

Kinetic Global Analysis of Alfvén Eigenmodes in Toroidal Plasmas

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Abstract. Systematic study on low to medium n (toroidal mode number) Alfvén eigenmodes (AE) in tokamaks has been carried out. Linear stability of AE in the presence of energetic ions was studied using the kinetic full-wave code TASK/WM. We have reproduced the destabilizing effect of toroidal co-rotation on TAE for JT-60U parameters. We have found the existence of reversed-shear-induced Alfvén eigenmode (RSAE) which localizes near the q minimum in a reversed magnetic shear configuration. Two kinds of mode structures are identified for energetic particle mode (EPM) below the TAE frequency gap.

Linear Stability Analysis of Alfvén Eigenmode

- **MHD Analysis** (Ideal, Resistive)
- **MHD including Kinetic Effect** (perturbative)
 - Eigen function from MHD analysis, Growth rate including kinetic effects
- **Kinetic Analysis** (Electron thermal motion, Ion gyromotion, Drift motion)
 - PENN code (Jaun, Alfvén Lab)
 - TASK/WM (Fukuyama)
- **Ballooning Expansion** (High n mode)
 - HINST (Gorelenkov, Cheng)
 - 2D-WKB (Vlad, Chen, Zonka)
- **3D Full Wave Code: TASK/WM**
 - Magnetic surface coordinates from MHD Equilibrium Analysis
 - Boundary value problem of Maxwell's equation. Dielectric tensor
 - Fourier mode expansion in poloidal and toroidal direction, FDM in radius
 - Looking for complex eigen frequency which maximize the integral of wave field.

3D Full Wave Code: TASK/WM

- **Magnetic Flux Coordinates:** (ψ, θ, φ)
 - **Non-orthogonal system**
- **Maxwell's Equation** for stationary wave electric field \mathbf{E}

$$\nabla \times \nabla \times \mathbf{E} = \frac{\omega^2}{c^2} \overleftrightarrow{\epsilon} \cdot \mathbf{E} + i\omega\mu_0 \mathbf{j}_{\text{ext}}$$

- $\overleftrightarrow{\epsilon}$: **Dielectric tensor with kinetic effects**
- **Fourier Expansion** in poloidal and toroidal directions
 - **Exact parallel wave number:** $k_{\parallel}^{m,n} = (mB^{\theta} + nB^{\varphi})/B$
- **Destabilization by Energetic Ion** include in $\overleftrightarrow{\epsilon}$

- **Drift kinetic equation**

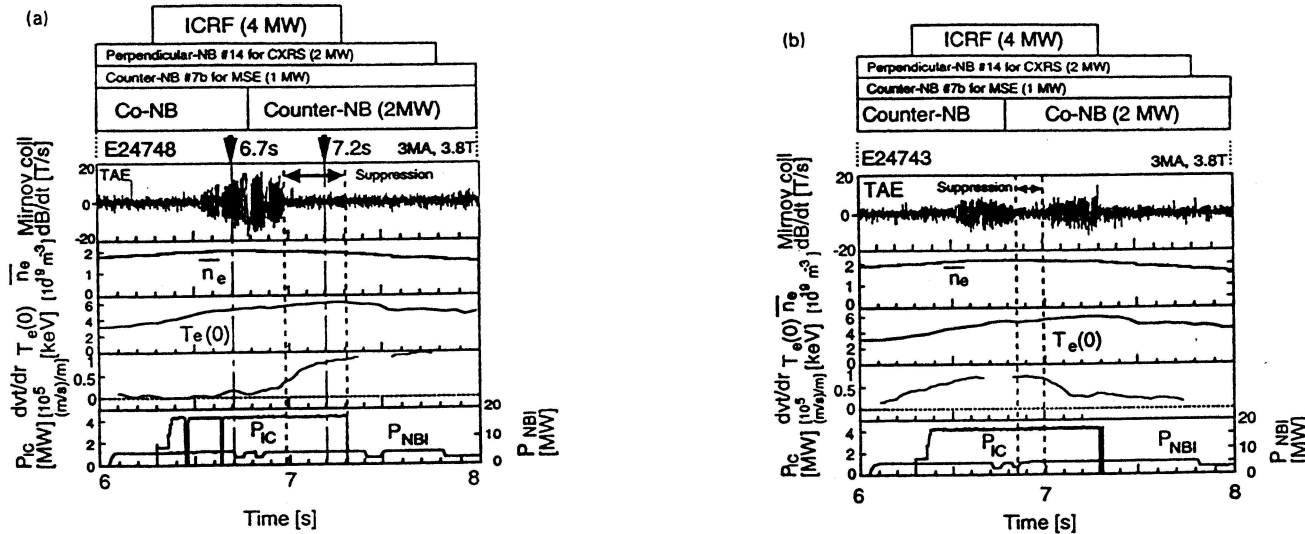
$$\left[\frac{\partial}{\partial t} + v_{\parallel} \nabla_{\parallel} + (\mathbf{v}_d + \mathbf{v}_E) \cdot \nabla + \frac{e_{\alpha}}{m_{\alpha}} (v_{\parallel} E_{\parallel} + \mathbf{v}_d \cdot \mathbf{E}) \frac{\partial}{\partial \varepsilon} \right] f_{\alpha} = 0$$

