

# Integrated Simulation of Tokamak Plasmas by TASK Code System

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# Motivation and Status

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- **Predictive Simulation of Tokamak Plasma**

- Integrated code with modular structure
- Various heating and current drive schemes
- Portability and compactness
- **Future extension for helical system**
- **Parallel processing using MPI library**

- **Status**

- Most of modules work for given spatial profiles.
- Transport module is under reorganization.
- Results of integrated simulation will be available soon.
- **Subjects of this presentation:** Results of modules
  - **Transport simulation of current hole formation:** TR
  - **Beam tracing analysis of ECCD:** EQ+WR+DP+FP
  - **Linear stability of Alfvén eigenmodes:** EQ+DP+WM

# TASK code system

- **T**ransport **A**nalyzing **S**ystem for tokama**K**
- **I**ntegrated **C**ode

<b>TASK/</b>	<b>EQ</b>	<b>Fixed boundary equilibrium</b>	toroidal rotation
	<b>PL</b>	<b>Profile data interface</b>	Exp. data, ITPA Profile DB
	<b>TR</b>	<b>Diffusive radial transport</b>	$n_s, u_{s\phi}, T_s, B_\theta, E_\phi$
	<b>DP</b>	<b>Wave dispersion relation</b>	various velocity distributions
	<b>WR</b>	<b>Ray and beam tracing</b>	EC, LH
	<b>WM</b>	<b>3D full wave analysis</b>	IC, AW, eigenmodes
	<b>FP</b>	<b>Velocity distribution analysis</b>	3D, relativistic, bounce averaged
	<b>EX</b>	<b>Free boundary equilibrium</b>	Start up, Shut down
	<b>TX</b>	<b>Fluid-like transport analysis</b>	$n, \mathbf{u}, T, \mathbf{E}, \mathbf{B}$ , including SOL

# Interaction between TASK modules

- Output\Input variables

	EQ	PL	TR	DP	WR	WM	FP
EQ	—	$\psi(R, Z)$					
PL	$p, j, u_\phi(\psi)$	—	metric	$n, T, u_\parallel, \mathbf{B}(\mathbf{r})$		metric	metric
TR		$n, T, j, u_\phi, \psi(\rho)$	—				
DP				—	$\leftrightarrow$ $\epsilon$	$\leftrightarrow$ $\epsilon$	
WR			$P_{\text{abs}}(\rho)$		—		$\tilde{\mathbf{E}}, \tilde{\mathbf{B}}(\mathbf{r})$
WM			$P_{\text{abs}}(\rho)$			—	$\tilde{\mathbf{E}}, \tilde{\mathbf{B}}(\mathbf{r})$
FP			$P_{\text{abs}}, j_{\text{CD}}(\rho)$	$f(v_\parallel, v_\perp, \rho)$			—

- Modules with 3D configuration for helical system: WM, WR
- Modules with MPI parallelization for computer cluster: WM, FP

# TASK/TR

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- **Diffusive Transport Simulation**

- **Variables:**

Density  $n_s$  for  $s = \text{D, T, He, Impurity}$

Rotation  $u_s$  for  $s = \text{D, T, He, Impurity}$

Temperature  $T_s$  for  $s = \text{Electron, D, T, He, Impurity}$

Energy density  $W_s$  for  $s = \text{Alpha, Beam ion}$

Neutral density  $n_{fs}$  (fast) ,  $n_{ss}$  (slow) for  $s = \text{D, T}$

Poloidal flux  $\Psi$

- **Diffusion equation**

- **Transport coefficients**

Neoclassical model

Turbulent transport models (CDBM, ...)

- **Source:**

Ionization

Collisional momentum and energy transfer

RF heating and current drive

Fusion reaction

# CDBM Turbulence Model

- **Marginal Stability Condition** ( $\gamma = 0$ )

$$\chi_{\text{TB}} = F(s, \alpha, \kappa, \omega_{E1}) \alpha^{3/2} \frac{c^2}{\omega_{pe}^2} \frac{v_A}{qR}$$

**Magnetic shear**

$$s \equiv \frac{r}{q} \frac{dq}{dr}$$

**Pressure gradient**

$$\alpha \equiv -q^2 R \frac{d\beta}{dr}$$

**Magnetic curvature**

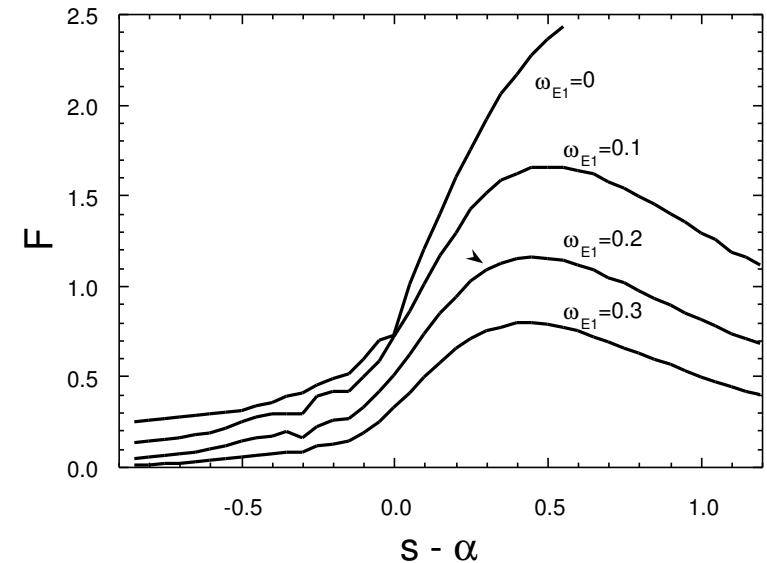
$$\kappa \equiv -\frac{r}{R} \left(1 - \frac{1}{q^2}\right)$$

**$E \times B$  rotation shear**

$$\omega_{E1} \equiv \frac{r^2}{sv_A} \frac{d}{dr} \frac{E}{rB}$$

- **Weak and negative magnetic shear,  
Shafranov shift and  
 $E \times B$  rotation shear  
reduce thermal diffusivity.**

