2002-11-15
44th APS DPP UP1-008
Orlando, USA

Simulation of RF Plasma Production by Adaptive FEM Code

A. Fukuyama and H. Kobayashi
Department of Nuclear Engineering,
Kyoto University, Kyoto 606-8501, Japan

Supported by Grant-in-Aid for Scientific Research of the Ministry of Education, Culture, Sports, Science and Technology, Japan.

Introduction

- **Modeling of RF Plasma Production**
  - **Self-consistent analysis of**
    Propagation and absorption of RF wave and
    Time evolution of produced plasma
  - **Especially at the initial stage of plasma production**
    — **Rapid change of plasma density and temperature**

- **Various models with different applicable ranges**
  - **RF field analysis**:
    — **Stationary wave**: $e^{i\omega}$
    — **Electrostatic field**: $\tilde{\boldsymbol{B}} = 0$
    — **Electromagnetic field**: speed of light
  - **Plasma analysis**:
    — **Diffusive transport model**: Collision-dominated plasma
    — **Dynamic fluid model**: Collisional plasma
    — **Vlasov kinetic model**: Week collision plasma
    — **PIC kinetic model**: Number of particles

- **Quantitative comparison between the models is necessary.**

Simulation code: PAF (Plasma analysis by FEM)

- **Features**:
  - **Based on Finite Element method with triangular elements**
  - **2D (rectangular and axisymmetric) and extendable to 3D**
  - **Full implicit method for time evolution**
  - **Parallel processing with MPI (in progress)**
- **Modules**:

WF	Stationary wave propagation analysis	2D/3D	available
TF	Transport analysis of plasma	2D	available
ES	Electrostatic field analysis	2D	available
PF	PIC simulation with triangular elements	2D	preliminary
EM	Electromagnetic field analysis	2D	underway
FF	Fluid analysis of plasma	2D	near future
DIV	Simple mesh generator	2D	available
GRF	Graphics	2D/3D	available

Neutral Loop Discharge Plasma

- **Neutral Loop Discharge (NLD)** [*T. Uchida, Jpn. J. Appl. Phys.* **33** *L43 (1994)*]
  - Magnetic configuration with a neutral loop of $|B| = 0$
  - Electron cyclotron resonance (ECR) in a cusp configuration
  - Suppression of fast electron generation [*T. Sakoda et al., Jpn. J. Appl. Phys.* **36** *6981 (1997)*]
  - Profile control by neutral loop radius [*W. Chen et al., J. Plasma and Fusion Res.* **74** *258 (1998)*]
- **Chaos of electron motion near the neutral loop** [*Z. Yoshida, J. Plasma and Fusion Res.* **73** *757 (1997)*]
  - Magnetic moment changes near $|B| = 0$.
  - Kinetic energy changes near the ECR.
  - Reduction of correlation time (increase of effective collision time)
- **Previous analyses: given RF field**
  - Applicable to a low density plasma ($\omega_{\rm pe}^2 \ll \omega_{\rm ce}^2$)
  - High density plasma ($\omega_{\rm pe}^2 \gg \omega_{\rm ce}^2$) ?

Wave Analysis

- **Wave propagation in an inhomogeneous plasma**

  Boundary-value problem of Maxwell's equation

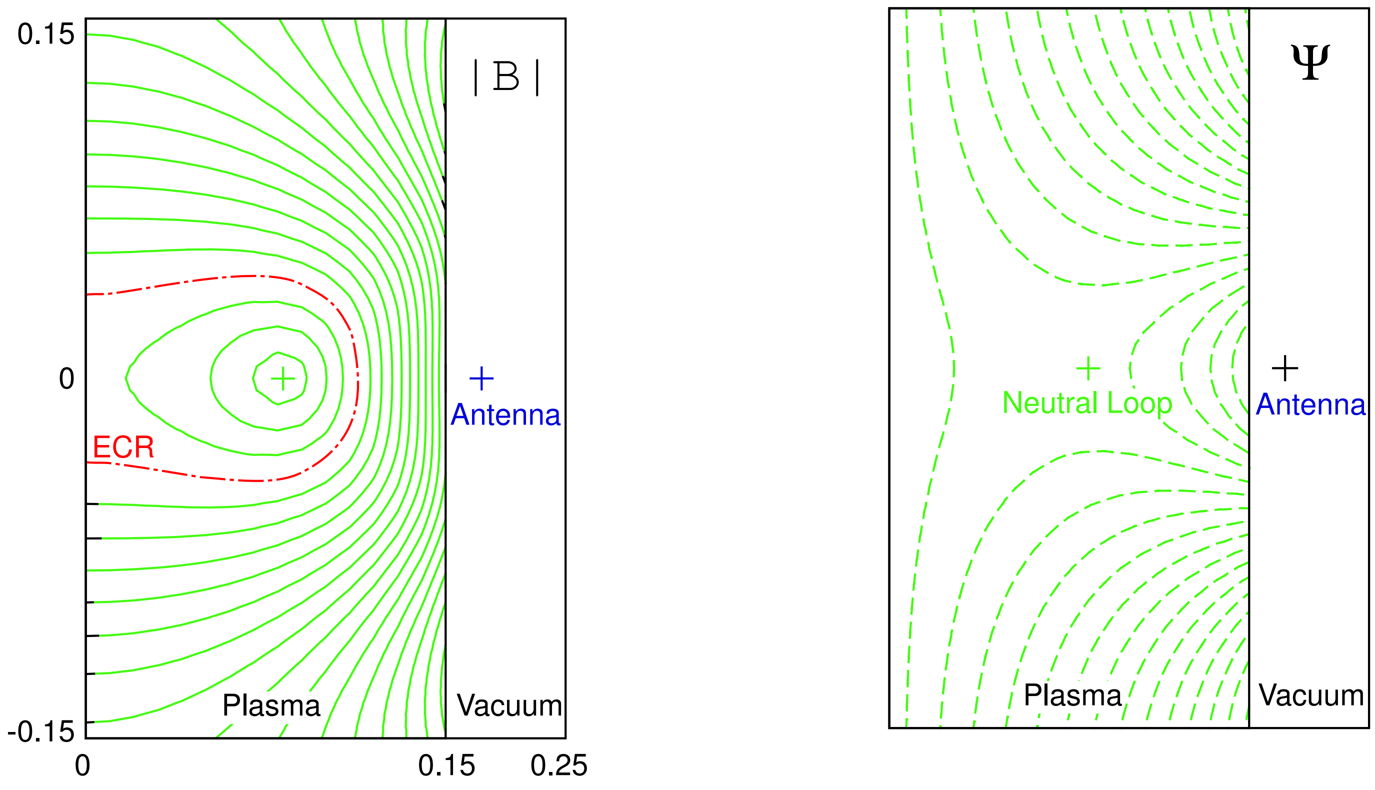
- **Analysis by the finite element method (FEM)**
  - Maxwell's equation for the electric field $\longrightarrow$ spurious solution in a low density plasma
  $$\nabla \times \nabla \times \boldsymbol{E} - \frac{\omega^2}{c^2} \overleftrightarrow{\epsilon} \cdot \boldsymbol{E} = \mathrm{i}\,\omega\mu_0 \boldsymbol{J}_{\rm ext}$$
  - We solve Maxwell's solutions for the vector and scalar potentials. $\mathrm{i}\boldsymbol{E} = \mathrm{i}\,\omega\boldsymbol{A} - \nabla\phi$j
  $$-\nabla^2 \boldsymbol{A} - \frac{\omega^2}{c^2} \overleftrightarrow{\epsilon} \cdot \left(\boldsymbol{A} + \frac{\mathrm{i}}{\omega} \nabla\phi \right) = \mu_0 \boldsymbol{J}_{\rm ext}$$
  $$-\nabla \cdot \overleftrightarrow{\epsilon} \cdot (\nabla\phi - \mathrm{i}\,\omega\boldsymbol{A}) = \frac{1}{\mathrm{i}\,\omega\epsilon_0} \nabla \cdot \boldsymbol{J}_{\rm ext}$$

