.**	**Reduce field-line bending effect**
Ion viscosity	μm^2	**Stab.**	**Enhance viscosity damping**
Thermal diffusivity	χm^2	**Stab.**	**Reduce pressure gradient effect**

CDBM Turbulence

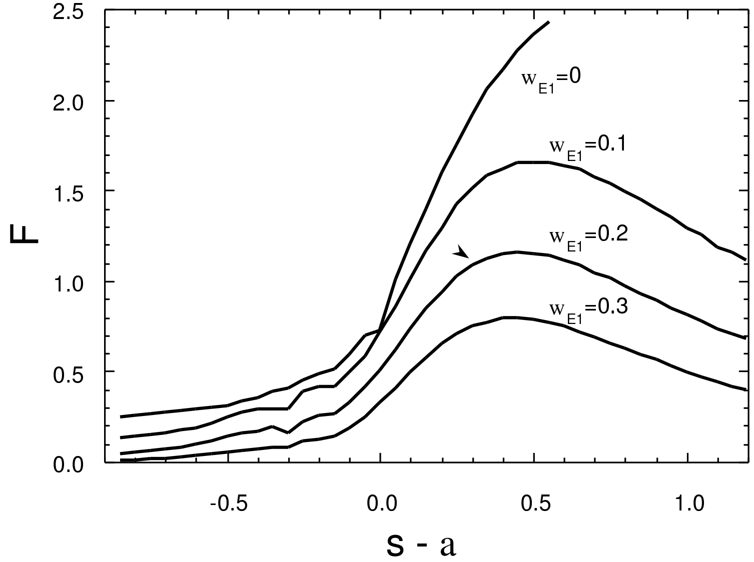
- **Marginal Stability Condition** $(\gamma = 0)$

$$\chi_{\rm TB} = F(s,\alpha,\kappa,\omega_{\rm E1})\,\alpha^{3/2}\,\frac{c^2}{\omega_{\rm pe}^2}\,\frac{v_{\rm A}}{qR}$$

Magnetic shear	$s \equiv \dfrac{r}{q}\dfrac{{\rm d}q}{{\rm d}r}$
Pressure gradient	$\alpha \equiv -q^2R\dfrac{{\rm d}\beta}{{\rm d}r}$
Magnetic curvature	$\kappa \equiv -\dfrac{r}{R}\left(1-\dfrac{1}{q^2}\right)$
$E\times B$ **rotation shear**	$\omega_{\rm E1} \equiv \dfrac{r^2}{sv_{\rm A}}\dfrac{\rm d}{{\rm d}r}\dfrac{E}{rB}$

- **Weak and negative magnetic shear,**
Shafranov shift and
$E\times B$ **rotation shear**
reduce thermal diffusivity.

$s-\alpha$ **dependence of** $F(s,\alpha,\kappa,\omega_{\rm E1})$



Fitting Formula

$$F = \begin{cases} \dfrac{1}{1+G_1\omega_{\rm E1}^2}\dfrac{1}{\sqrt{2(1-2s')(1-2s'+3s'^2)}} & \textbf{for}\quad s'=s-\alpha<0 \\[2ex] \dfrac{1}{1+G_1\omega_{\rm E1}^2}\dfrac{1+9\sqrt{2}s'^{5/2}}{\sqrt{2}(1-2s'+3s'^2+2s'^3)} & \textbf{for}\quad s'=s-\alpha>0 \end{cases}$$

Heat Transport Simulation

- **Simple One-Dimensional Analysis**
  - **No impurity, No neutral, No sawtooth**
  - **Fixed density profile:** $n_\mathrm{e}(r) \propto (1 - r^2/a^2)^{1/2}$
  - **Thermal diffusivity (adjustable parameter** $C = 12$**)**

$$\chi_\mathrm{e} = C\chi_\mathrm{TB} + \chi_\mathrm{NC,e}$$
$$\chi_\mathrm{i} = C\chi_\mathrm{TB} + \chi_\mathrm{NC,i}$$

- **Transport Equation**

$$\frac{\partial}{\partial t}\frac{3}{2}n_\mathrm{e}T_\mathrm{e} = -\frac{1}{r}\frac{\partial}{\partial r} r\, n_\mathrm{e}\chi_\mathrm{e}\frac{\partial T_\mathrm{e}}{\partial r} + P_\mathrm{OH} + P_\mathrm{ie} + P_\mathrm{He}$$

$$\frac{\partial}{\partial t}\frac{3}{2}n_\mathrm{i}T_\mathrm{i} = -\frac{1}{r}\frac{\partial}{\partial r} r\, n_\mathrm{i}\chi_\mathrm{i}\frac{\partial T_\mathrm{i}}{\partial r} - P_\mathrm{ie} + P_\mathrm{Hi}$$

$$\frac{\partial}{\partial t}B_\theta = \frac{\partial}{\partial r}\eta_\mathrm{NC}\left[\frac{1}{\mu_0}\frac{1}{r}\frac{\partial}{\partial r} r B_\theta - J_\mathrm{BS} - J_\mathrm{LH}\right]$$

- **Standard Plasma Parameter**

$R = 3\,\mathrm{m}$ $B_\mathrm{t} = 3\,\mathrm{T}$ **Elongation** $= 1.5$

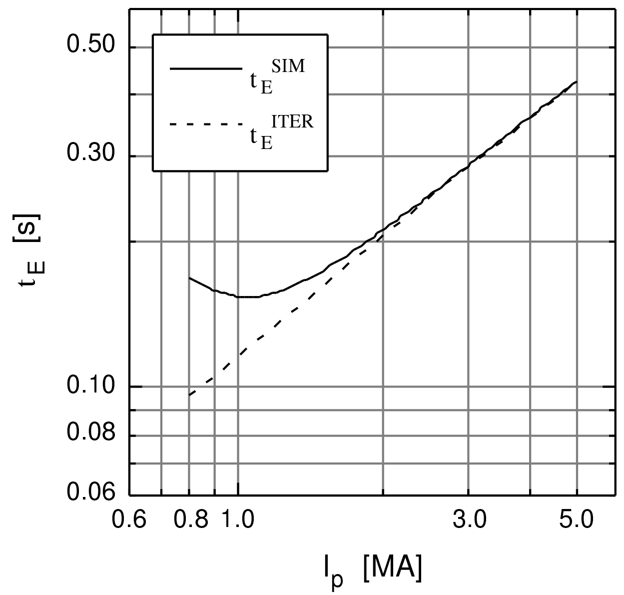
$a = 1.2\,\mathrm{m}$ $I_\mathrm{p} = 3\,\mathrm{MA}$ $n_\mathrm{e0} = 5 \times 10^{19}\,\mathrm{m}^{-3}$

Simulation of L-mode and Improved Confinement

- **Zero-Dimensional Analysis** with fixed $F(s, \alpha, \kappa)$ (gyro-Bohm scaling)

$$\tau_{\mathrm{E}} \propto \; F^{-0.4} A_{\mathrm{i}}^{0.2} I_{\mathrm{P}}^{0.8} n^{0.6} B^{0} a^{1.0} R_{0}^{1.2} P^{-0.6}$$

- **Deviation from L-mode scaling at low I_{p}**
  - Increase of P_{in}
  - Increase of pressure gradient
    $\longrightarrow$ Increase of α
  - Increase of bootstrap current
    $\longrightarrow$ Decrease of s
  - Reduction of χ

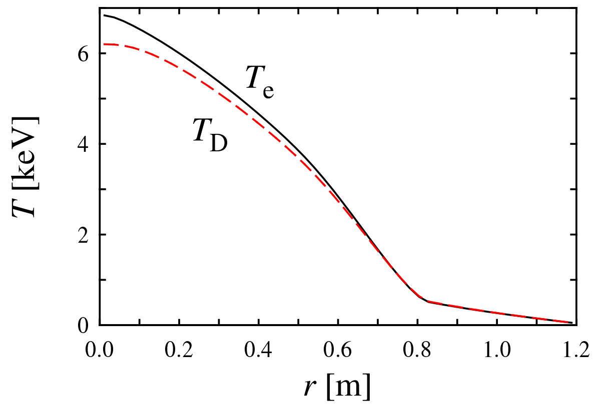


- **Various improved core confinement modes have been reproduced**
  - **High-β_{p}** mode
  - **PEP** (Pellet Enhanced Performance) mode
  - **LHEP** (Lower Hybrid Enhanced Performance) mode
  - **Negative Magnetic Shear** mode

