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Analysis of EC Wave Using Beam Tracing Method

A. Fukuyama and S. Nishina
Department of Nuclear Engineering,
Kyoto University, Kyoto 606-8501, Japan

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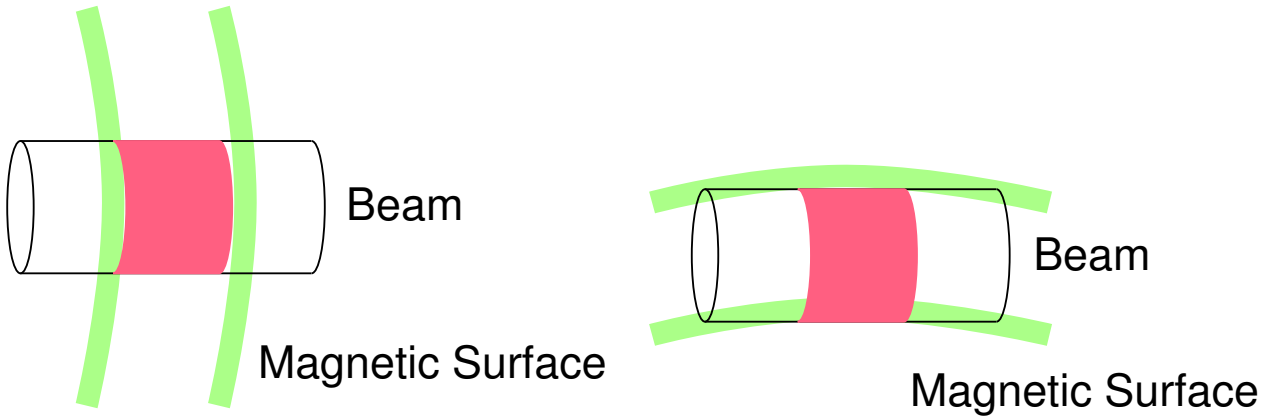
Motivation

- **Suppression of Tearing Modes by ECCD**
  - Necessary power depends on the driven current profile
- **Evaluation of Current Profile Width**
  - **Doppler broadening**: decay length
  - **Beam radius**: focusing and defraction



- **Limitation of Ray Tracing** (Geometrical Optics)
  - **Plane wave**: Beam size d is sufficiently large
  - **Finite size beam**: Effect of defraction: $L \ll d^2/\lambda$

Beam Tracing Method

- **Ref: e.g., G. V. Pereverzev, Phys. Plasmas 4 (1998) 3529**
- **Expansion parameter**: $\delta = \sqrt{c/\omega L} \ll 1$
- **Beam electric field** : Gaussian beam with $m = 0, n = 0$

$$\boldsymbol{E}(\boldsymbol{r}) = \mathrm{Re}\left[\sum_{mn} C_{mn}(\delta^2\boldsymbol{r})\boldsymbol{e}(\delta^2\boldsymbol{r})H_m(\delta\xi_1)H_n(\delta\xi_2)\,\mathrm{e}^{\,\mathrm{i}\,s(\boldsymbol{r})-\phi(\boldsymbol{r})}\right]$$

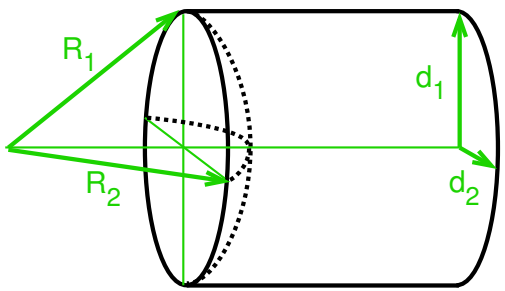
  - **Amplitude**: C_{mn}, **Polarization**: $\boldsymbol{e}$, **Beam shape**: Hermite polynomial, H_n

- **Phase**: $s(\boldsymbol{r}) + \mathrm{i}\,\phi(\boldsymbol{r})$

$$s(\boldsymbol{r}) = s_0(\tau) + k_\alpha^0(\tau)[r^\alpha - r_0^\alpha(\tau)] + \frac{1}{2}s_{\alpha\beta}[r^\alpha - r_0^\alpha(\tau)][r^\beta - r_0^\beta(\tau)]$$

$$\phi(\tau) = \frac{1}{2}\phi_{\alpha\beta}[r^\alpha - r_0^\alpha(\tau)][r^\beta - r_0^\beta(\tau)]$$