- **Excitation by Antenna**

  Loop antenna, parallel antenna, waveguide

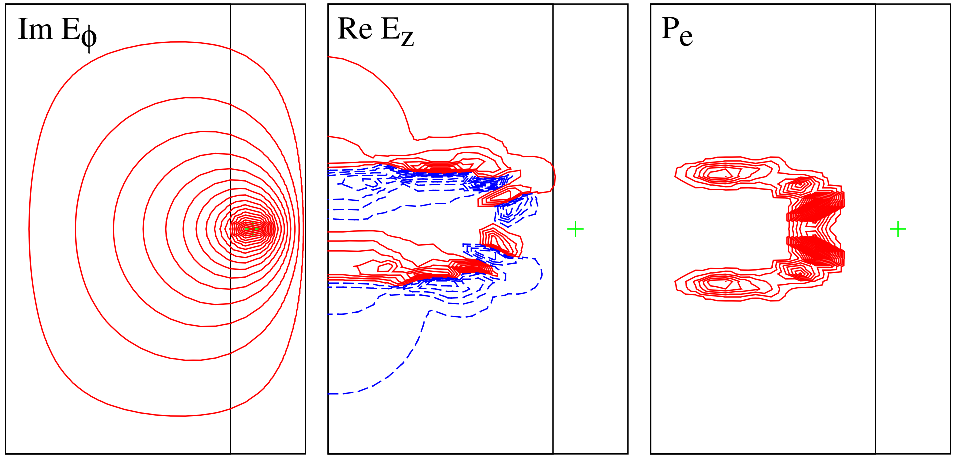
Electromagnetic Field model

- **Static magnetic field** : 3 loop coils (r= 0.3 m)
- **RF field** : Loop antenna (r= 0.165 m, f = 13.56 MHz)
- **Plasma** : Cylindrical plasma (r= 0.15 m, parabolic density profile)

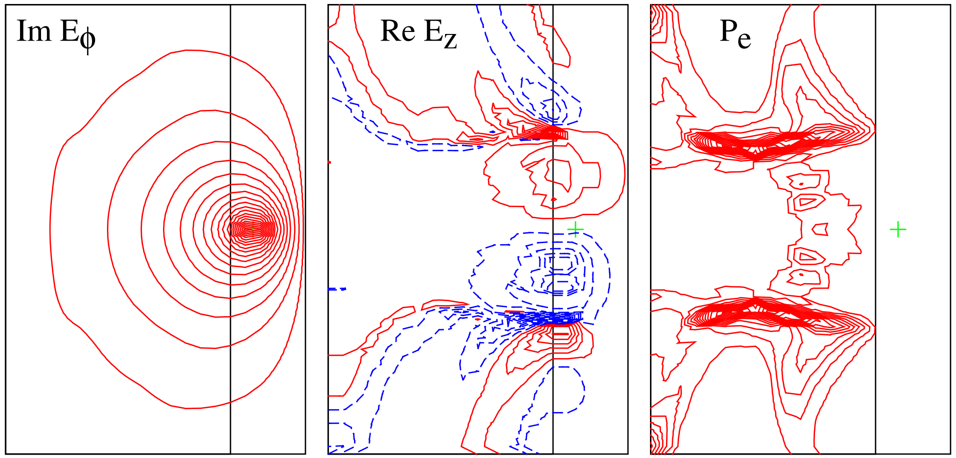


Low and High Density Plasmas

- **Low density plasma** ($10^{12}\,\mathrm{m}^{-3}$) ....Absorption near the ECR layer