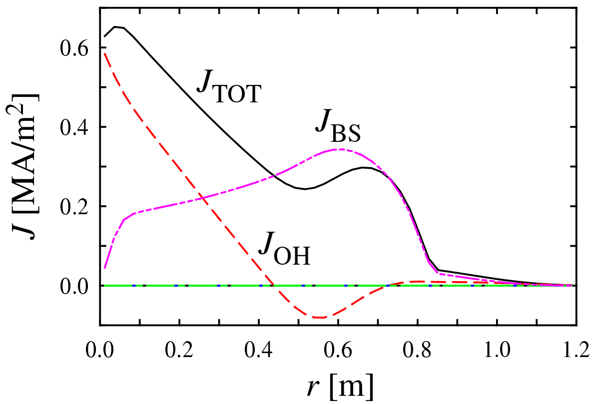
High β_p mode (1)

- $R = 3\,\mathrm{m},\ a = 1.2\,\mathrm{m},\ \kappa = 1.5,\ B_0 = 3\,\mathrm{T},\ I_p = 1\,\mathrm{MA}$
- **one second after heating power of $P_H = 20\,\mathrm{MW}$ was switched on**

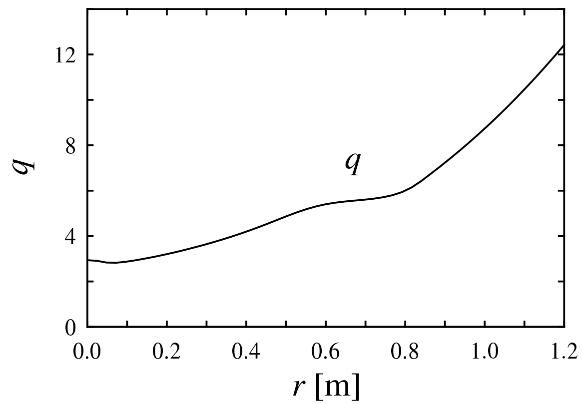
Temperater profile



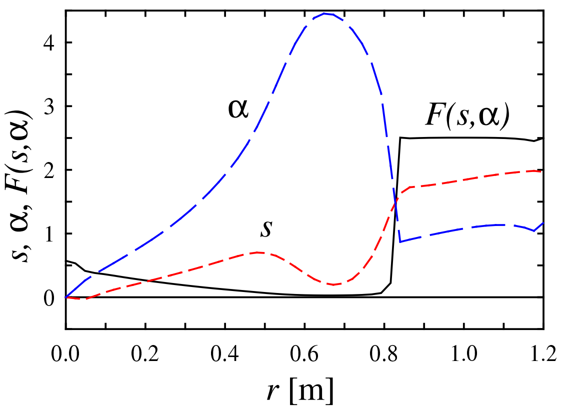
Current profile



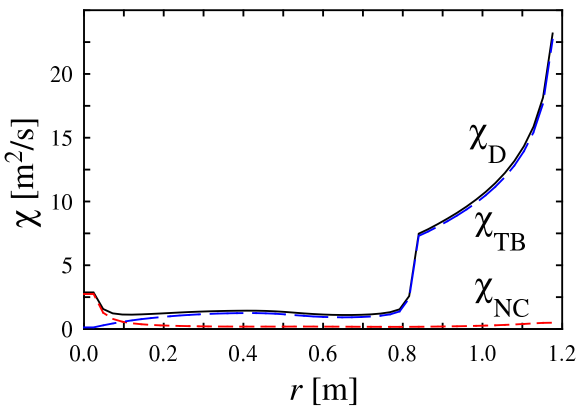
Safety factor



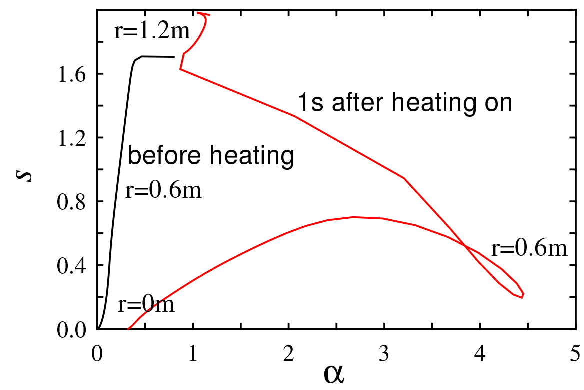
Shear and pressure



Thermal diffusivity



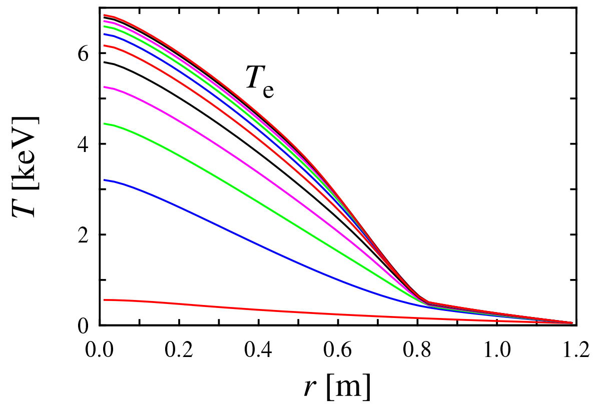
$s - \alpha$ diagram



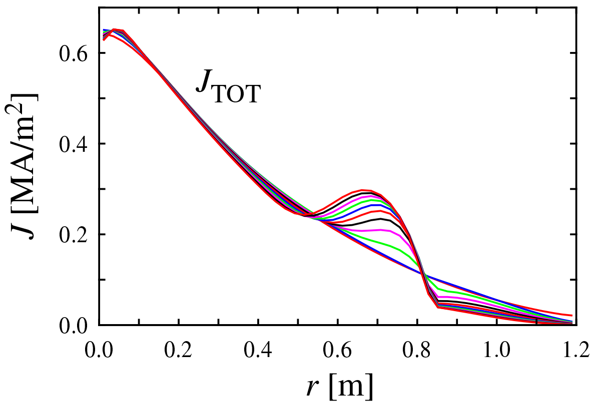
High β_p mode (2)