$s - \alpha$  dependence of  
 $F(s, \alpha, \kappa, \omega_{E1})$

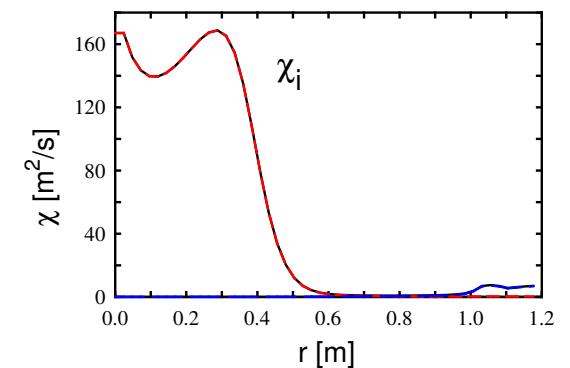
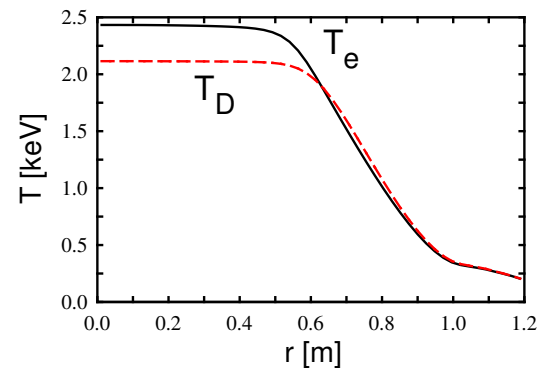
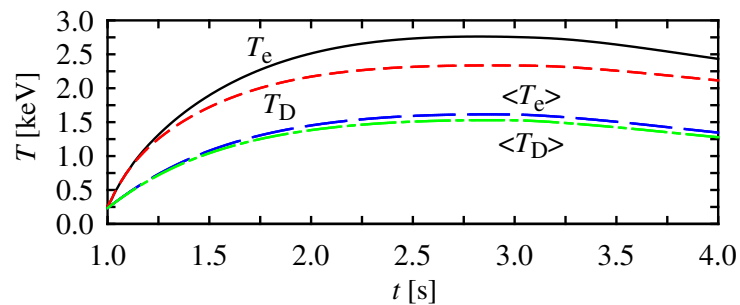
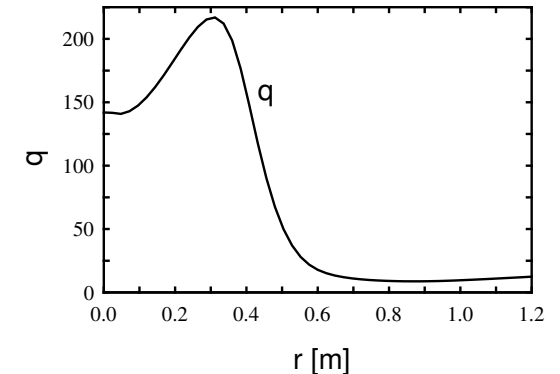
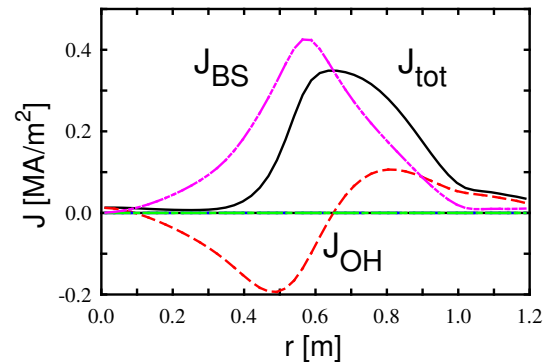
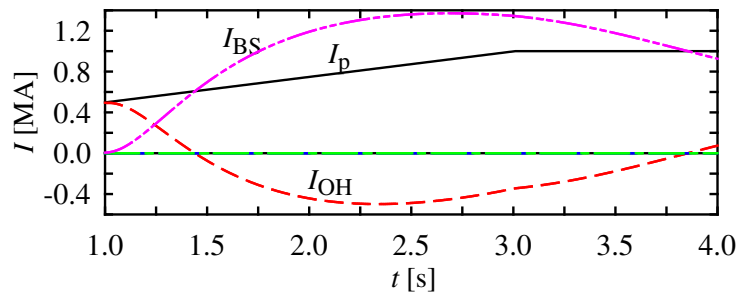


**Fitting Formula**

$$F = \begin{cases} \frac{1}{1 + G_1 \omega_{E1}^2} \frac{1}{\sqrt{2}(1 - 2s')(1 - 2s' + 3s'^2)} & \text{for } s' = s - \alpha < 0 \\ \frac{1}{1 + G_1 \omega_{E1}^2} \frac{1 + 9\sqrt{2}s'^{5/2}}{\sqrt{2}(1 - 2s' + 3s'^2 + 2s'^3)} & \text{for } s' = s - \alpha > 0 \end{cases}$$

# Simulation of Current Hole

- Current ramp up:  $I_p = 0.5 \rightarrow 1.0$  MA
- Moderate heating:  $P_H = 5$  MW
- **Current hole** is formed.
- The formation is sensitive to the edge temperature.



# TASK/DP

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- **Various Models of Dielectric Tensor:** (available, underway)
  - Resistive MHD plasma model
  - Cold plasma model with collision
  - Warm plasma model with collision
  - Kinetic plasma model (Maxwellian)
  - Kinetic plasma model (Relativistic Maxwellian)
  - Kinetic plasma model (Numerical, Relativistic/Non-Rel.)
  - Gyrokinetic plasma model (Maxwellian)
  - Gyrokinetic plasma model (Numerical, Relativistic/Non-Rel.)
- **for**
  - given  $n, u_{\parallel}, T, B, \nabla_{\perp} p, \nabla_{\perp} B$
  - given  $n, u_{\parallel}, T, B, \nabla_{\perp} n, \nabla_{\perp} T, \nabla_{\perp} B, E_{\perp}$



# TASK/WR

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- **Ray Tracing** (Geometrical Optics)
  - Wave length  $\lambda \ll$  Characteristic scale length  $L$  of the medium
  - Plane wave: Beam size  $d$  is sufficiently large
    - **Fresnel condition**:  $L \ll d^2/\lambda$
  - Beam : Diffraction effect determines the beam size  $d$
- **Beam Tracing**
  - Propagation of beam with finite size
    - Spatial evolution of beam size
  - References
    - G. V. Pereverzev, in *Reviews of Plasma Physics*, Vol. 19, p. 1.
    - A. G. Peeters, *Phys. Plasmas* **3** (1996) 4386.
    - G. V. Pereverzev, *Phys. Plasmas* **4** (1998) 3529.