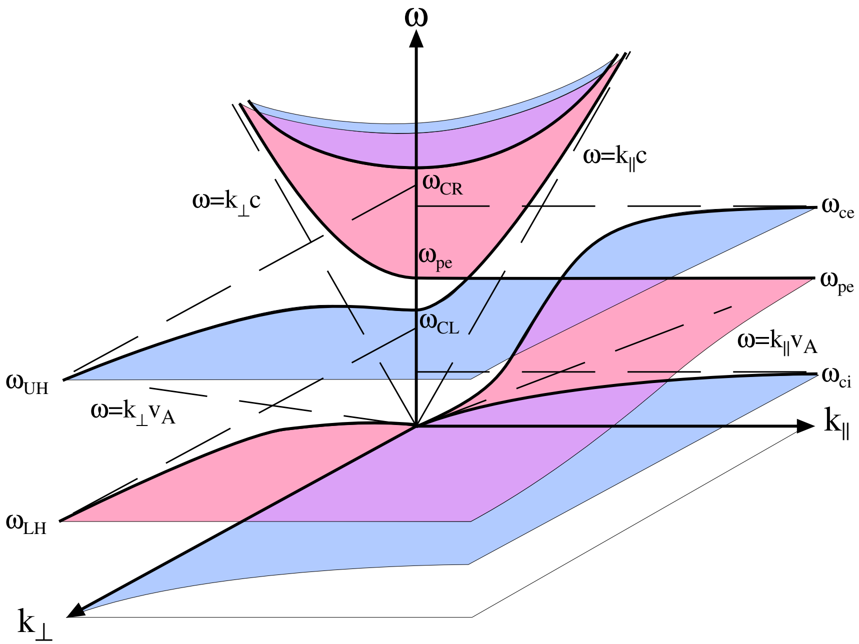


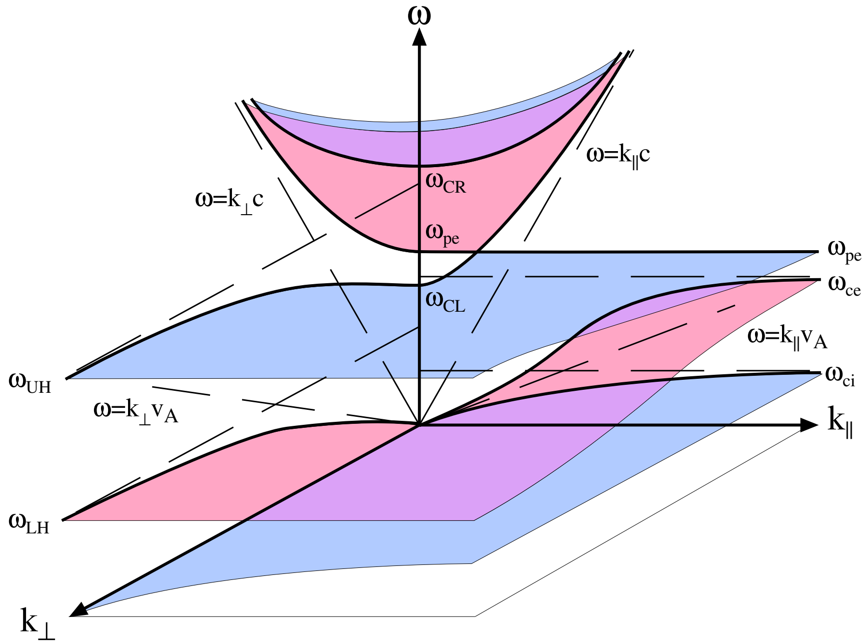
- **High density plasma** ($10^{16}\,\mathrm{m}^{-3}$) ....Absorption near the LHR layer



Dispersion Relation

- **Low density plasma**i$\omega_{\text{pe}}^2 < \omega_{\text{ce}}^2$)
  - Electron cyclotron wave
  - Oblique electron plasma wave
- **High density plasma**i$\omega_{\text{pe}}^2 > \omega_{\text{ce}}^2$)
  - Helicon wave
  - Lower hybrid wave





Obliquely Propagating Helicon Wave

- **High density plasma**:

$$\omega_{\rm pe}^2 \gg \omega_{\rm ce}^2, \qquad N_\parallel^2 \ll N_\perp^2$$

- **Dispersion relation**

$$(S - N_\parallel^2)(S - N_\perp^2 - N_\parallel^2) - D^2 = 0$$

$$S = 1 - \sum_s \frac{\omega_{\rm ps}^2}{\omega^2 - \omega_{\rm cs}^2}, \qquad D = \sum_s \frac{\omega_{\rm cs}}{\omega}\frac{\omega_{\rm ps}^2}{\omega^2 - \omega_{\rm cs}^2}$$

- **Resonance condition**: $N_\perp \to \infty$

$$S - N_\parallel^2 = 0 \quad \Longrightarrow \quad \omega^2 = \omega_{\rm ce}^2 - \frac{\omega_{\rm pe}^2}{N_\parallel^2 - 1}$$