  - **Position** of beam axis : $\boldsymbol{r}_0$, **Wave number** on beam axis: $\boldsymbol{k}^0$
  - **Curvature radius** of isophase surface: $R_\alpha = \dfrac{1}{\lambda s_{\alpha\alpha}}$
  - **Beam radius**: $d_\alpha = \sqrt{\dfrac{2}{\phi_{\alpha\alpha}}}$



Beam Propagation Equation

- **Solvable condition** of Maxwell's equation with beam electric field

$$\frac{\mathrm{d}r_0^\alpha}{\mathrm{d}\tau} = \frac{\partial K}{\partial k_\alpha}$$

$$\frac{\mathrm{d}k_\alpha^0}{\mathrm{d}\tau} = -\frac{\partial K}{\partial r^\alpha}$$

$$\frac{\mathrm{d}s_{\alpha\beta}}{\mathrm{d}\tau} = -\frac{\partial^2 K}{\partial r^\alpha \partial r^\beta} - \frac{\partial^2 K}{\partial r^\beta \partial k_\gamma} s_{\alpha\gamma} - \frac{\partial^2 K}{\partial r^\alpha \partial k_\gamma} s_{\beta\gamma} - \frac{\partial^2 K}{\partial k^\gamma \partial k^\delta} s_{\alpha\gamma} s_{\beta\delta} + \frac{\partial^2 K}{\partial k^\gamma \partial k^\delta} \phi_{\alpha\gamma} \phi_{\beta\delta}$$

$$\frac{\mathrm{d}\phi_{\alpha\beta}}{\mathrm{d}\tau} = -\left(\frac{\partial^2 K}{\partial r^\alpha \partial k^\gamma} + \frac{\partial^2 K}{\partial k^\gamma \partial k_\delta} s_{\alpha\delta}\right)\phi_{\beta\gamma} - \left(\frac{\partial^2 K}{\partial r^\beta \partial k^\gamma} + \frac{\partial^2 K}{\partial k^\gamma \partial k_\delta} s_{\beta\delta}\right)\phi_{\alpha\gamma}$$

- By integrating this set of 18 ordinary differential equations, we obtain trace of the beam axis, wave number on the beam axis, curvature of isophase surface, and beam size.