# Beam Tracing Method

- Beam size perpendicular to the beam direction: first order in  $\delta$
- **Beam shape** : Weber function Hermite polynomial:  $H_n$ )

$$\mathbf{E}(\mathbf{r}) = \text{Re} \left[ \sum_{mn} C_{mn}(\delta^2 \mathbf{r}) \mathbf{e} e(\delta^2 \mathbf{r}) H_m(\delta \xi_1) H_n(\delta \xi_2) e^{i s(\mathbf{r}) - \phi(\mathbf{r})} \right]$$

- Amplitude :  $C_{mn}$ , Polarization :  $\mathbf{e}e$ , Phase :  $s(\mathbf{r}) + i\phi(\mathbf{r})$

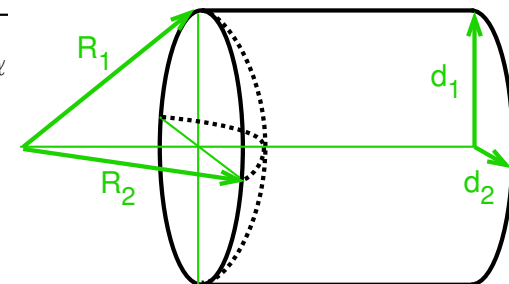
$$s(\mathbf{r}) = s_0(\tau) + k_\alpha^0(\tau)[r^\alpha - r_0^\alpha(\tau)] + \frac{1}{2} s_{\alpha\beta} [r^\alpha - r_0^\alpha(\tau)][r^\beta - r_0^\beta(\tau)]$$

$$\phi(\tau) = \frac{1}{2} \phi_{\alpha\beta} [r^\alpha - r_0^\alpha(\tau)][r^\beta - r_0^\beta(\tau)]$$

- Position of beam axis :  $\mathbf{r}_0$ , Wave number on beam axis:  $\mathbf{k}^0$

- **Curvature radius** of equi-phase surface:  $R_\alpha = \frac{1}{\lambda s_{\alpha\alpha}}$

- **Beam radius**  $d_\alpha = \sqrt{\frac{2}{\phi_{\alpha\alpha}}}$



- Gaussian beam : case with  $m = 0, n = 0$

# Beam Propagation Equation

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- Solvable condition for Maxwell's equation with beam field

$$\frac{dr_0^\alpha}{d\tau} = \frac{\partial K}{\partial k_\alpha}$$

$$\frac{dk_\alpha^0}{d\tau} = -\frac{\partial K}{\partial r^\alpha}$$

$$\frac{ds_{\alpha\beta}}{d\tau} = -\frac{\partial^2 K}{\partial r^\alpha \partial r^\beta} - \frac{\partial^2 K}{\partial r^\beta \partial k_\gamma} s_{\alpha\gamma} - \frac{\partial^2 K}{\partial r^\alpha \partial k_\gamma} s_{\beta\gamma} - \frac{\partial^2 K}{\partial k_\gamma \partial k_\delta} s_{\alpha\gamma} s_{\beta\delta} + \frac{\partial^2 K}{\partial k_\gamma \partial k_\delta} \phi_{\alpha\gamma} \phi_{\beta\delta}$$

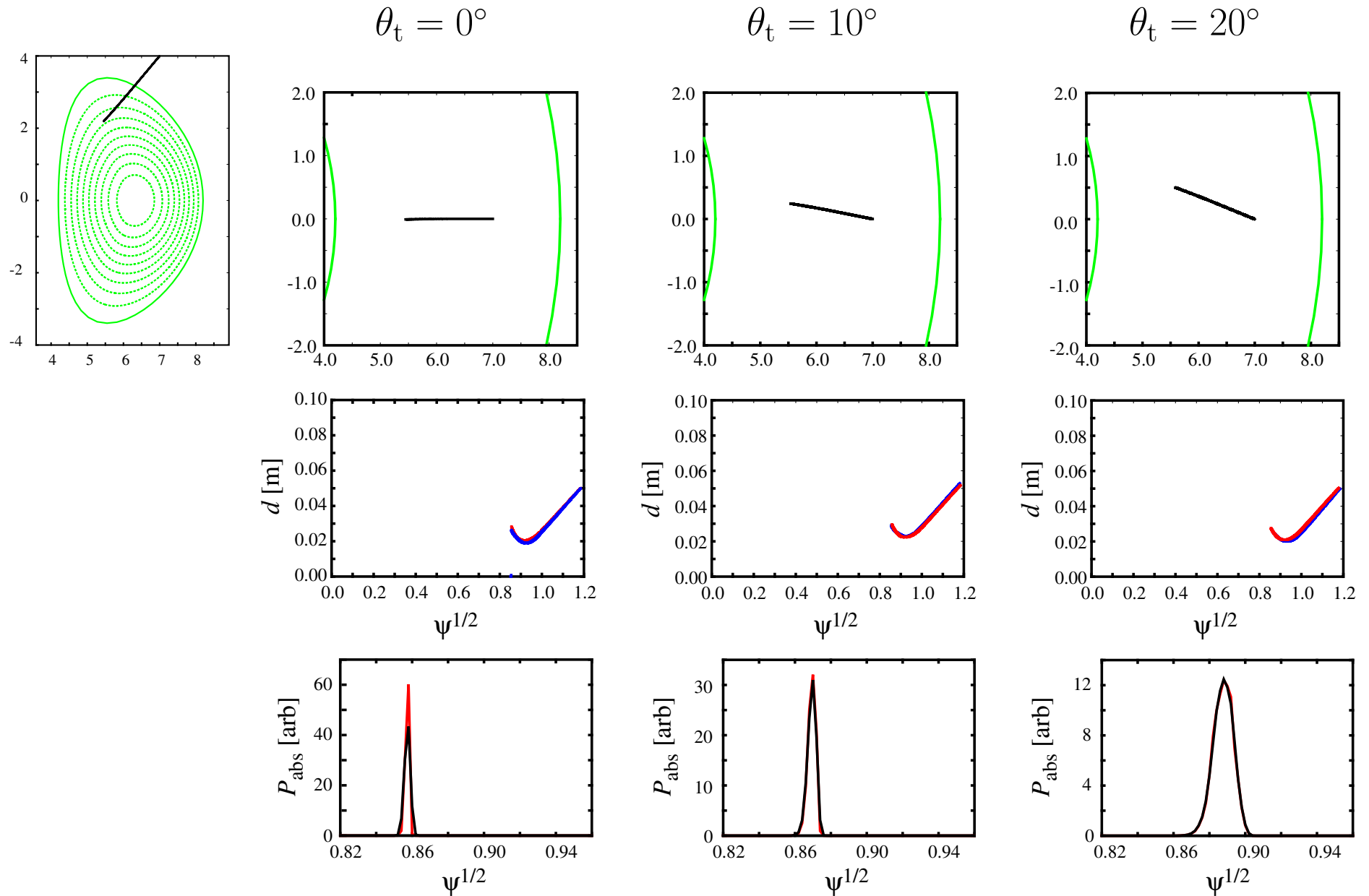
$$\frac{d\phi_{\alpha\beta}}{d\tau} = -\left( \frac{\partial^2 K}{\partial r^\alpha \partial k_\gamma} + \frac{\partial^2 K}{\partial k_\gamma \partial k_\delta} s_{\alpha\delta} \right) \phi_{\beta\gamma} - \left( \frac{\partial^2 K}{\partial r^\beta \partial k_\gamma} + \frac{\partial^2 K}{\partial k_\gamma \partial k_\delta} s_{\beta\delta} \right) \phi_{\alpha\gamma}$$