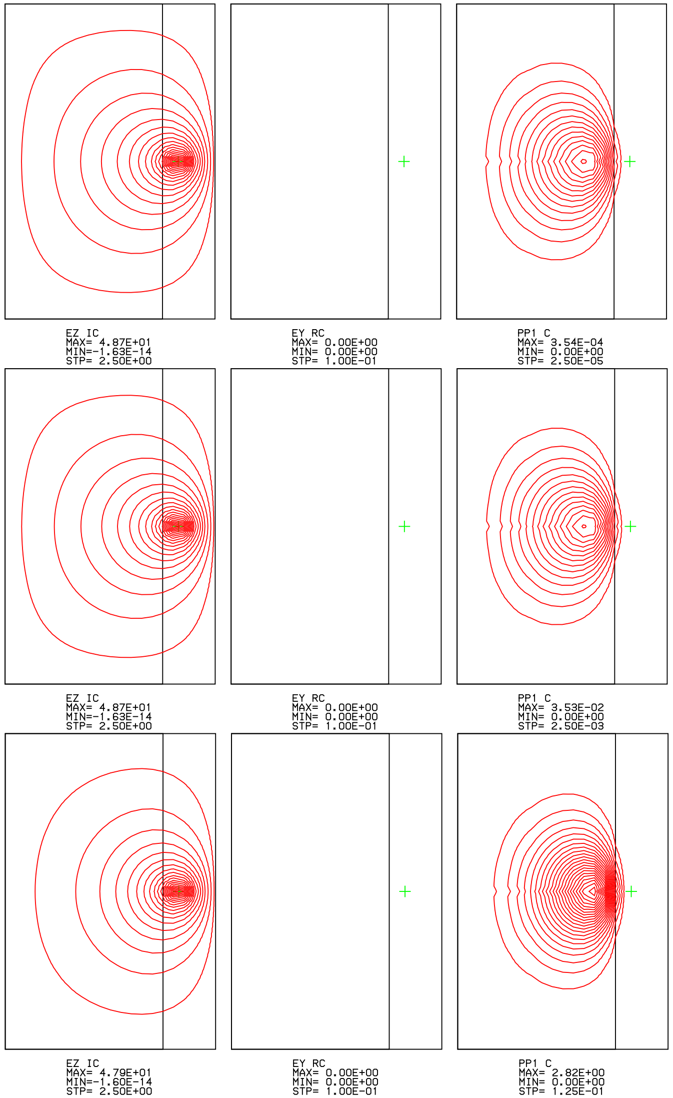
- **Parallel propagation**: $N_\perp = 0$ (Ordinary Helicon)

$$N_\parallel^2 = S - D \quad \Longrightarrow \quad \omega = \frac{|\omega_{\rm ce}|}{\omega_{\rm pe}^2} k_\parallel^2 c^2$$

ICP Plasma and NLD Plasma

ICP plasma

$\mathrm{Im}E_\phi$ $\mathrm{Re}E_z$ P_e



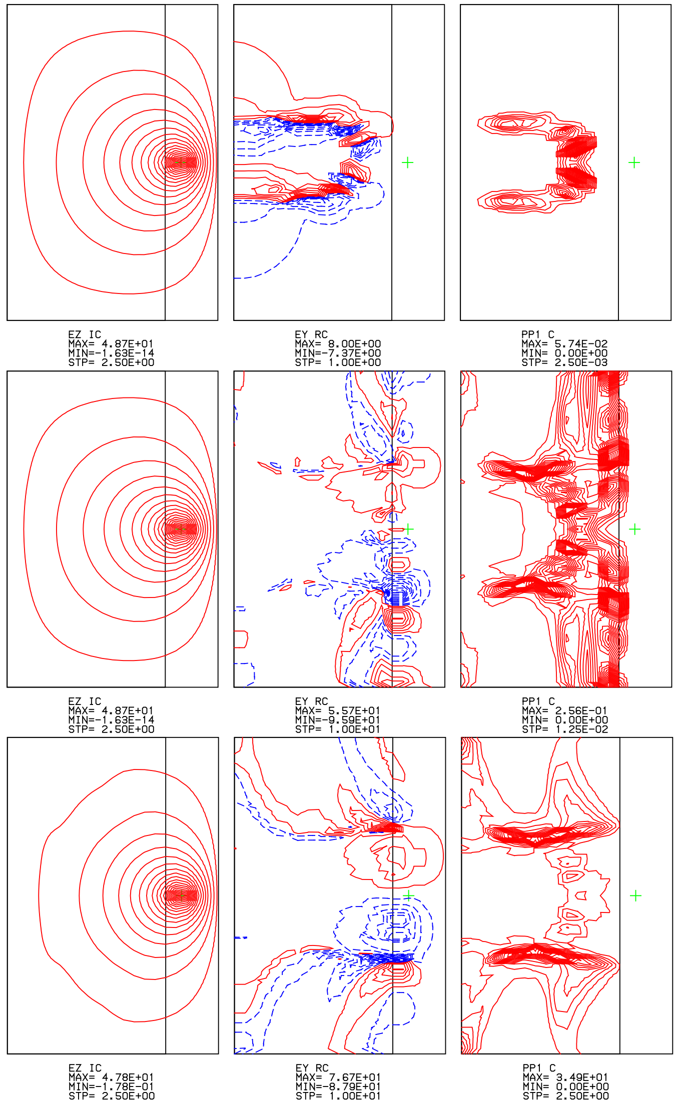
$10^{12}\,\mathrm{m}^{-3}$

$10^{14}\,\mathrm{m}^{-3}$

$10^{16}\,\mathrm{m}^{-3}$

NLD Plasma

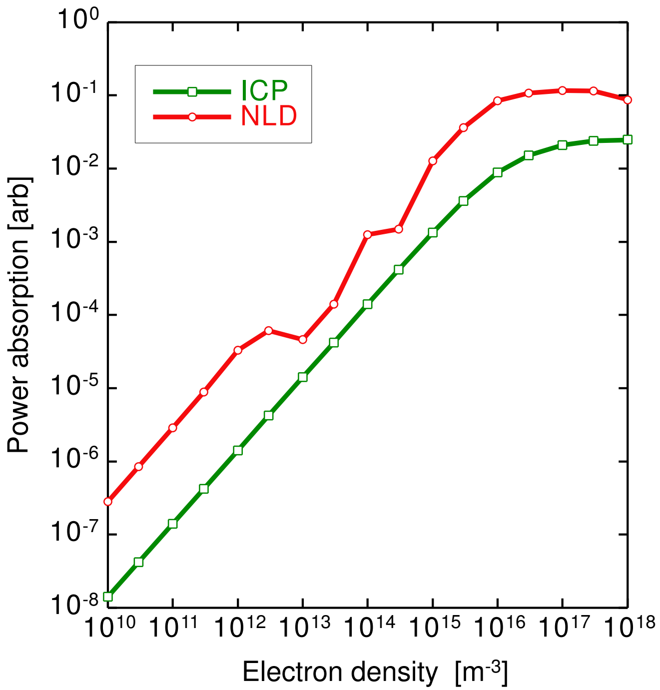
$\mathrm{Im}E_\phi$ $\mathrm{Re}E_z$ P_e



Absorption by Helicon Wave

- In a high density plasma, helicon waves are excited on the high field side of ECR.
- The excited helicon wave is absorbed near the LHR resonance $S - N_{\parallel}^2 = 0$.
- In the reegion away from ECR, the helicon wave propagates along the field line.