- **Time evolution during the first one second after heating switched on**

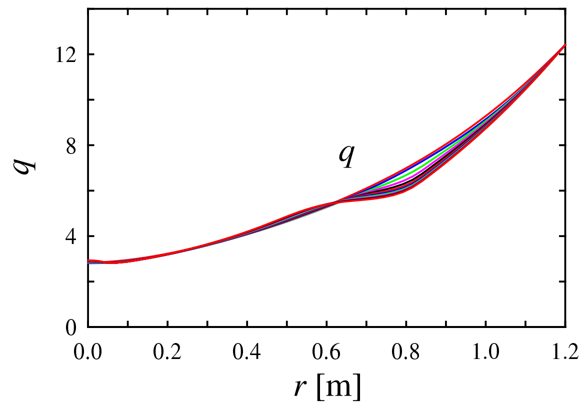
Temperater



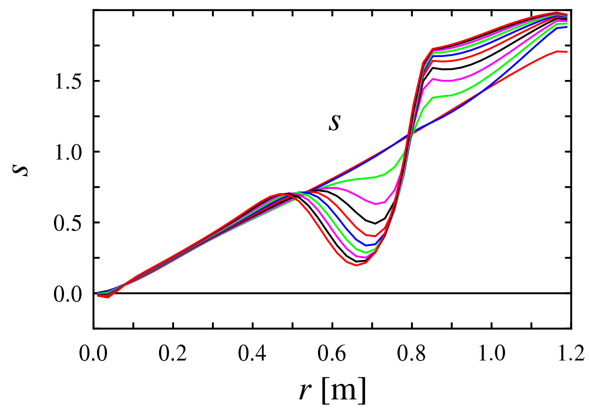
Current



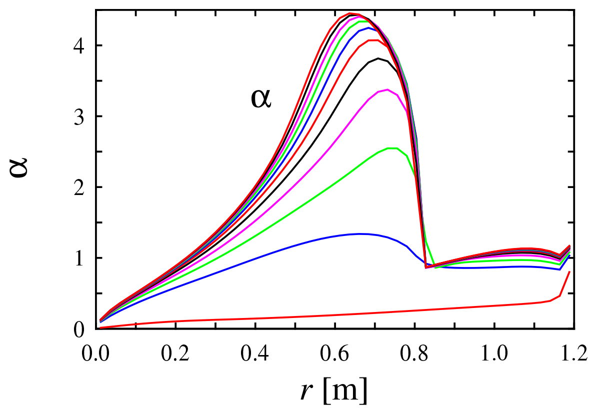
Safety factor



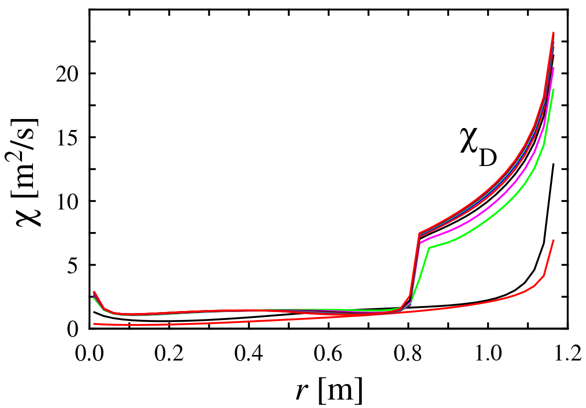
Shear



Normalized Pressure

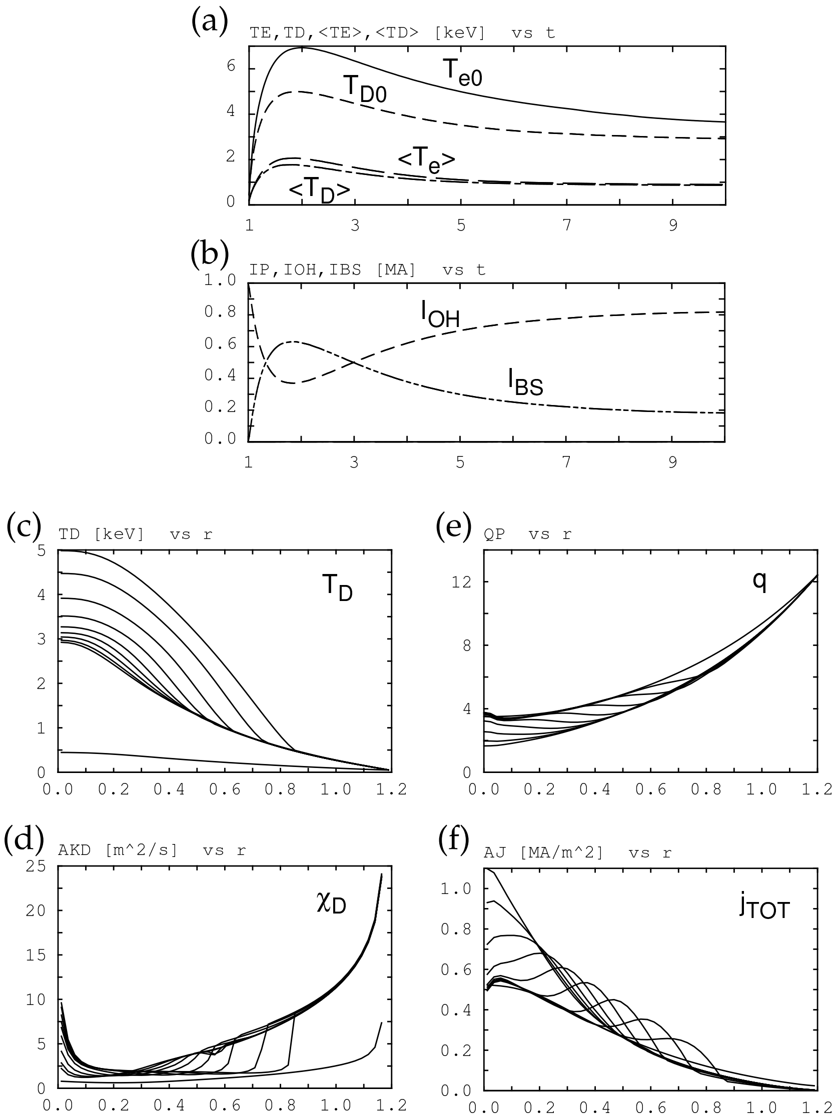


Thermal diffusivity

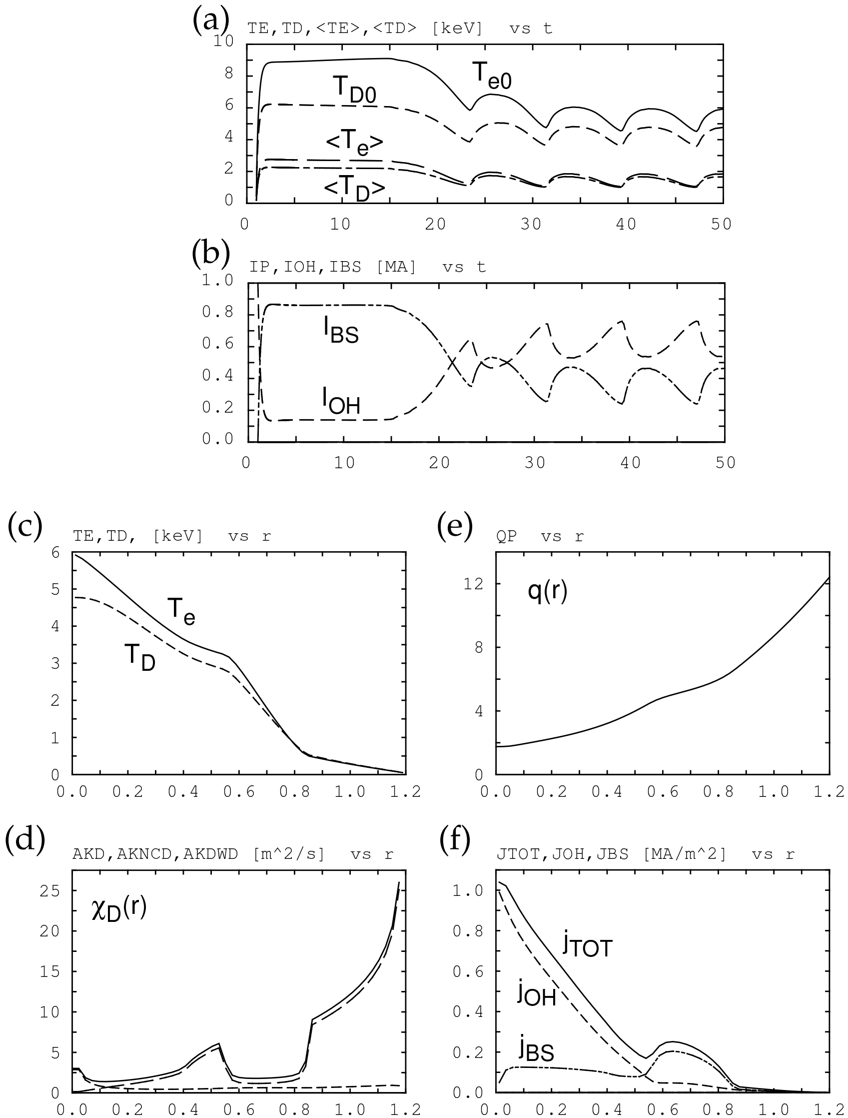


High β_p mode (3)