- By integrating this set of 18 ordinary differential equations, we obtain trace of the beam axis, wave number on the beam axis, curvature of equi-phase surface, and beam size.
- Equation for the wave amplitude  $C_{mn}$

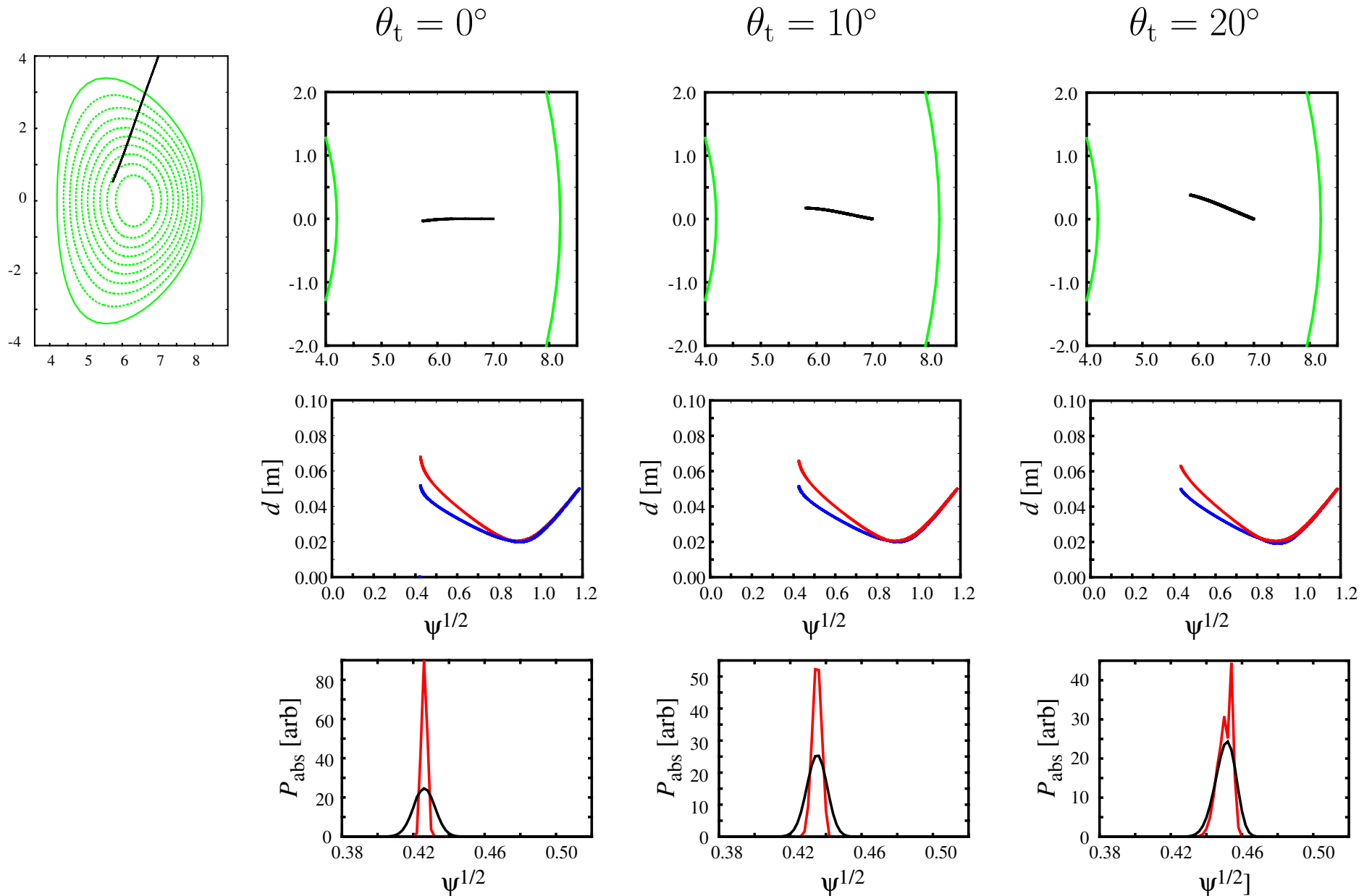
$$\nabla \cdot (\mathbf{v}_{g0} |C_{mn}|^2) = -2 (\gamma |C_{mn}|^2)$$

Group velocity:  $\mathbf{v}_{g0}$ , Damping rate:  $\gamma \equiv (\mathbf{e}^* \cdot \overleftrightarrow{\epsilon}_A \cdot \mathbf{e}) / (\partial K / \partial \omega)$

# Beam Tracing in ITER-FEAT Plasma: $R_c = 2$ m, $d_{ini} = 0.05$ m



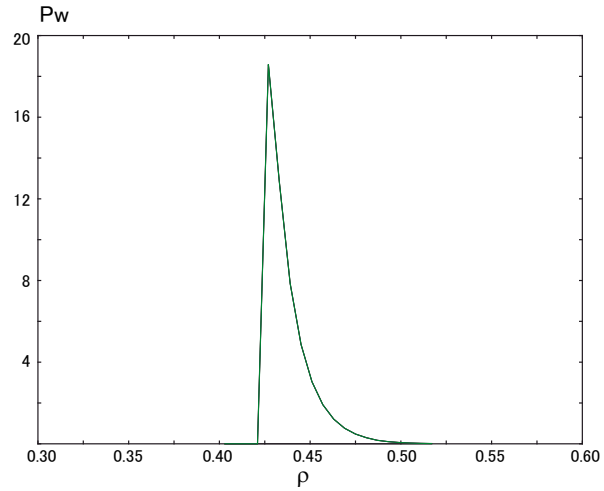
# Beam Tracing in ITER-FEAT Plasma: $R_c = 2$ m, $d_{ini} = 0.05$ m