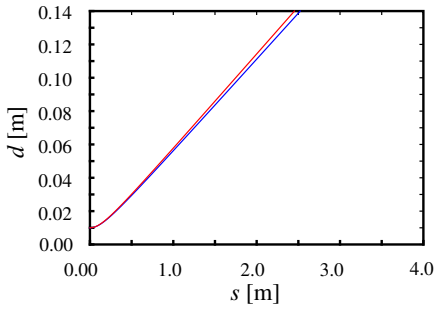
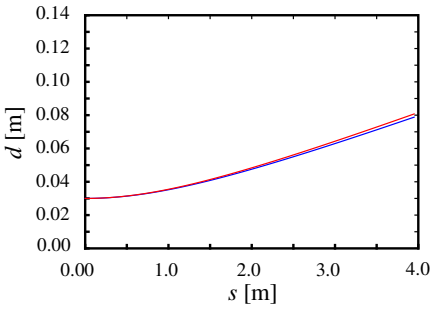
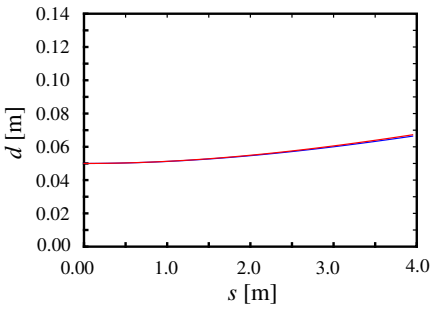
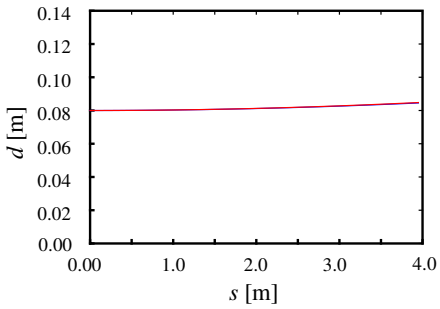
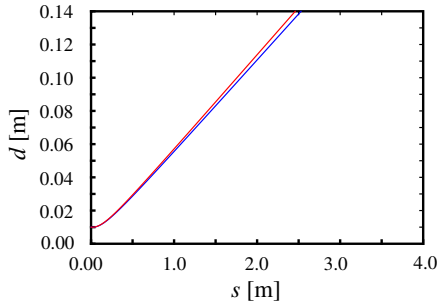
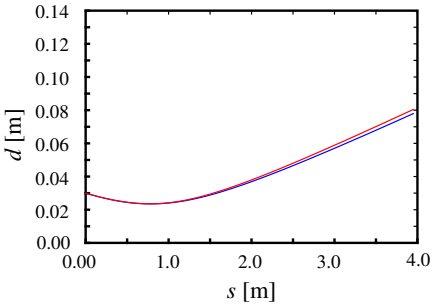
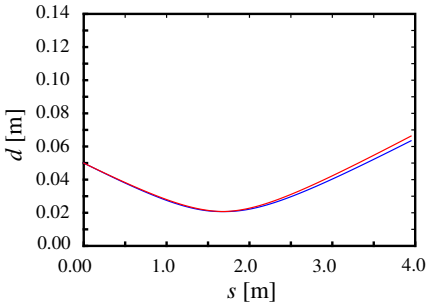
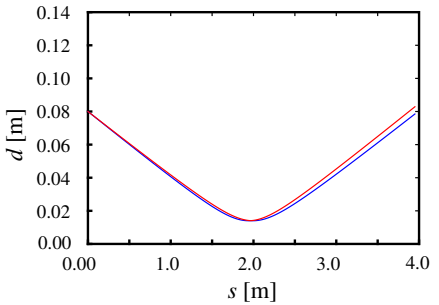
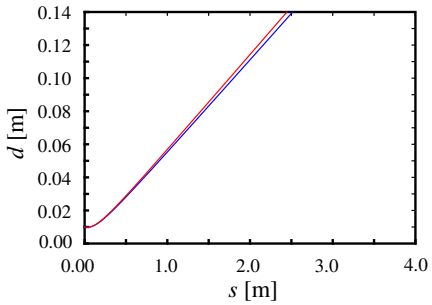
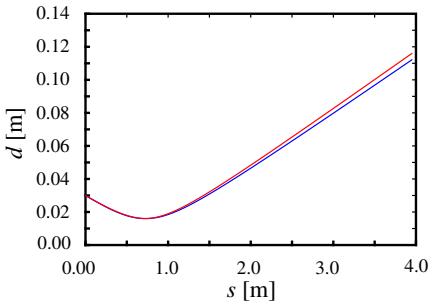
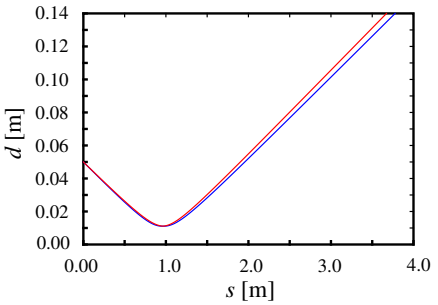
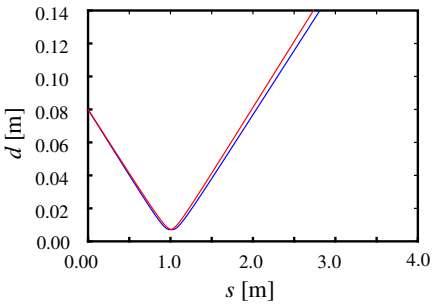
- **Equation for the wave amplitude**: C_{mn}

$$\boldsymbol{\nabla} \cdot \left(\boldsymbol{v}_{\mathrm{g}0} |C_{mn}|^2\right) = -2\left(\gamma |C_{mn}|^2\right)$$

Group velocity: $\boldsymbol{v}_{\mathrm{g}0}$, Damping rate: $\gamma \equiv (\boldsymbol{e}^* \cdot \overleftrightarrow{\epsilon}_{\mathrm{A}} \cdot \boldsymbol{e})/(\partial K/\partial\omega)$

Beam Tracing in a Uniform Plasma

- **170 GHz, Ordinary Mode, Perpendicular Injection**

$R_{c,ini}$	d_{ini} = 0.01 m	d_{ini} = 0.03 m	d_{ini} = 0.05 m	d_{ini} = 0.08 m
∞				
2 m				
1 m				