Electron density dependence of the antenna loading resistance



Plasma Transport Analysis

- **Plasma model**
  - Diffusive transport equation (collision dominant)
  - Time evolution of density n_s, temperature T_s and electrostatic potential ϕ ($s =$ electron, ion)
  - Axisymmetric two-dimensional analysis
  - RF enhanced ionization, RF heating, Collisional heat and particle transport
- **Numerical method**
  - **Spatial structure**: Finite element method
  - **Time evolution**: Full implicit method
  - At every time step, wave propagation is solved.
    (assuming that time evolution is much slower than the oasscillation of wave field)

Model Transport Equation

- **Time evolution of density n_s, temperature T_s, ES potential Φ**

$$\frac{\partial}{\partial t} n_s = \nabla \cdot \left[n_s \overleftrightarrow{\mu}_s \cdot \nabla\Phi + \overleftrightarrow{D}_s \cdot \nabla n_s \right] + \nu_{Is} n_s$$

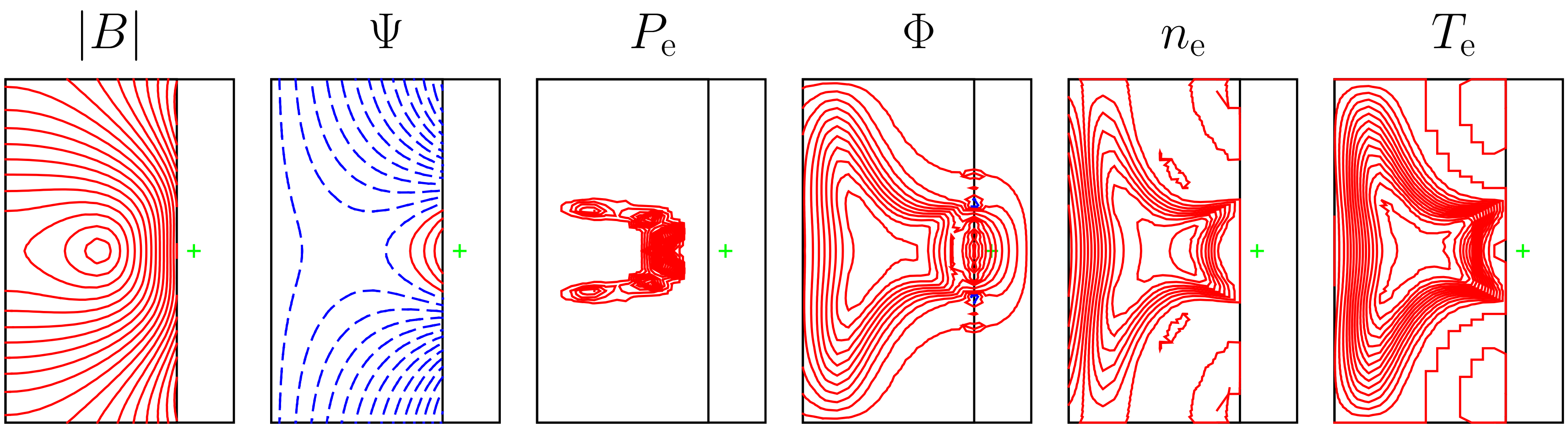
$$\frac{\partial}{\partial t}\frac{3}{2} n_s T_s = \nabla \cdot \left[\frac{5}{2} T_s \left(n_s \overleftrightarrow{\mu}_s \cdot \nabla\Phi + \overleftrightarrow{D}_s \cdot \nabla n_s \right) + \frac{3}{2} n_s \overleftrightarrow{\chi}_s \cdot \nabla T_s \right] - \frac{3}{2}\nu_{s\mathrm{n}} n_s (T_s - T_\mathrm{n}) - \frac{3}{2}\sum_{s'} \nu_{ss'} n_s (T_s - T_{s'}) + P_s$$