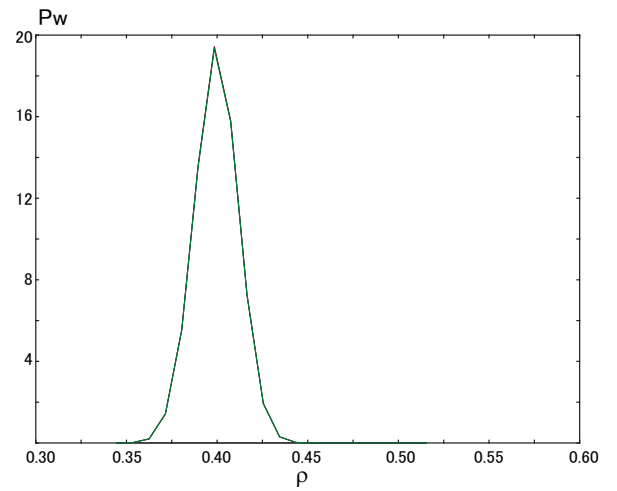
# ECCD Driven Current Profile

Power deposition

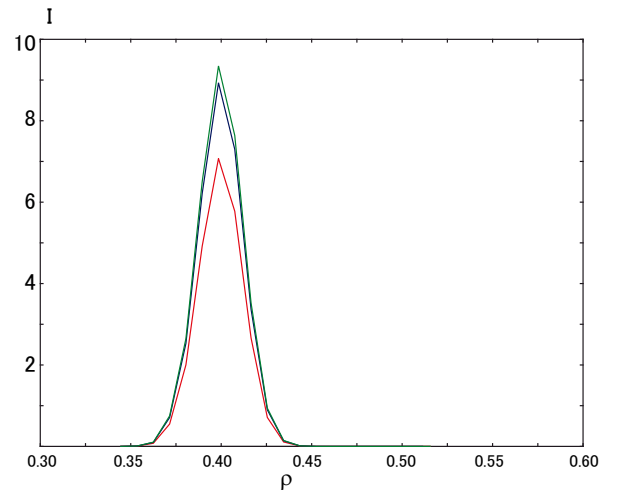
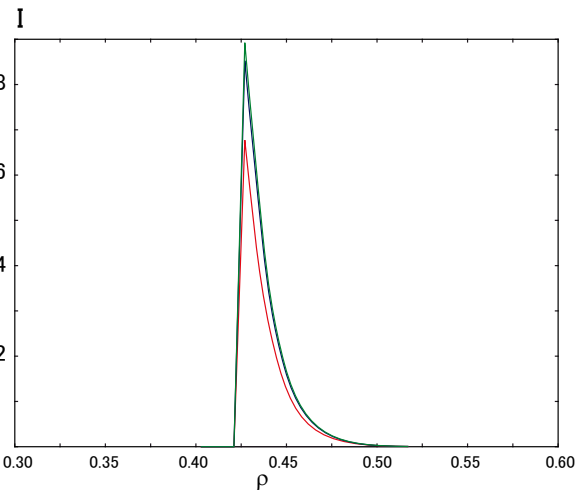
Single-Ray Tracing



Beam Tracing



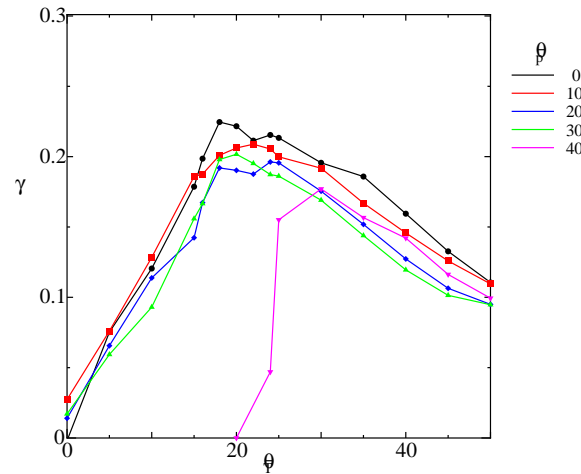
Driven current density



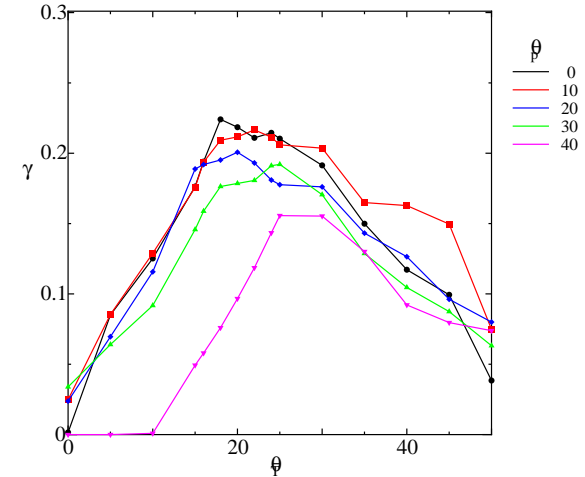
# Current Drive Efficiency (Preliminary)

Horizontal port

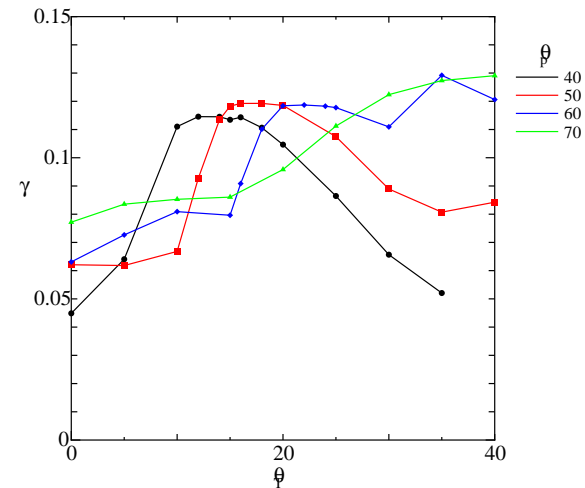
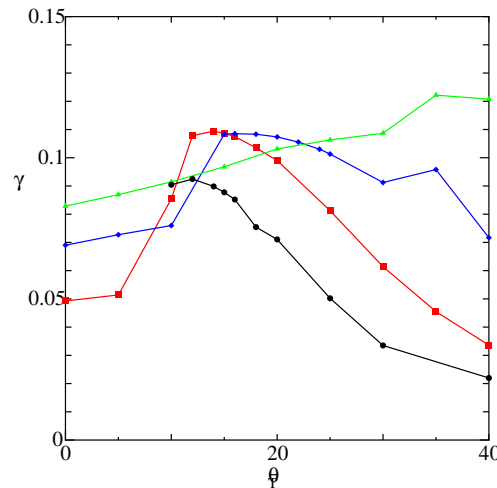
## Single-Ray Tracing