- $P_H = 20\,\mathrm{MW}$

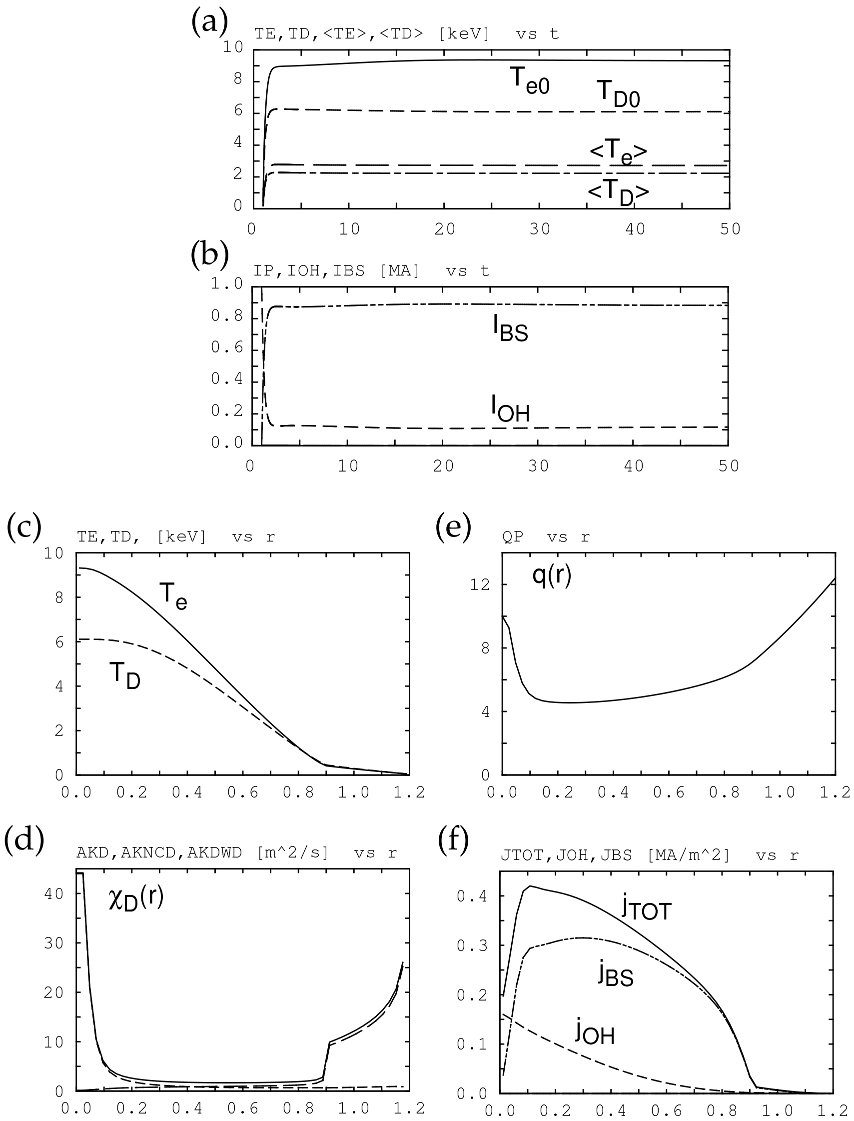


- $P_H = 24.2\,\mathrm{MW}$

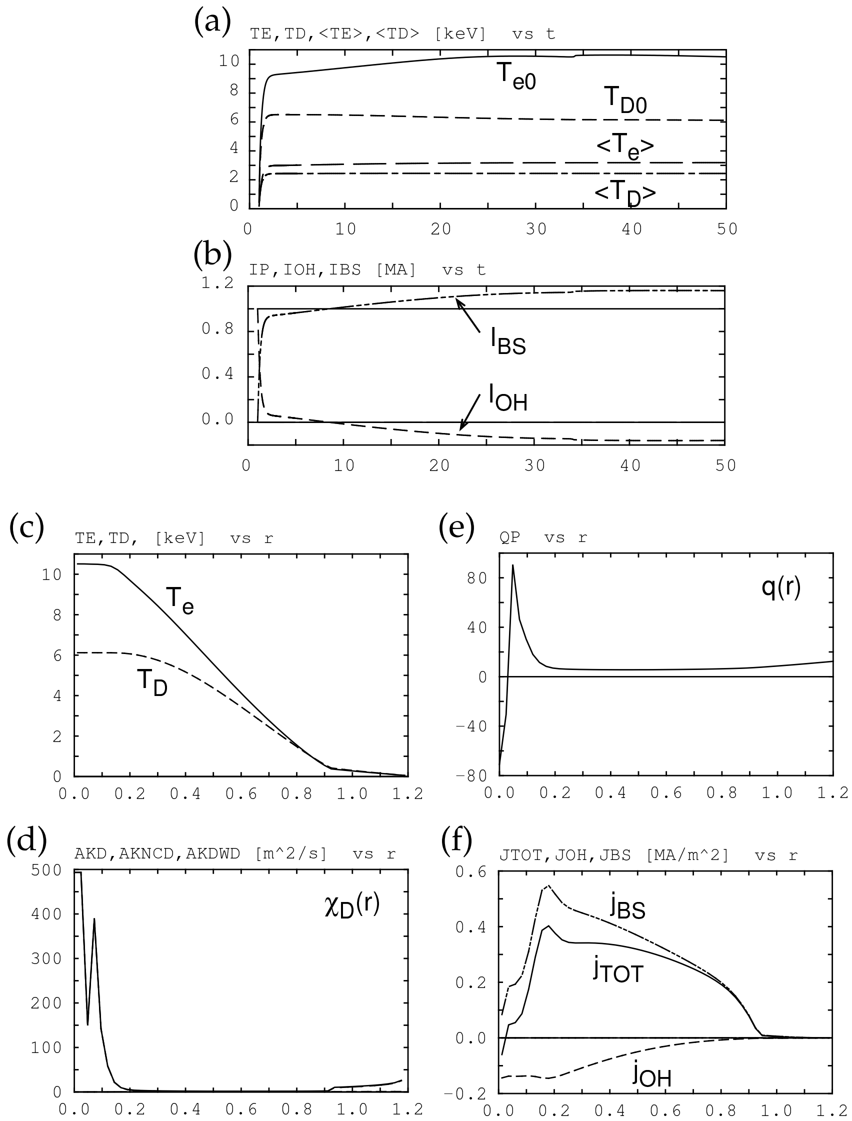


High β_p mode (4)

- $P_H = 24.4\,\mathrm{MW}$



- $P_H = 24.6\,\mathrm{MW}$



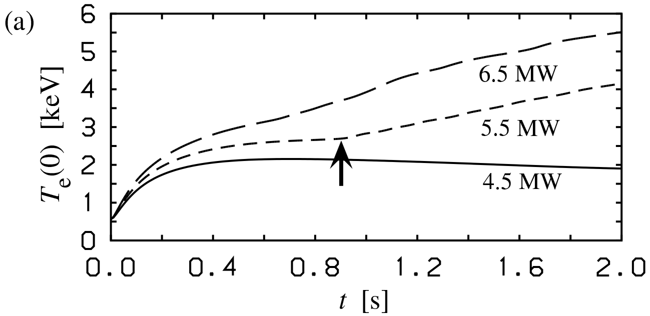
Effect of $\boldsymbol{E} \times \boldsymbol{B}$ Rotation Shear

- **Reduction of transport** due to small s and large α: $F(s, \alpha, \kappa)$

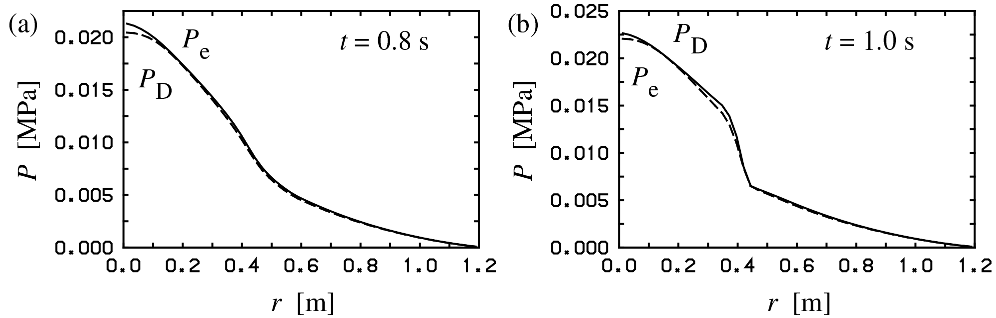
  $\Longrightarrow$ **Rapid increase of rotation shear**: $1/[1 + G(s, \alpha)\omega_{E1}^2]$