- **Eigenvalue Problem** for complex wave frequency
 - **Maximize wave amplitude** for finite excitation proportional to nne

Effect of Toroidal Plasma Rotation

- **Experimental Results on JT-60U:** Saigusa et al., NF 37 (1997) 1559.

Counter-NBI: Stabilization Co-NBI: Destabilization



- **Dispersion relation:** $k_{||m} = (m + nq)/qR$

$$\left(k_{||m}^2 - \frac{(\omega - k_{||m}u)^2}{v_A^2} \right) \left(k_{||m+1}^2 - \frac{(\omega - k_{||m+1}u)^2}{v_A^2} \right) - \epsilon^2 \frac{(\omega - k_{||m}u)^2 (\omega - k_{||m+1}u)^2}{v_A^4} = 0$$

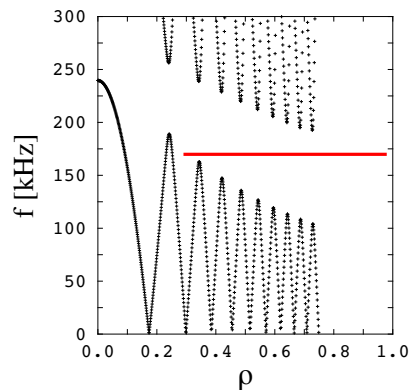
- **Safety factor at TAE gap:** $q = -\frac{m + 1/2}{n} - \frac{1}{2n} \frac{u}{v_A}$: linear u -dependence

- **TAE gap frequency:** $\omega = \frac{v_A}{2qR} \left(1 - \frac{u^2}{v_A^2} \right)$: dominant u -dep. comes from q

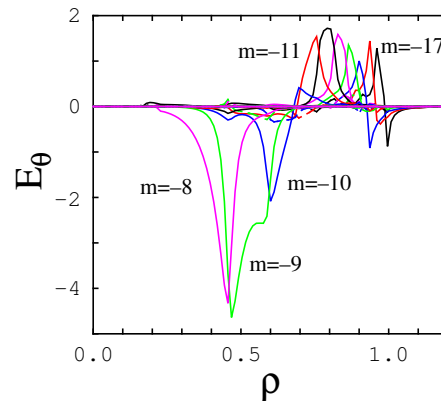
Effect of Rotation on $n = 7$ mode

- Sensitive dependence on poloidal mode number
- $m : -17.. -3$, $f = 223$ kHz : Stabilizing for $u > 0$ (contradict with exp.)