## Beam Tracing



Upper port



- Multiple-ray tracing is underway.

# TASK/WM

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- **Magnetic flux coordinates:**  $(\psi, \theta, \varphi)$

- **Non-orthogonal system**

- **Maxwell's equation** for stationary wave electric field  $\mathbf{E}$

$$\nabla \times \nabla \times \mathbf{E} = \frac{\omega^2}{c^2} \overleftrightarrow{\epsilon} \cdot \mathbf{E} + i\omega\mu_0 \mathbf{j}_{\text{ext}}$$

- $\overleftrightarrow{\epsilon}$  : **Dielectric tensor with kinetic effects:**  $Z[(\omega - n\omega_c)/k_{\parallel}]$

- **Fourier expansion** in poloidal and toroidal directions

- **Exact parallel wave number:**  $k_{\parallel}^{m,n} = (mB^{\theta} + nB^{\varphi})/B$

- **Destabilization by energetic ions** include in  $\overleftrightarrow{\epsilon}$

- **Drift kinetic equation**

$$\left[ \frac{\partial}{\partial t} + v_{\parallel} \nabla_{\parallel} + (\mathbf{v}_d + \mathbf{v}_E) \cdot \nabla + \frac{e_{\alpha}}{m_{\alpha}} (v_{\parallel} E_{\parallel} + \mathbf{v}_d \cdot \mathbf{E}) \frac{\partial}{\partial \epsilon} \right] f_{\alpha} = 0$$

- **Eigenvalue problem** for complex wave frequency

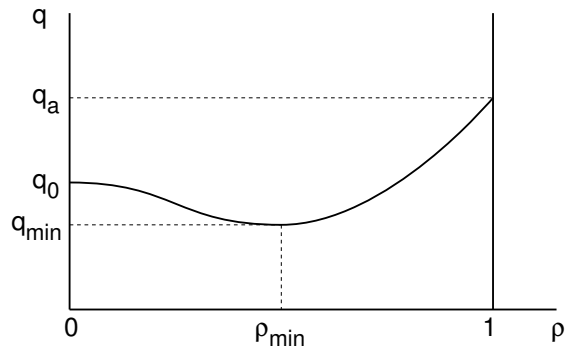
- **Maximize wave amplitude** for finite excitation proportional to  $nne$



# Analysis of TAE in Reversed Shear Configuration

## $q_{\min}$ Dependence of Eigenmode Frequency

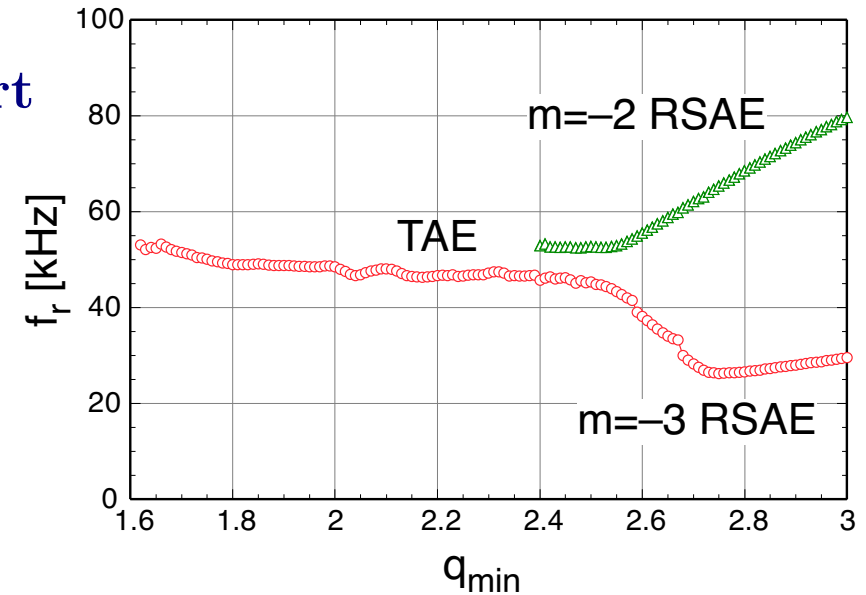
### Assumed $q$ profile



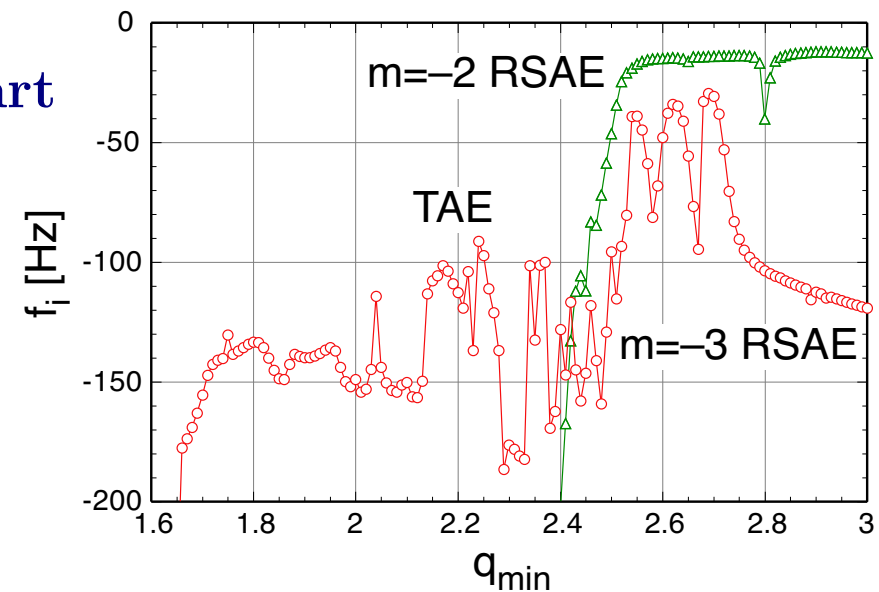
### Plasma Parameters

$R_0$	3 m
$a$	1 m
$B_0$	3 T
$n_e(0)$	$10^{20} \text{ m}^{-3}$
$T(0)$	3 keV
$q(0)$	3
$q(a)$	5
$\rho_{\min}$	0.5
$n$	1
Flat density profile	