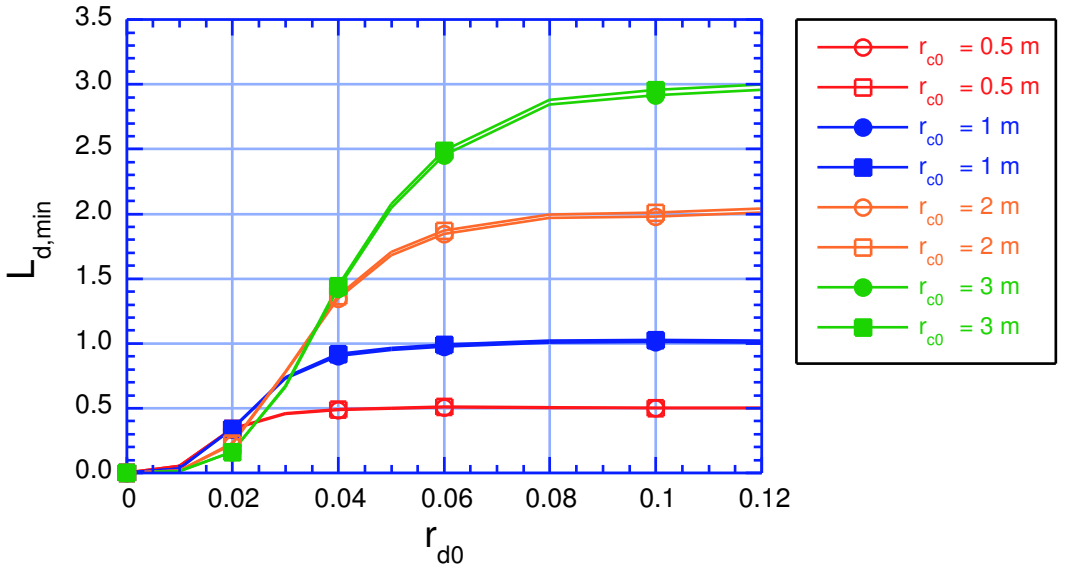
Dependence of Initial Beam Radius, d_{ini}