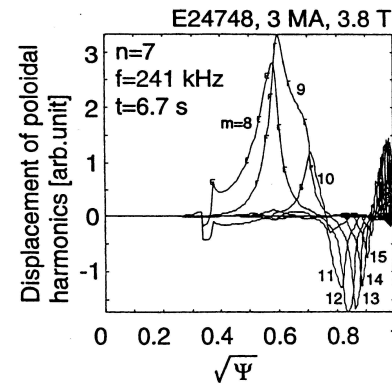
Alfvén resonance



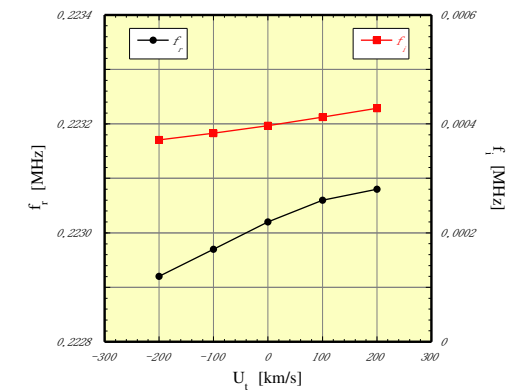
Radial structure



Nova-K result

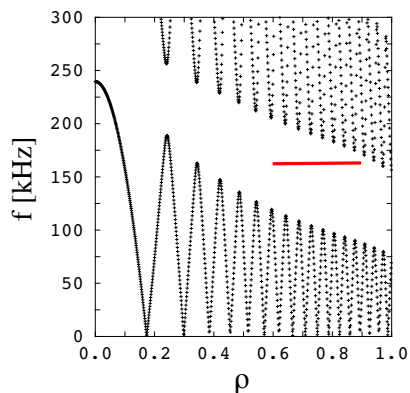


Eigen frequency

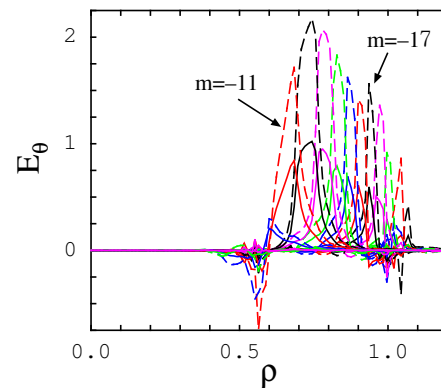


- $m : -21.. -7$, $f = 238$ kHz : Destabilizing for $u > 0$ (agree with exp.)