### Real part

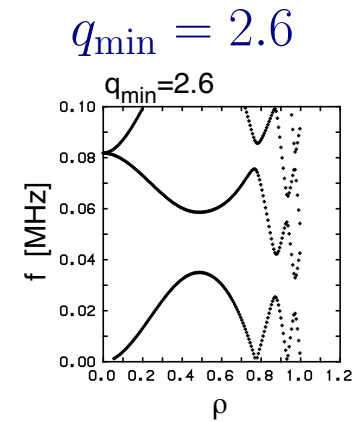
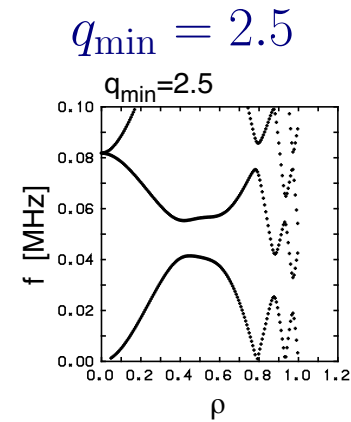
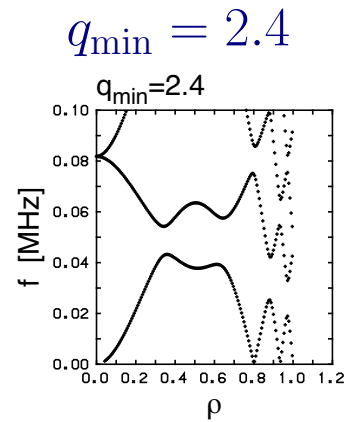


### Imag part

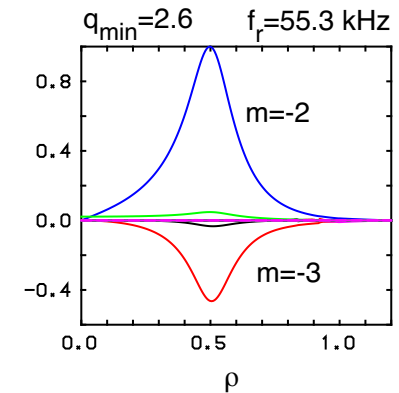
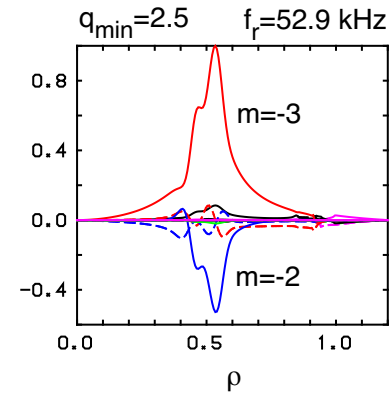
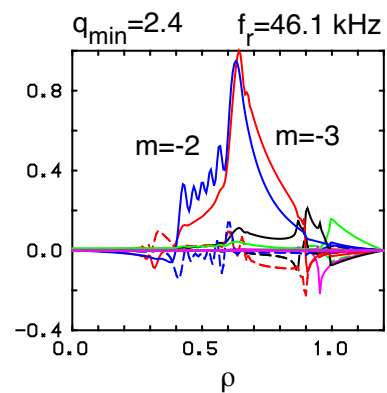


# Eigenmode Structure

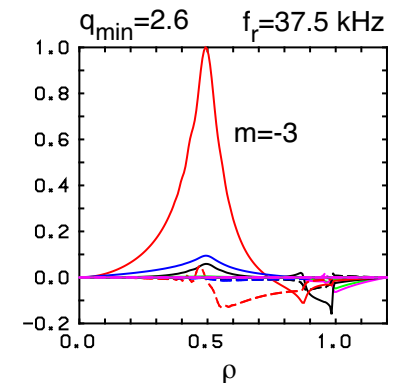
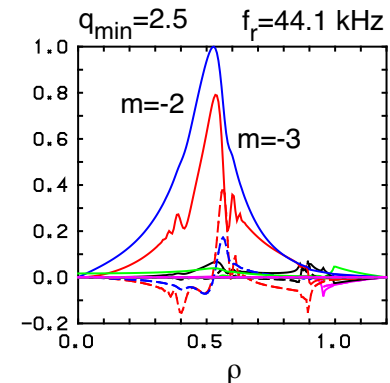
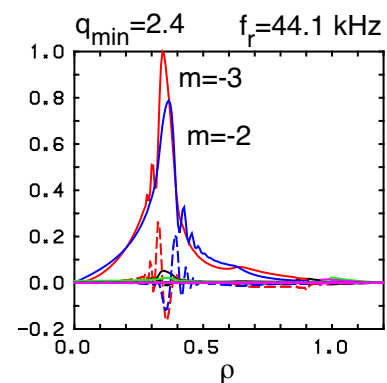
Alfvén resonance



Higher freq.



Lower freq.