$$-\nabla^2\Phi = \frac{1}{\epsilon_0}\sum_s Z_s e n_s$$

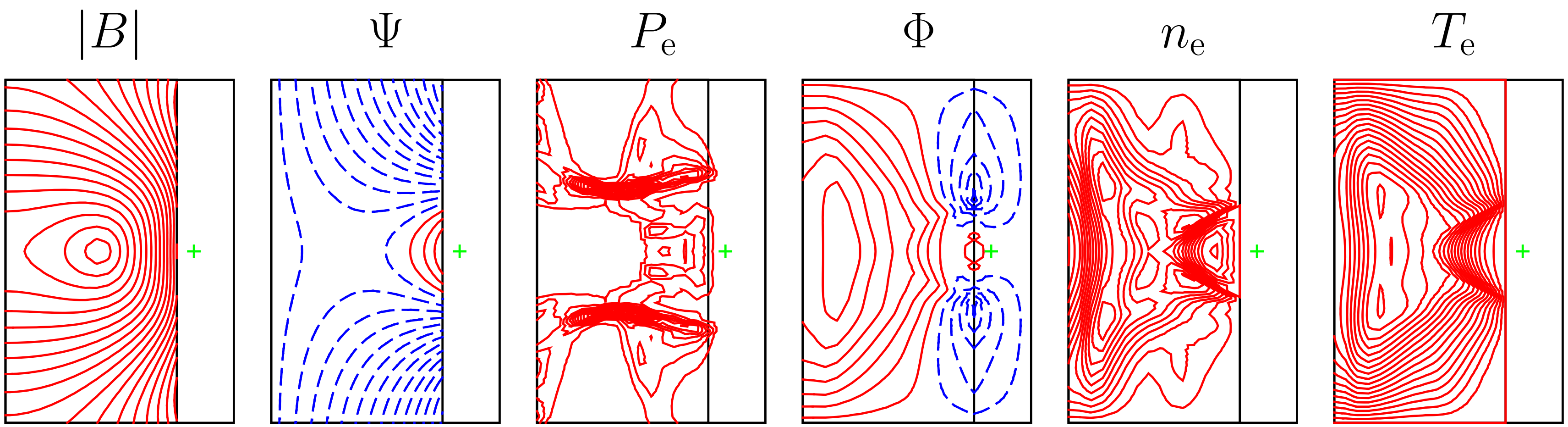
$\overleftrightarrow{\mu}_s$:	Mobility tensor	$\overleftrightarrow{D}_s$:	Partile diffusion tensor
$\overleftrightarrow{\chi}_s$:	Heat diffusion tensor	Z_s	Charge number of species s
P_s:	Heating power density	ν_{Is}:	Ionizaion frequency
$\nu_{s\mathrm{n}}$:	Collision frequency with neutrals	$\nu_{ss'}$:	Coulomb collision frequency

Preliminary Analysis of Plasma Production

- **Low density case** ($n_\mathrm{e} = 10^{12}\,\mathrm{m}^{-3}$)

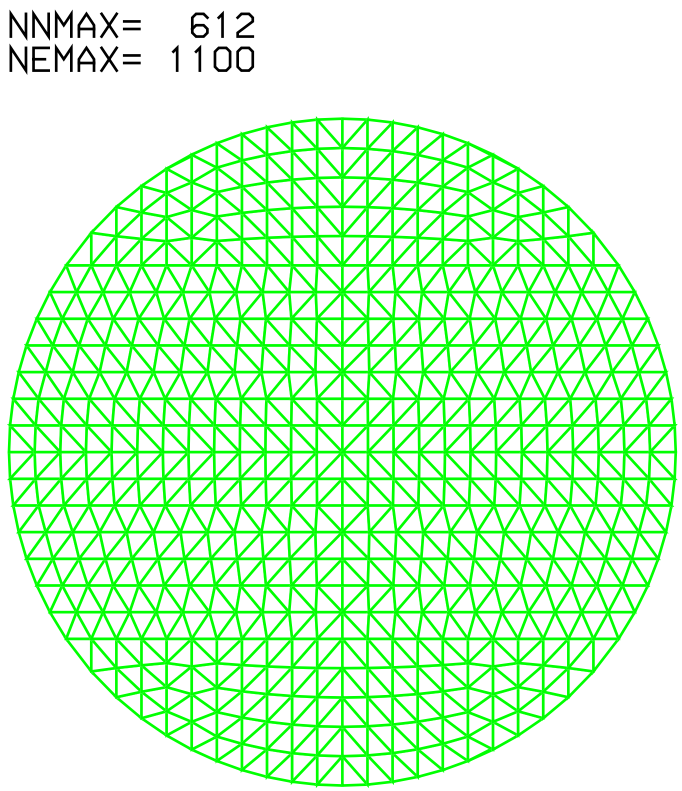


- **Higher density case** ($n_\mathrm{e} = 10^{14}\,\mathrm{m}^{-3}$)



PIC Simulation with Triangular Elements

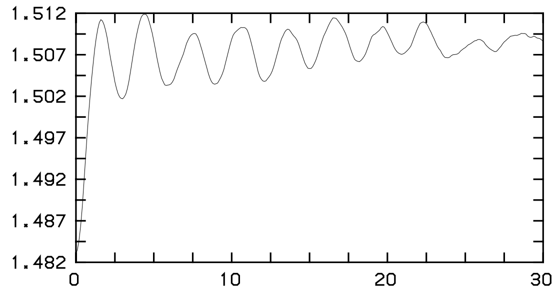
- **Why triangular mesh?**
  - Adaptive to any boundary shape
  - Mash accumulation at any location
- **However,**
  - How to find which element the particle is
  - How to do particle shaping



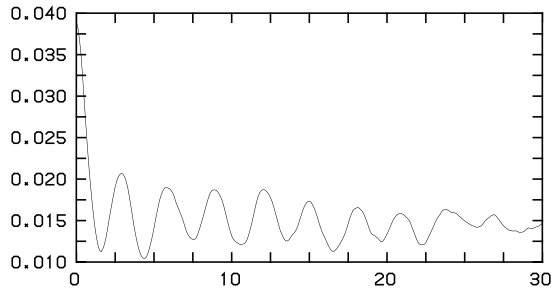
Very Preliminary Result

- $R = 25,\ \Delta = 2,\ \Delta t = 0.1,\ N_{\mathrm{p}} = 10000$
- **Time evolution**