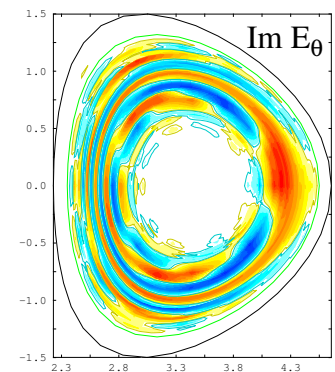
Alfvén resonance



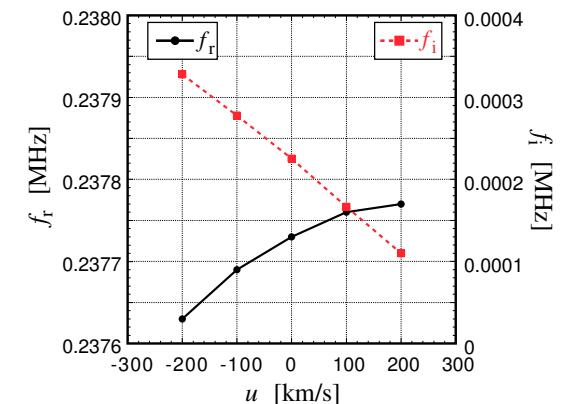
Radial structure



Poloidal structure



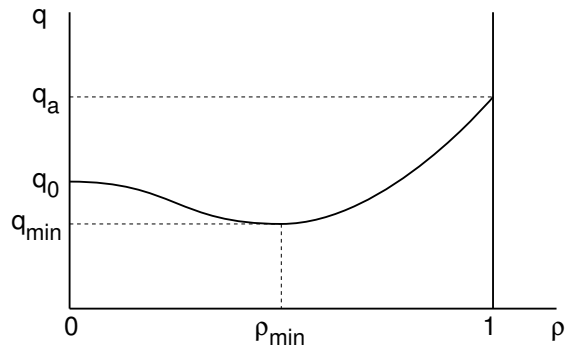
Eigen frequency



Analysis of TAE in Reversed Shear Configuration

q_{\min} Dependence of Eigenmode Frequency

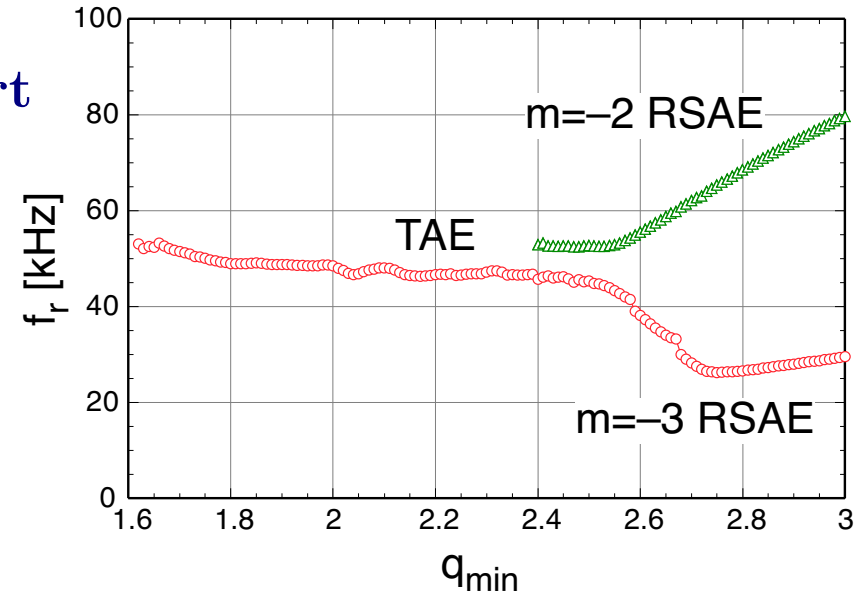
Assumed q profile



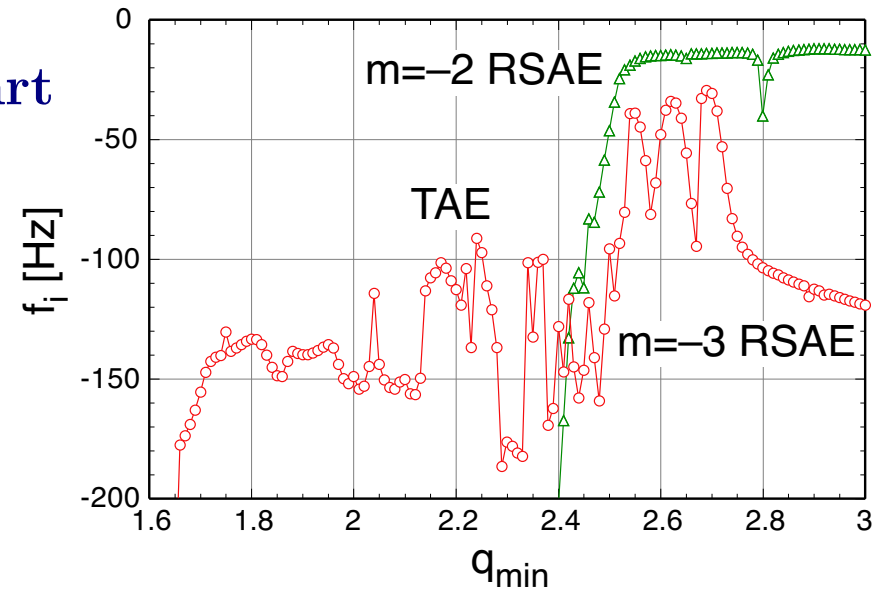
Plasma Parameters

R_0	3 m
a	1 m
B_0	3 T
$n_e(0)$	10^{20} m^{-3}
$T(0)$	3 keV
$q(0)$	3
$q(a)$	5
ρ_{\min}	0.5
n	1
Flat density profile	