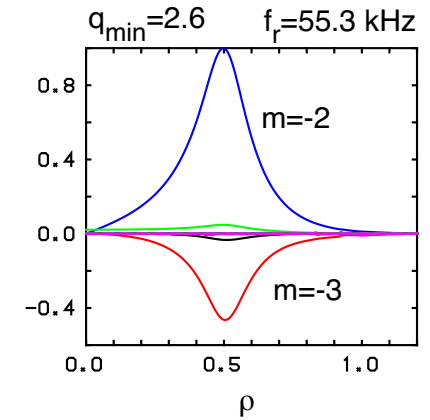
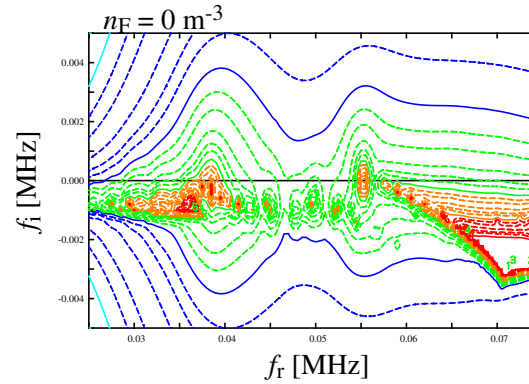
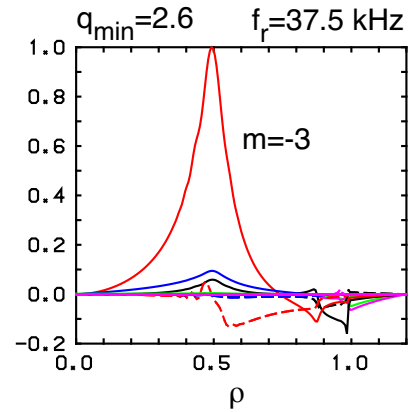
TAEs

Double TAE

RSAE

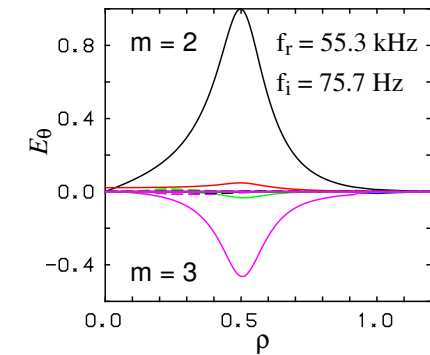
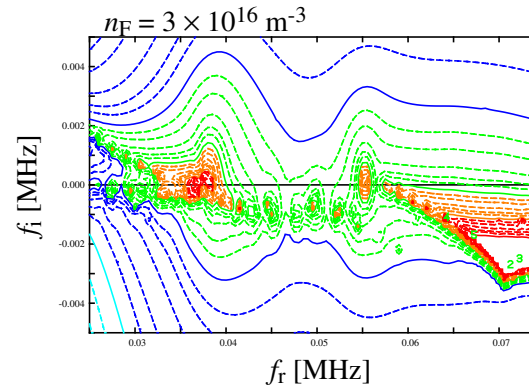
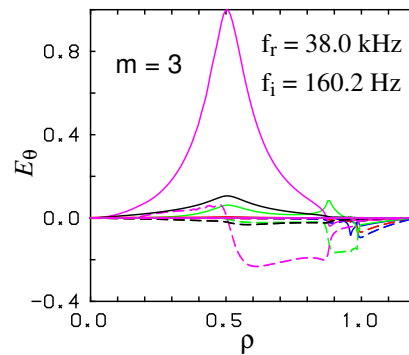
# Excitation by Energetic Particles ( $q_{\min} = 2.6$ )

- Without EP



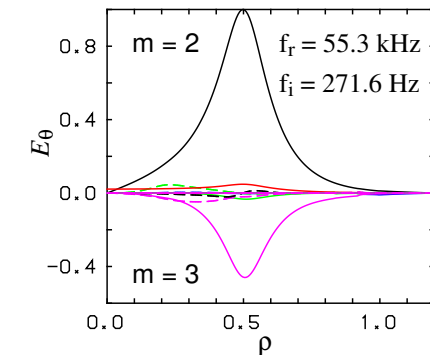
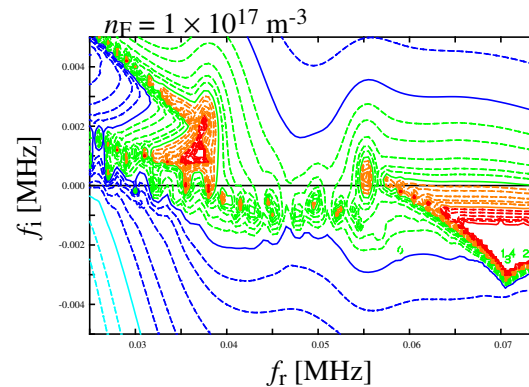
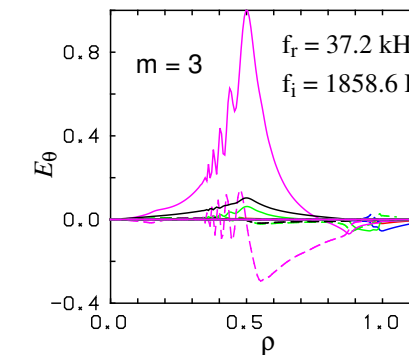
- With EP

$3 \times 10^{16}$  m<sup>-3</sup>  
 360 keV  
 0.5 m



- With EP

$1 \times 10^{17}$  m<sup>-3</sup>  
 360 keV  
 0.5 m



# Summary

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- Development of TASK code system for integrated simulation of tokamak plasmas in underway.
- **New results obtained by the modules:**
  - **TR:** Heat transport simulation using CDBM transport model reproduced the current hole formation. The obtained radial profiles are similar to the experimental ones, though the ITB formation occurs in earlier phase.
  - **WR:** In order to estimate the driven current profile in ITER, the beam tracing method was applied to the analysis of ECCD. In the case of toroidally oblique injection, the effect of Doppler shift seems dominant for the current width.
  - **WM:** In the reversed magnetic shear configuration, reversed-shear induced Alfvén eigenmode (RSAE) can be excited by energetic ions. The lower frequency mode (slightly above the lower end of frequency gap) which has a larger damping rate is more easily excited by the energetic beam.
- **Near future works:**

- Complete the update to include toroidal rotation in TR module
- Test integrated simulation code with experimental observations

# Burning Plasma Simulation Initiative

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- **Purpose:**
  - Integrated simulation of burning plasma
  - Interaction between various codes (Universities, NIFS, JAERI)
  - Consistent analysis of phenomena with different time/space scales
- **Activity:** (in consideration)
  - **Level 0:** Development of open-source core code based on TASK
  - **Level 1:** Unified data interface between various codes  
Enhancement and extension of modules  
Parallel processing and grid computing
  - **Level 2:** Innovative model including different time/space scales
  - **Level 3:** Interaction with direct numerical simulation

- **Collaborations:**

- **Theory group:** New approach  
Critical issues
- **Experimental group:** Experimental data handling  
User-oriented interface
- **Computer science:** New Algorithm  
Grid computing
- **ITPA:** International collaboration

- **Framework:**

- **Voluntary work** from November 2002
- **Research collaboration** of NIFS
- Proposal for **Grant-in-Aid for Scientific Research**