  $\Longrightarrow$ **Transition to enhanced ITB**

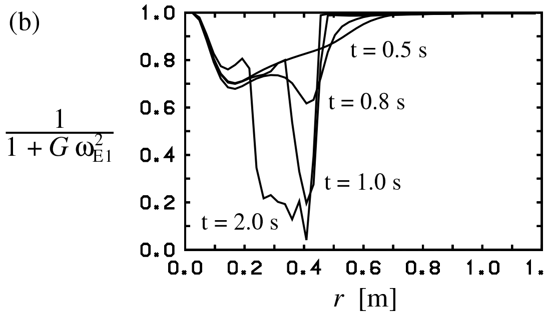
Time evolution of $T_e(0)$



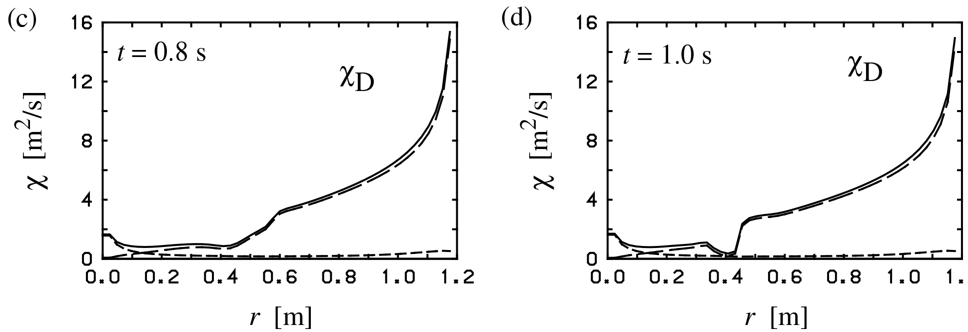
Pressure Profile Before and After Transition