Real part

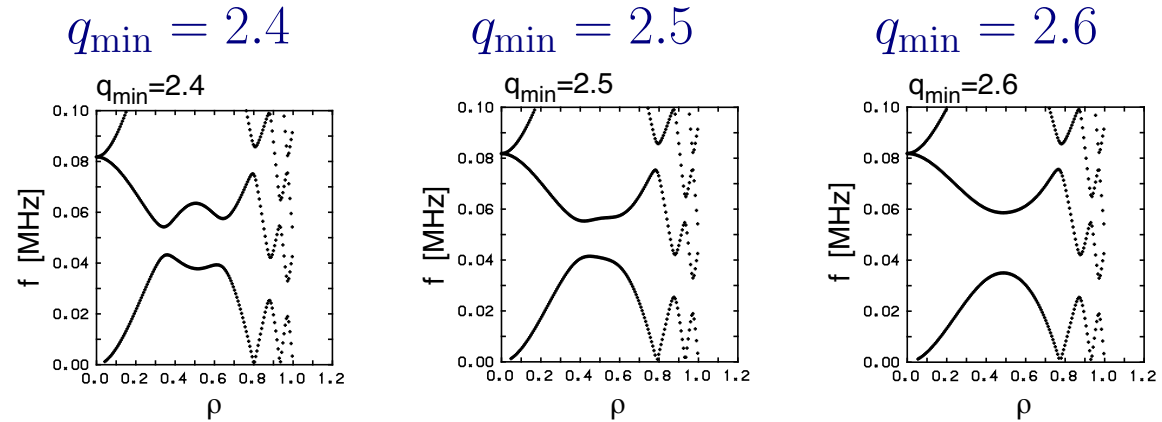


Imag part

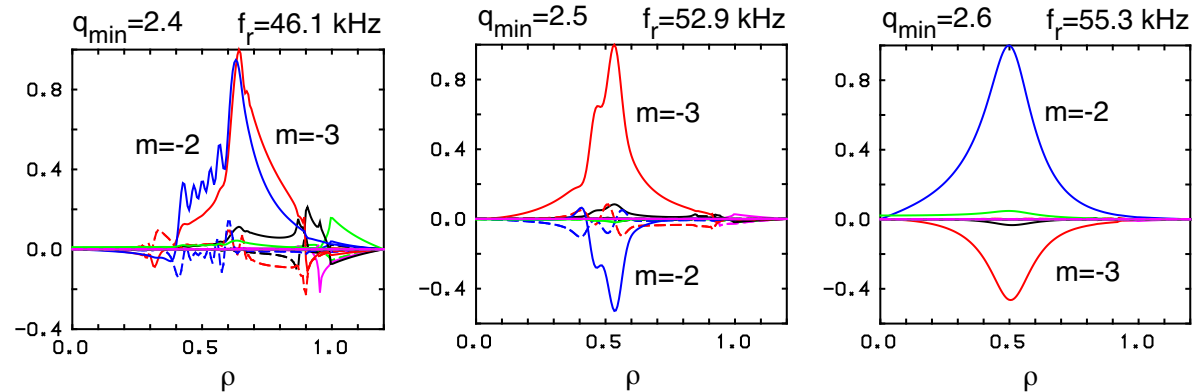


Eigenmode Structure

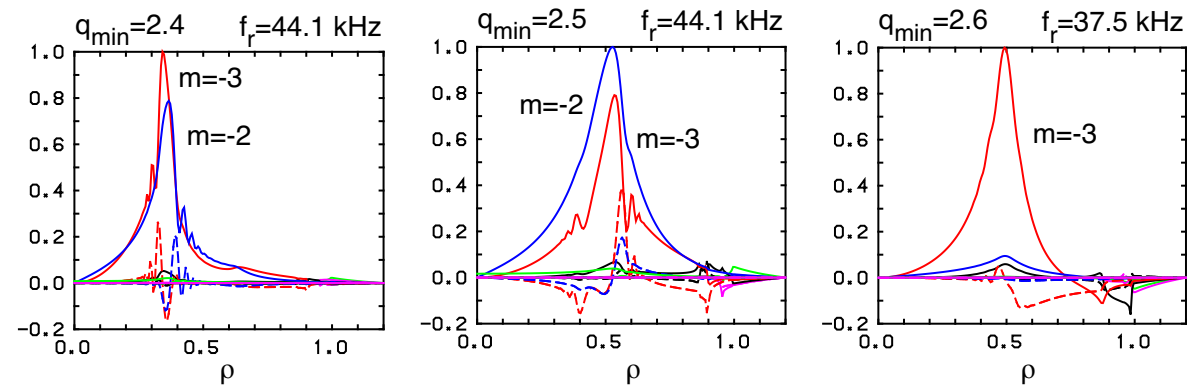
Alfvén resonance



Higher freq.



Lower freq.