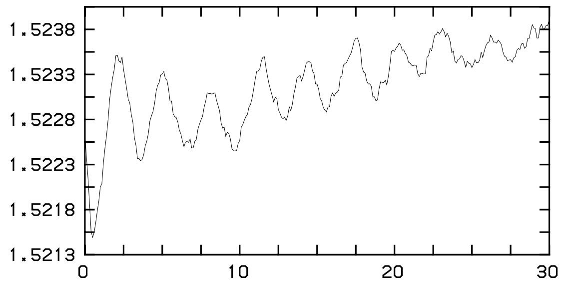
Kinetic Energy



Potential Energy

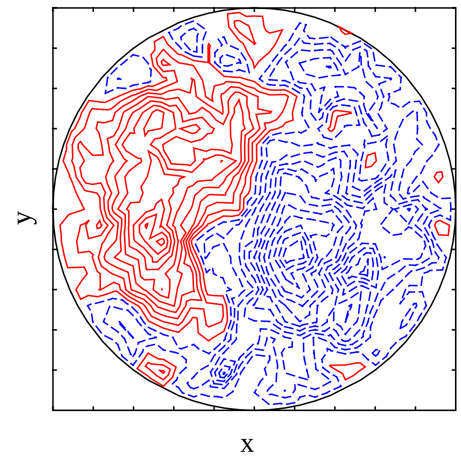


Total Energy

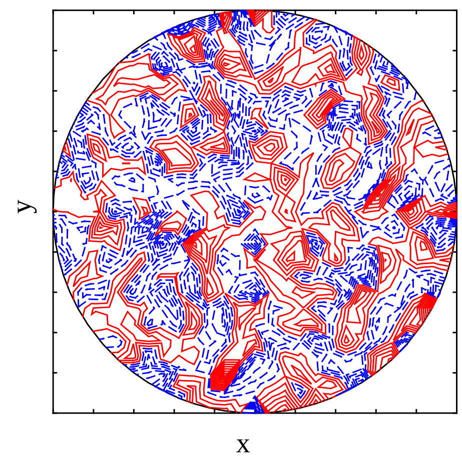


- **Spatial Profile**

Potential



Density



- Plasma oscillation can be described.

Summary

- **Wave propagation and absorption in the NLD configuration was numerically studied.**
- **In the case of n_e lower than $10^{12}\,\mathrm{m}^{-3}$, absorption at ECR is dominant.**
- **When $n_\mathrm{e} > 10^{12}\,\mathrm{m}^{-3}$, the excited helicon wave may contribute the plasma production.**
- **Quantitative analysis of collisionless absorption is left for future work.**
- **Self-consistent analysis of plasma production is ongoing.**
- **PIC simulation with triangular element is in very preliminary phase.**

Wave Equation

- **Maxwell equation for electric field** ($\overleftrightarrow{\epsilon}$: Dielectric tensorj

$$\boldsymbol{\nabla} \times \boldsymbol{\nabla} \times \boldsymbol{E} - \frac{\omega^2}{c^2} \overleftrightarrow{\epsilon} \cdot \boldsymbol{E} = \mathrm{i}\,\omega\mu_0 \boldsymbol{J}_{\mathrm{ext}}$$

- Vector potential $\boldsymbol{A}$ and scalar potential ϕ

$$\boldsymbol{E} = \mathrm{i}\,\omega\boldsymbol{A} - \boldsymbol{\nabla}\phi$$

- **Maxwell equation for potentials** with Coulomb gauge: $\boldsymbol{\nabla} \cdot \boldsymbol{A} = 0$

$$-\nabla^2 \boldsymbol{A} - \frac{\omega^2}{c^2} \overleftrightarrow{\epsilon} \cdot \left(\boldsymbol{A} + \frac{\mathrm{i}}{\omega} \boldsymbol{\nabla}\phi \right) = \mu_0 \boldsymbol{J}_{\mathrm{ext}}$$

$$-\boldsymbol{\nabla} \cdot \overleftrightarrow{\epsilon} \cdot (\boldsymbol{\nabla}\phi - \mathrm{i}\,\omega\boldsymbol{A}) = \frac{1}{\mathrm{i}\,\omega\epsilon_0} \boldsymbol{\nabla} \cdot \boldsymbol{J}_{\mathrm{ext}}$$

Boundary conditions and Excitation

- **Boundary conditions on conducting wall**
  Tangential component of $\boldsymbol{E} = 0$

$$\boldsymbol{A}_\mathrm{t} = 0, \qquad \phi = 0$$

- **Boundary conditions on axis** (cylindrical geometry: m = mode number)
  Finiteness of $\boldsymbol{B}$ and ρ

$$\begin{array}{ll} A_r = A_\theta = 0 & \text{for } m = 0 \\ A_r + \mathrm{i}\, m A_\theta = A_z = \phi = 0 & \text{for } m = \pm 1 \\ A_r = A_\theta = A_z = \phi = 0 & \text{for } |m| > 1 \end{array}$$

- **Excitation by antennas** (Electrostatic and electromagnetic coupling)
  - Loop antenna (fixed current profile)
  - Parallel antenna (fixed current profile)
  - Waveguide excitation (to be available)

Wave Ionization and Heating

- **Interaction with RF waves**
  - Increase of electron kinetic energy

    $\Longrightarrow$ Increase of collision with neutrals

    $\Longrightarrow$ Increase of Ionization

    $\Longrightarrow$ Increase of plasma density

  - Increase of electron thermal energy

    $\Longrightarrow$ Increase of electron temperature

Wave Ionization

- **Oscillation velocity of electrons (mass m_e, charge $-e$) due to electric field $\boldsymbol{E}$ with frequency ω**

$$\boldsymbol{v}_\mathrm{E} = -\frac{\mathrm{i}\, e}{m_\mathrm{e}\omega}\begin{pmatrix} \dfrac{\omega^2}{\omega^2-\omega_\mathrm{ce}^2} & \dfrac{\mathrm{i}\,\omega\omega_\mathrm{ce}}{\omega^2-\omega_\mathrm{ce}^2} & 0 \\ \dfrac{-\mathrm{i}\,\omega\omega_\mathrm{ce}}{\omega^2-\omega_\mathrm{ce}^2} & \dfrac{\omega^2}{\omega^2-\omega_\mathrm{ce}^2} & 0 \\ 0 & 0 & 1 \end{pmatrix}\cdot \boldsymbol{E}$$

  - This oscillation increases relative velocity to neutrals and contributes to increase effective temperature in ionization.