Rapid Change of Rotation Shear

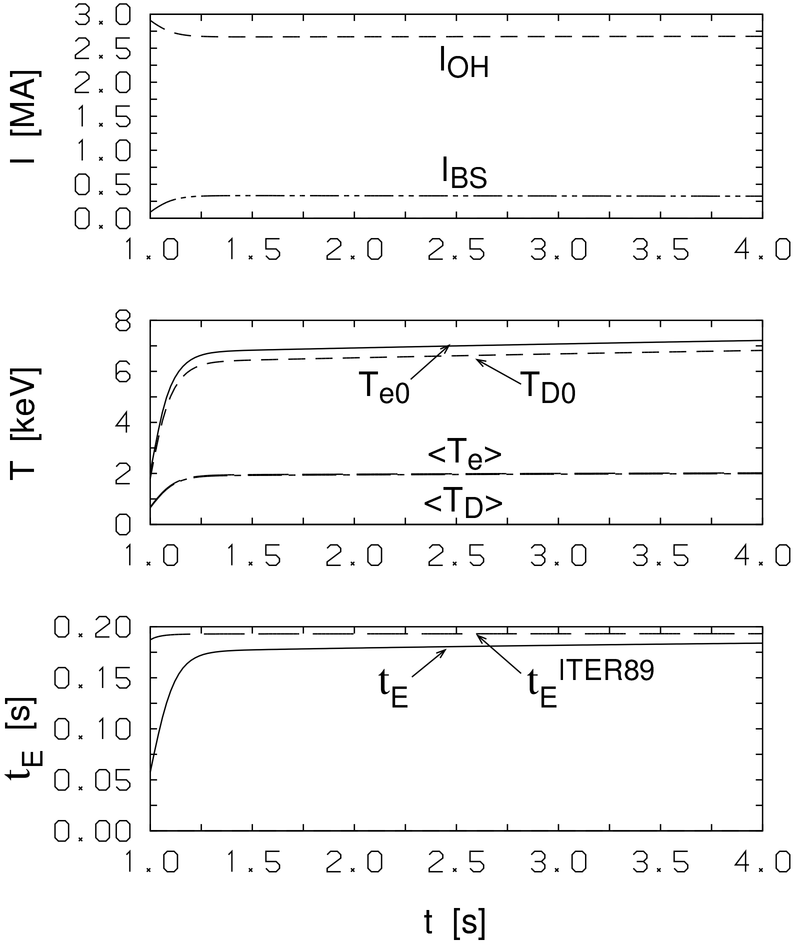


Thermal Diffusivity Before and After Transition

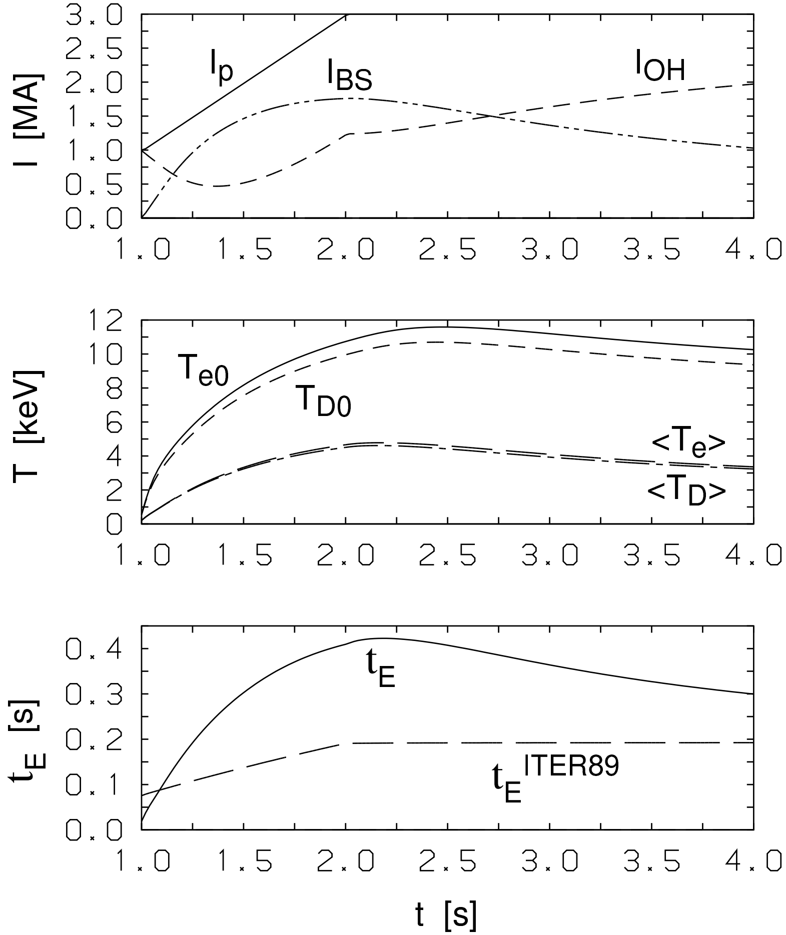


Simulation of Negative Shear Configuration

I_p : 3 MA constant
Heating : 20 MW
H factor ≃ 0.95

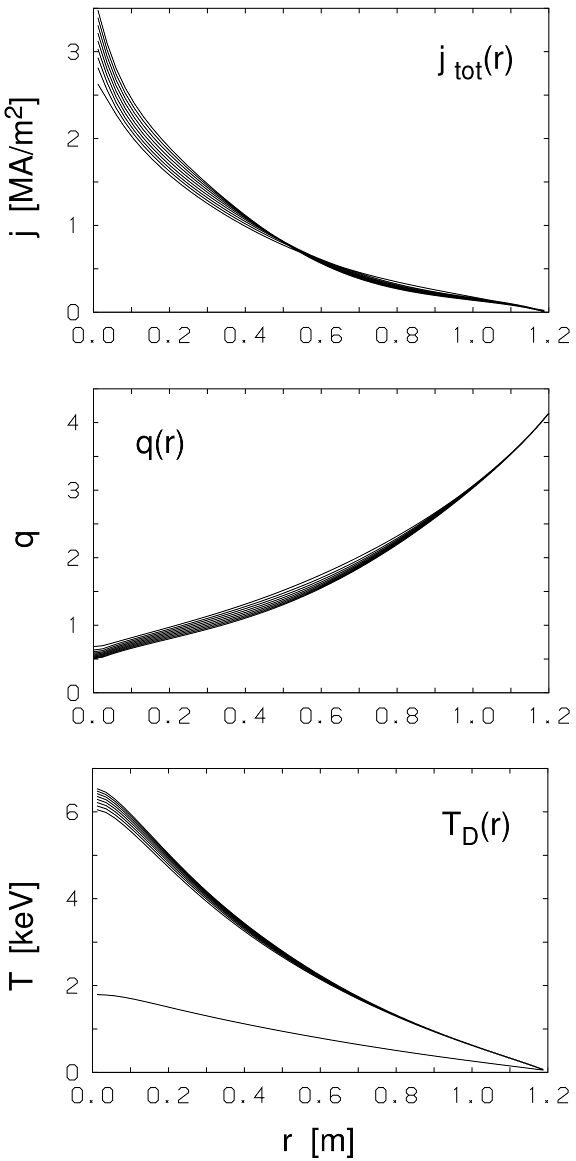


I_p : 1 MA ⟶ 3 MA/1 s
Heating : 20 MW
H factor ≃ 1.6

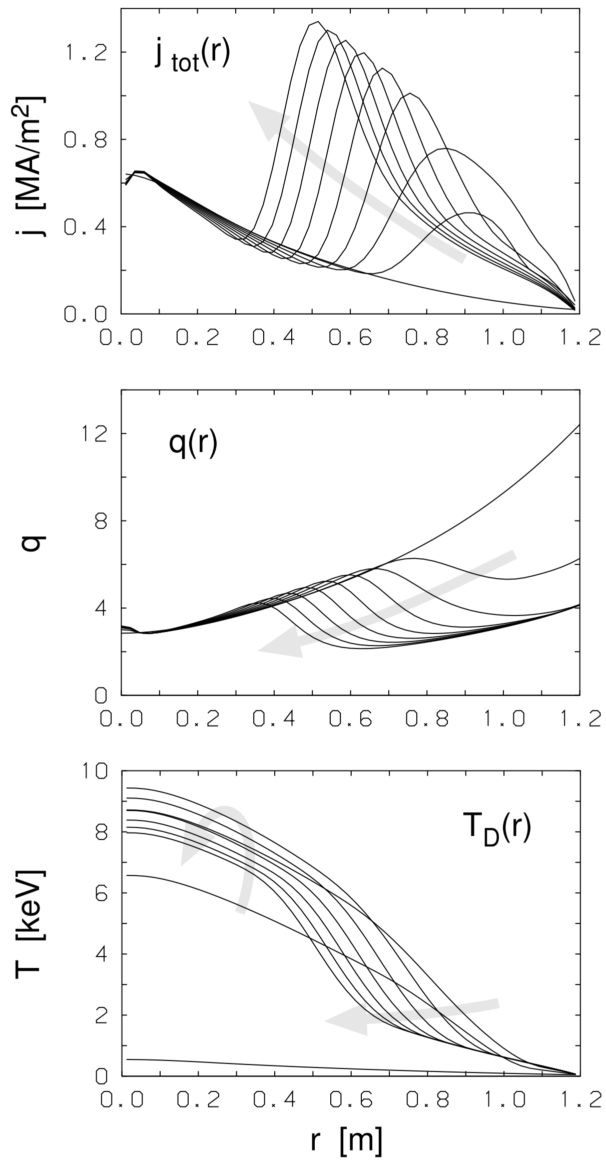


Evolution of Negative Shear Configuration

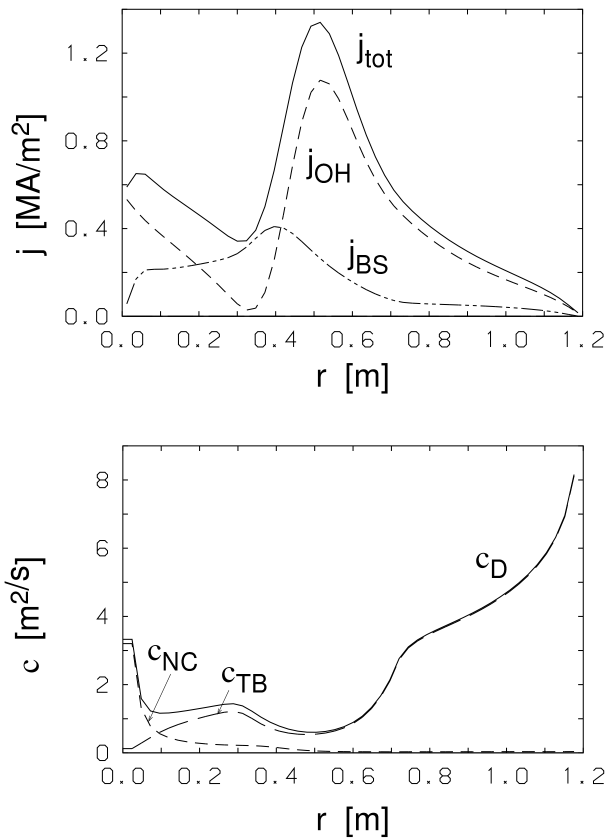
I_p : 3 MA **constant**



I_p : 1 MA $\longrightarrow$ 3 MA/1 s



$t = 5\,\text{s}$

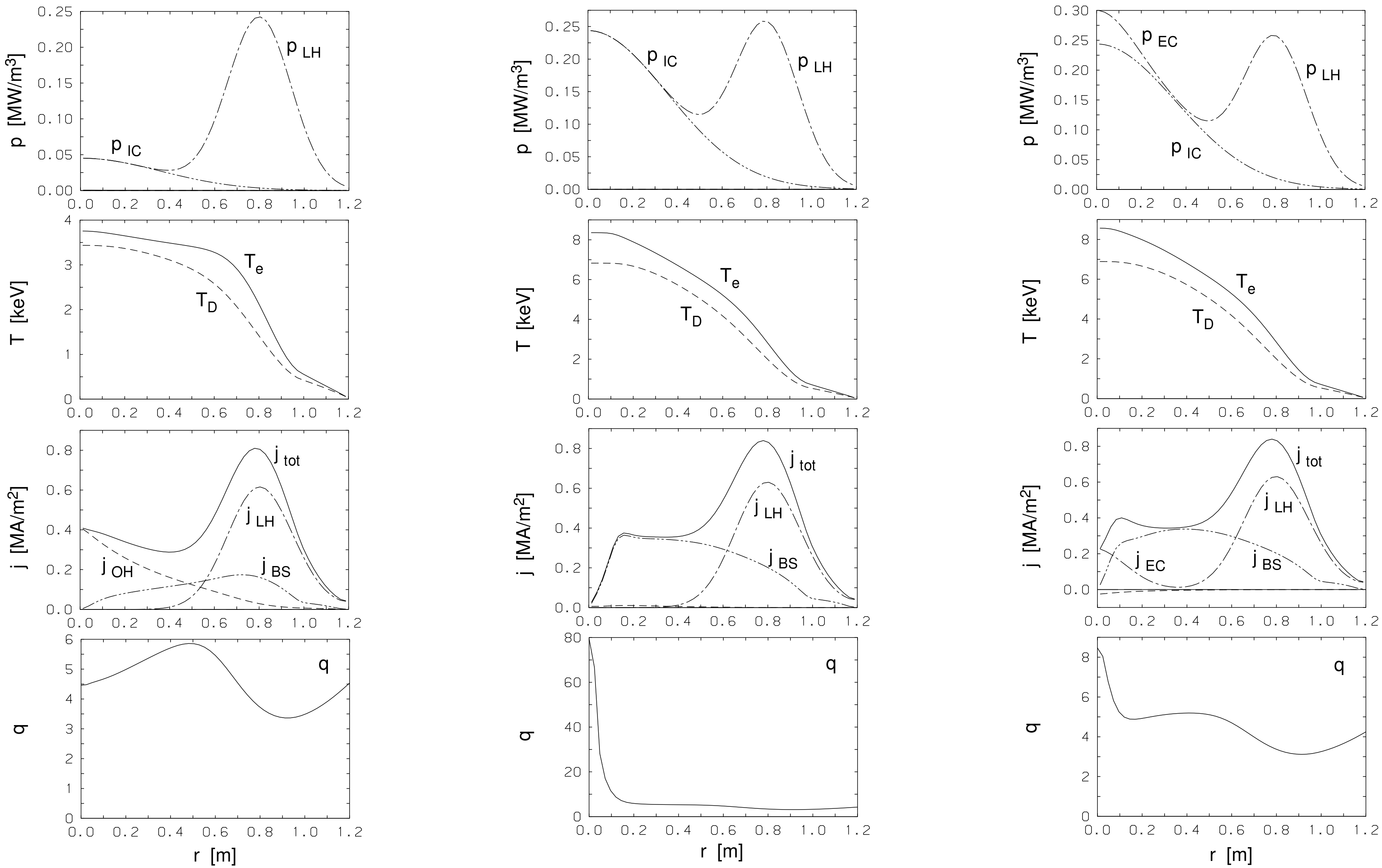


Sustainment of Negative Shear Configuration

Off-Axis CD ($H \simeq 1.2$)
$P_{\mathrm{LH}}/P_{\mathrm{IC}} = 12/2\,\mathrm{MW}$

Full CD ($H \simeq 1.6$)
$P_{\mathrm{LH}}/P_{\mathrm{IC}} = 12/10.8\,\mathrm{MW}$

Full CD + On-Axis CD
$P_{\mathrm{LH}}/P_{\mathrm{IC}}/P_{\mathrm{EC}} = 12/10.8/0.2\,\mathrm{MW}$



Bifurcation in Transport Barrier Formation

- **Transition in barrier formation is soft or hard?**
  - **ETB: Fast transition of $E_r \longrightarrow$ hard transition**
  - **ITB: Experimental observation ?; Theoretical approach**
- **Analysis of ITB based on CDBM model**
  - **Constraint: Constant heating power $P_{\rm H}$ inside ITB**
  - **Proposition: Two stable solutions may coexist?**
  - **Heat flux:**