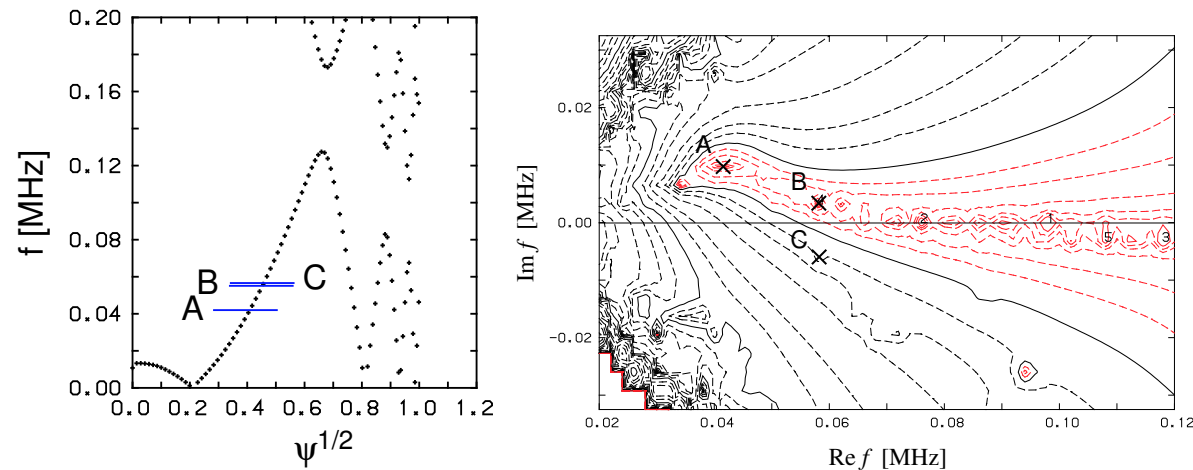
TAEs

Double TAE

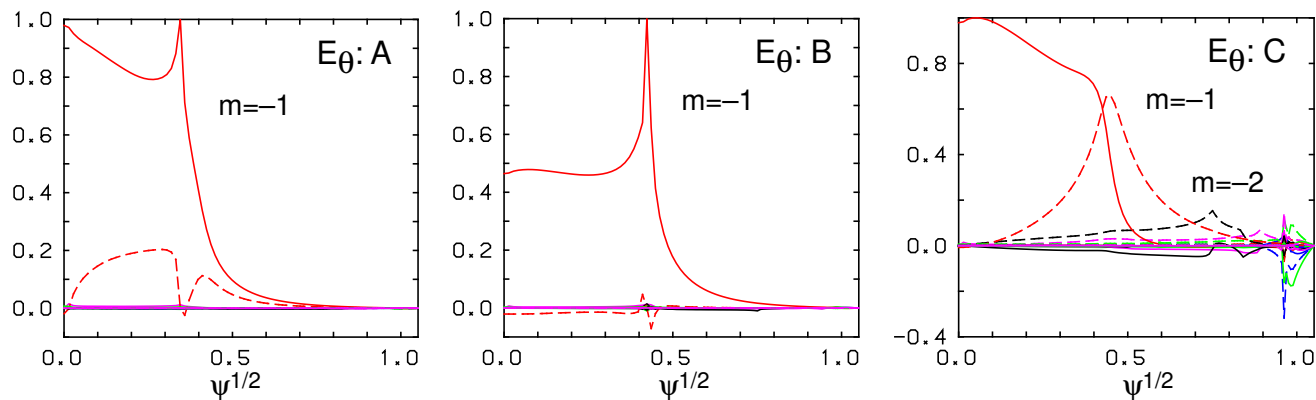
RSAE

Energetic Particle Mode (EPM)

- Energetic ions can excite EPM with frequency below the TAE frequency gap.
- With β of energetic ions about 0.5%, ω_A and contour of wave amplitude



- Eigenmode structure



A: $f_r = 41.8$ kHz

A: $f_i = 8.1$ kHz

B: $f_r = 57.7$ kHz

B: $f_i = 3.6$ kHz

C: $f_r = 58.0$ kHz

C: $f_i = -6.0$ kHz

Summary

- The toroidal rotation changes the TAE frequency mainly through the change of gap position and q value.
- Destabilization by co-rotation agrees with experimental observation in JT-60U, though the stability is sensitive to the Alfvén resonance near the plasma surface.
- Reversed magnetic configuration supports RSAE with single dominant poloidal mode number. The eigen frequency is close to the lower bound of the frequency gap and increases quickly with the decrease of q_{\min} .
- The mode structure of the EPM/RTAE below the gap frequency was studied. Two types of modes can be destabilized by the energetic ions; strongly damped TAE mode and weakly damped shear Alfvén mode.