- **Estimation of necessary electric field strength**
  - Thermal velocity of electron of 1 eV: $7.3 \times 10^5\,\mathrm{m/s}$
  - For 13.56 MHz, $E \sim 350\,\mathrm{V/m}$ gives comparable $\boldsymbol{v}_{\mathrm{E}\parallel}$
  - Near ECR ($\omega \sim \omega_\mathrm{ce}$), $\boldsymbol{v}_{\mathrm{E}\perp}$ is strongly enhanced.

Wave Heating

- **Absorption Mechanism**
  - Wave-Particle Resonant Interaction: $v_\parallel = \dfrac{\omega}{k_\parallel}$
  - Collisional Interaction
    - Non-Resonant Interaction: $\dfrac{1}{2}\nu m v^2$
    - Approximation of Resonant Interaction (ECR)

- **Frequency**
  - Electron Cyclotron Resonance (ECR) $\Longrightarrow$ Increase of $\boldsymbol{v}_{\mathrm{E}\perp}$
  - Lower Hybrid Resonance (LHR) $\Longrightarrow$ Increase of $E_\perp$ $\Longrightarrow$ Increase of $\boldsymbol{v}_{\mathrm{E}\perp}$
  - Upper Hybrid Resonance (UHR) $\Longrightarrow$ Increase of $E_\perp$ $\Longrightarrow$ Increase of $\boldsymbol{v}_{\mathrm{E}\perp}$
  - Increase of $E_\parallel$ $\Longrightarrow$ Increase of $\boldsymbol{v}_{\mathrm{E}\parallel}$

Plasma Transport Model

- **Features**
  - Time Evolution of Density n_s, Temperature T_s and Electrostatic Potential ϕ
  - Collisional Fluid Equation
  - Two-Dimensional Analysis
  - Ionization, Heating, Heat Transfer through Collisions
- **Numerical Method** (TASK/TP)
  - Finite Element Method (A part of elements in WF)
  - Full-Implicit Time Advancing
  - Coupling with Wave Code

Model Transport Equation

- **Time evolution of density n_s, temperature T_s and ES potential ϕ**

$$\frac{\partial}{\partial t} n_s = \nabla \cdot \left[n_s \overleftrightarrow{\mu}_s \cdot \nabla \Phi + \overleftrightarrow{D}_s \cdot \nabla n_s \right] + \nu_{Is} n_s$$

$$\frac{\partial}{\partial t} \frac{3}{2} n_s T_s = \nabla \cdot \left[\frac{5}{2} T_s \left(n_s \overleftrightarrow{\mu}_s \cdot \nabla \Phi + \overleftrightarrow{D}_s \cdot \nabla n_s \right) + \frac{3}{2} n_s \overleftrightarrow{\chi}_s \cdot \nabla T_s \right]$$

$$- \frac{3}{2} \nu_{\mathrm{sn}} n_s (T_s - T_{\mathrm{n}}) - \frac{3}{2} \sum_{s'} \nu_{ss'} n_s (T_s - T_{s'}) + P_s$$

$$-\nabla^2 \Phi = \frac{1}{\epsilon_0} \sum_s Z_s e n_s$$

$\overleftrightarrow{\mu}_s$	Mobility tensor	$\overleftrightarrow{D}_s$	Particle diffusion tensor
$\overleftrightarrow{\chi}_s$	Thermal diffusion tensor	Z_s	Charge number of speces s
P_s	Heating power density	ν_{Is}	Ionization frequency
ν_{sn}	Collision frequency with neutrals	$\nu_{ss'}$	Coulomb collision frequency

Transport Coefficients

- Effective temperature including electron oscillation due to wave electric field

$$\frac{3}{2}T_{\rm eff} = \frac{3}{2}T_{\rm e} + \frac{1}{2}m_{\rm e}|\boldsymbol{v}_{\rm E}|^2$$

- Ionization frequency: Analytic formula + Effective temperature

$$\nu_{\rm I} = n_{\rm n}\frac{10^{-11}\left(T_{\rm eff}/E_{\rm I}\right)^{1/2}}{{E_{\rm I}}^{3/2}\left(6+T_{\rm eff}/E_{\rm I}\right)}\exp\left(-\frac{E_{\rm I}}{T_{\rm eff}}\right)$$

- Classical transport coefficients

$$\mu_\parallel = \frac{Z_s e}{m\nu} \qquad \mu_\perp = \frac{1}{1+\omega_{\rm c}^2/\nu^2}\,\mu_\parallel$$

$$D_\parallel = \frac{T}{m\nu} \qquad D_\perp = \frac{1}{1+\omega_{\rm c}^2/\nu^2}\,D_\parallel$$

$$\chi_\parallel = 3.16\frac{T}{m\nu} \qquad \chi_\perp = 4.66\frac{1}{1+\omega_{\rm c}^2/\nu^2}\,\frac{T}{m\nu}$$

- Collision frequency ν :
  Sum of collision with neutrals ($\nu_{\rm n} = 0.88 n_{\rm n}\sigma_{\rm n}v_{\rm T}$) and Coulomb collisions

Boundary Conditions

- **Boundary condition on the wall**
  - Density gradient scale length $\sim$ Mean free path
    (Particle flux due to density gradient: $-D\boldsymbol{\nabla}n \sim v_{\rm th}n$)
  - Temperature: Fixed to 0.03 eV
  - Potential:

Fixed to 0	(Conducting wall)
Zero current	(Insulated wall)
Potential proportional to electric charge	(Insulated wall)
Fixed to Finite value	(Biased wall)

- **Initial condition**
  - Density, Temperature: Flat profile
  - Potential: 0
  - Neutral gas pressure $P_{\rm n}$: Flat profile, Fixed in time