$$q_{\rm H} = -n\chi\frac{{\rm d}T}{{\rm d}r} = \frac{P_{\rm H}}{4\pi^2 rR}$$

  - **Pressure gradient:**

$$\alpha = -q^2R\frac{{\rm d}\beta}{{\rm d}r} = nq^2R\frac{2\mu_0}{B^2}\left(1+\frac{1}{\eta_{\rm T}}\right)\frac{{\rm d}T}{{\rm d}r}, \qquad \eta_{\rm T} = \frac{{\rm d}\ln T}{{\rm d}\ln n}$$

  - **Thermal diffusivity:**

$$\chi_{\rm TB} = C\frac{F(s,\alpha)}{1+G\omega_E^2}\,\alpha^{3/2}\,\frac{c^2}{\omega_{\rm pe}^2}\,\frac{v_{\rm A}}{qR}$$

Heat Flux Relation

- **Heat flux relation can be rewritten as**

$$\hat{P}_{\mathrm{H}} = [\hat{\chi}_{\mathrm{TB}} + \hat{\chi}_{\mathrm{NC}}]\,\alpha$$

- **Normalization: P_{H} and χ are normalized by P_{H0} and χ_0**

$$P_{\mathrm{H0}} = 2\pi^2 \frac{r}{qR}\frac{B^2}{\mu_0}\frac{\eta_{\mathrm{T}}}{1+\eta_{\mathrm{T}}}\chi_0, \qquad \chi_0 = C\,\frac{c^2}{\omega_{\mathrm{pe}}^2}\frac{v_{\mathrm{A}}}{qR}$$

- **Therefore**

$$\hat{P}_{\mathrm{H}} = \frac{P_{\mathrm{H}}}{P_{\mathrm{H0}}}, \qquad \hat{\chi}_{\mathrm{TB}} = \frac{\chi_{\mathrm{TB}}}{\chi_0} = \frac{F(s,\alpha)}{1+G\omega_{\mathrm{E}}^2}\,\alpha^{3/2}, \qquad \hat{\chi}_{\mathrm{NC}} = \frac{\chi_{\mathrm{NC}}}{\chi_0}$$

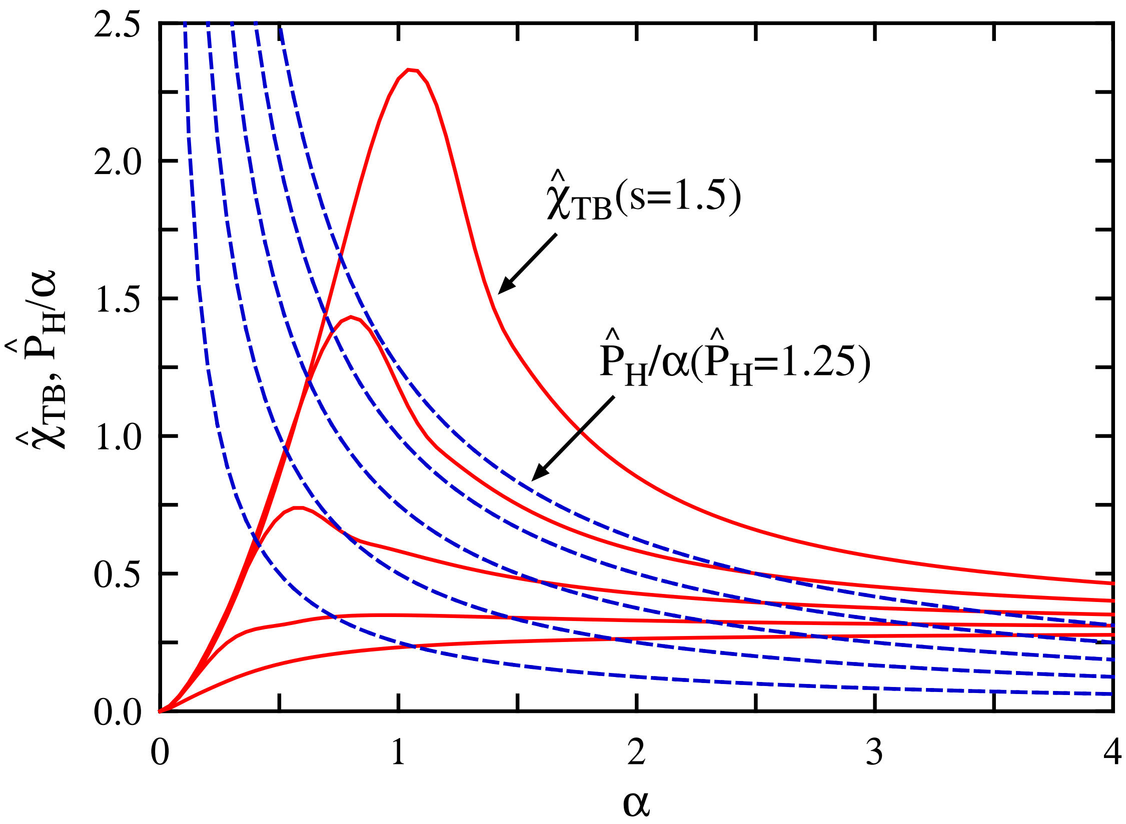
- **We plot**

$$\frac{\hat{P}_{\mathrm{H}}}{\alpha} = \hat{\chi}_{\mathrm{TB}} + \hat{\chi}_{\mathrm{NC}}$$

as a function of α for various values of $\hat{P}_{\mathrm{H}}$, s and G.

Condition of Bifurcation

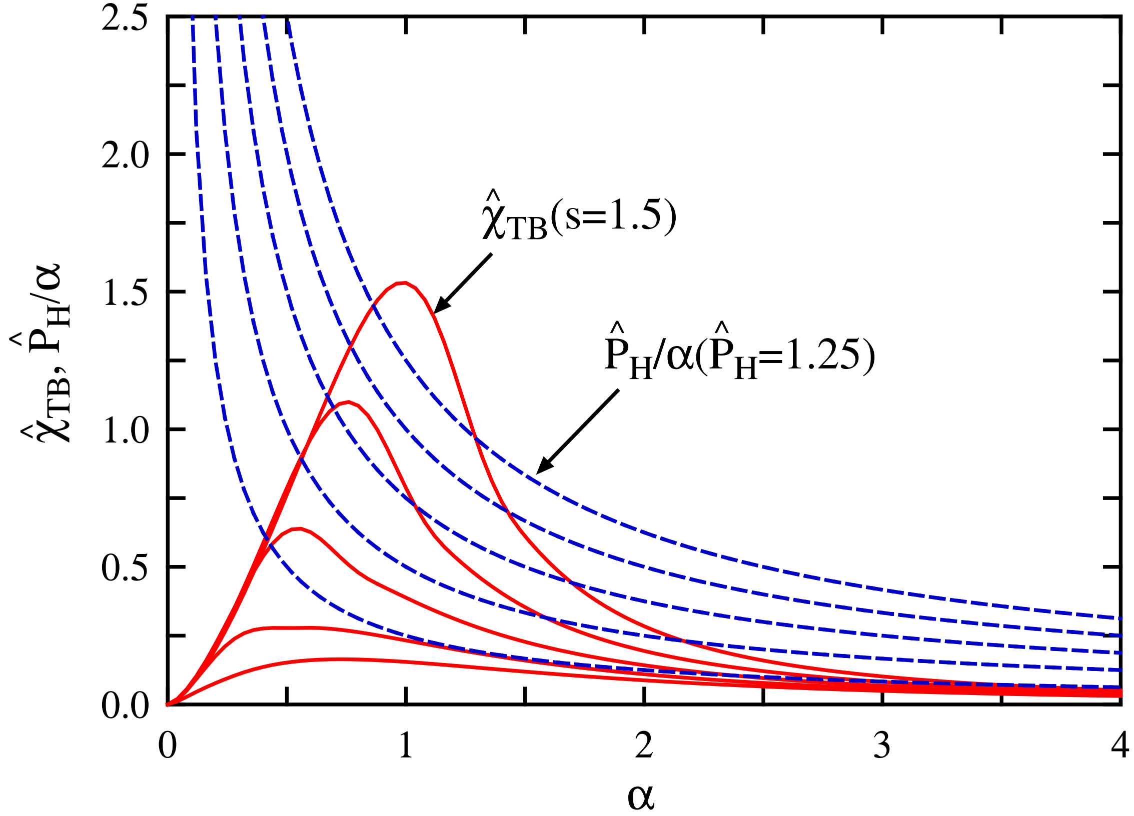
- **Effect of Shafranov shift ($G = 0$, $\hat{\chi}_{\rm NC} = 0$)**
- $s = 0.3$, 0.6, 0.9, 1.2, 1.5, $\hat{P}_{\rm H0} = 0.25$, 0.5, 0.75, 1.0, 1.25



- **For $s > 1.2$, bifurcation may occur.**
- **Threshold power: $\hat{P}_{\rm H0} = 1.25$**

Condition of Bifurcation

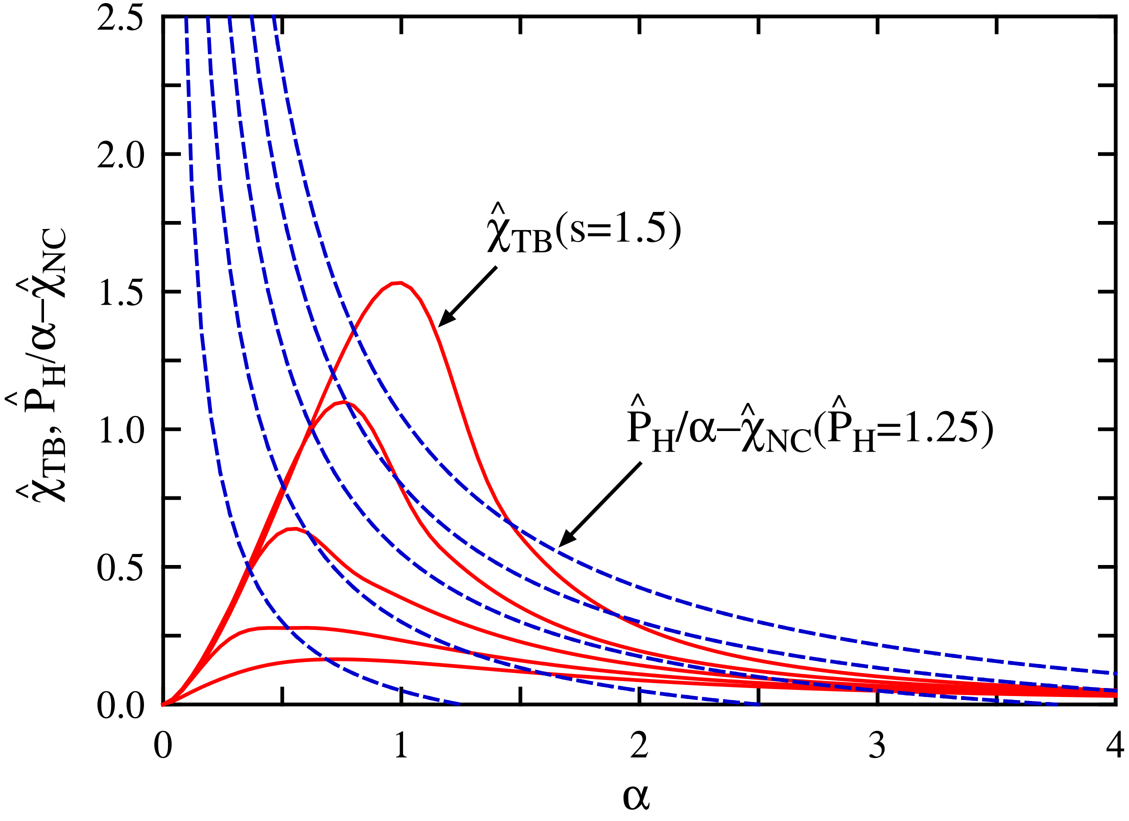
- **Effect of $E \times B$ rotation shear ($G' = 0.5$, $\hat{\chi}_{\rm NC} = 0$)**
- **Approximation:** $G\omega_{\rm E}^2 \simeq G'\alpha^2$
- $s = 0.3$, 0.6, 0.9, 1.2, 1.5, $\hat{P}_{\rm H0} = 0.25$, 0.5, 0.75, 1.0, 1.25



- **Thresholds of both s and $\hat{P}_{\rm H0}$ are reduced.**

Condition of Bifurcation

- **Effect of $E \times B$ rotation shear ($G' = 0.5$, $\hat{\chi}_{\rm NC} = 0.2$)**
- **Approximation:** $G\omega_{\rm E}^2 \simeq G'\alpha^2$
- $s = 0.3$, 0.6, 0.9, 1.2, 1.5, $\hat{P}_{\rm H0} = 0.25$, 0.5, 0.75, 1.0, 1.25



- α **after transition is finite but large.**

Summary

- **We have examined the possibility of bifurcation in transport barrier formation based on the CDBM transport model.**
- **In the high β_{p} mode, hard transition may occur for $s \gtrsim 1.2$.**
- **The effect of $E \times B$ rotation shear reduces the threshold of both s and $\hat{P}_{\mathrm{H}}$. The effect of $\hat{\chi}_{\mathrm{NC}}$ has to be taken into account to obtain finite α solution.**
- **In the case of low or negative magnetic shear, soft transition is dominant.**
- **These behaviors are consistent with simulation results.**
- **How to compare them with experimental observations?**