- **Focusing length**: $L_{d,min}$

  beam length to the minimum d

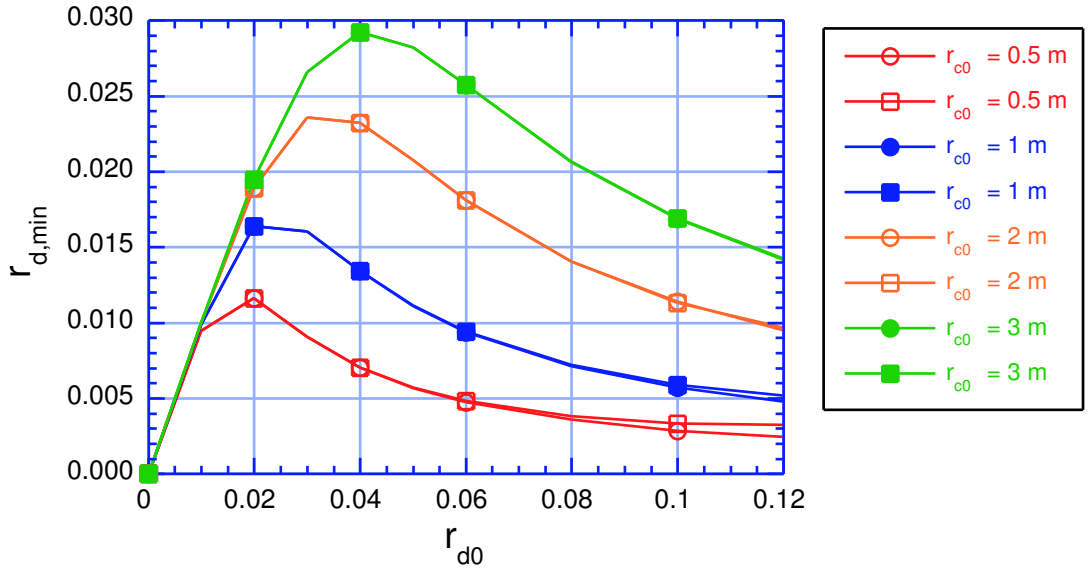
$$L_{d,min} \sim R_c$$

for $d_{ini} > \sqrt{R_c \lambda}$

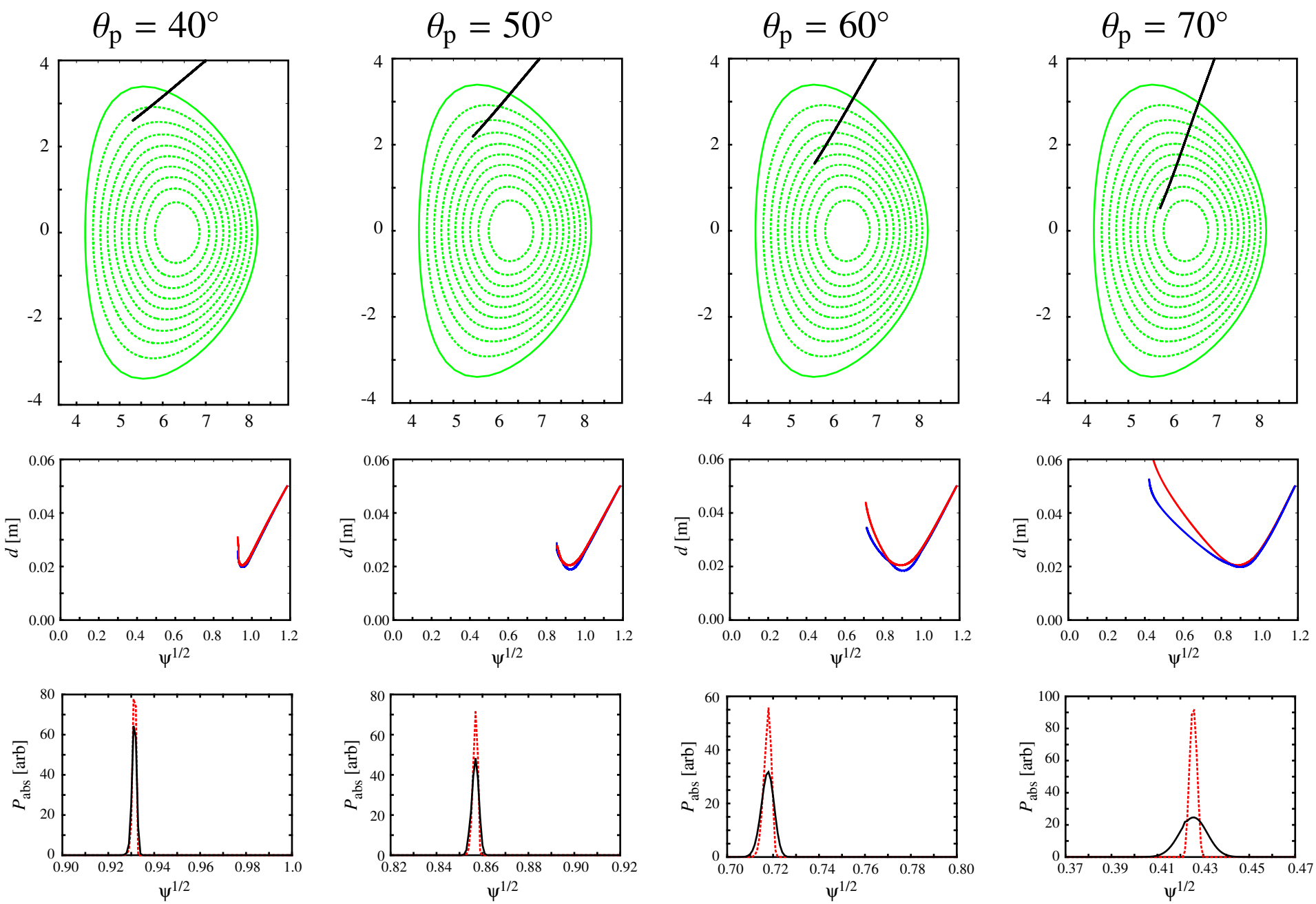


- **Minimum beam radius**: d_{min}

$$d_{min} \sim \frac{\lambda}{\pi d_{ini}} R_c$$



Beam Tracing in ITER-FEAT Plasma: $R_c = 2$ m, $d_{ini} = 0.05$ m



Beam Tracing in ITER-FEAT Plasma

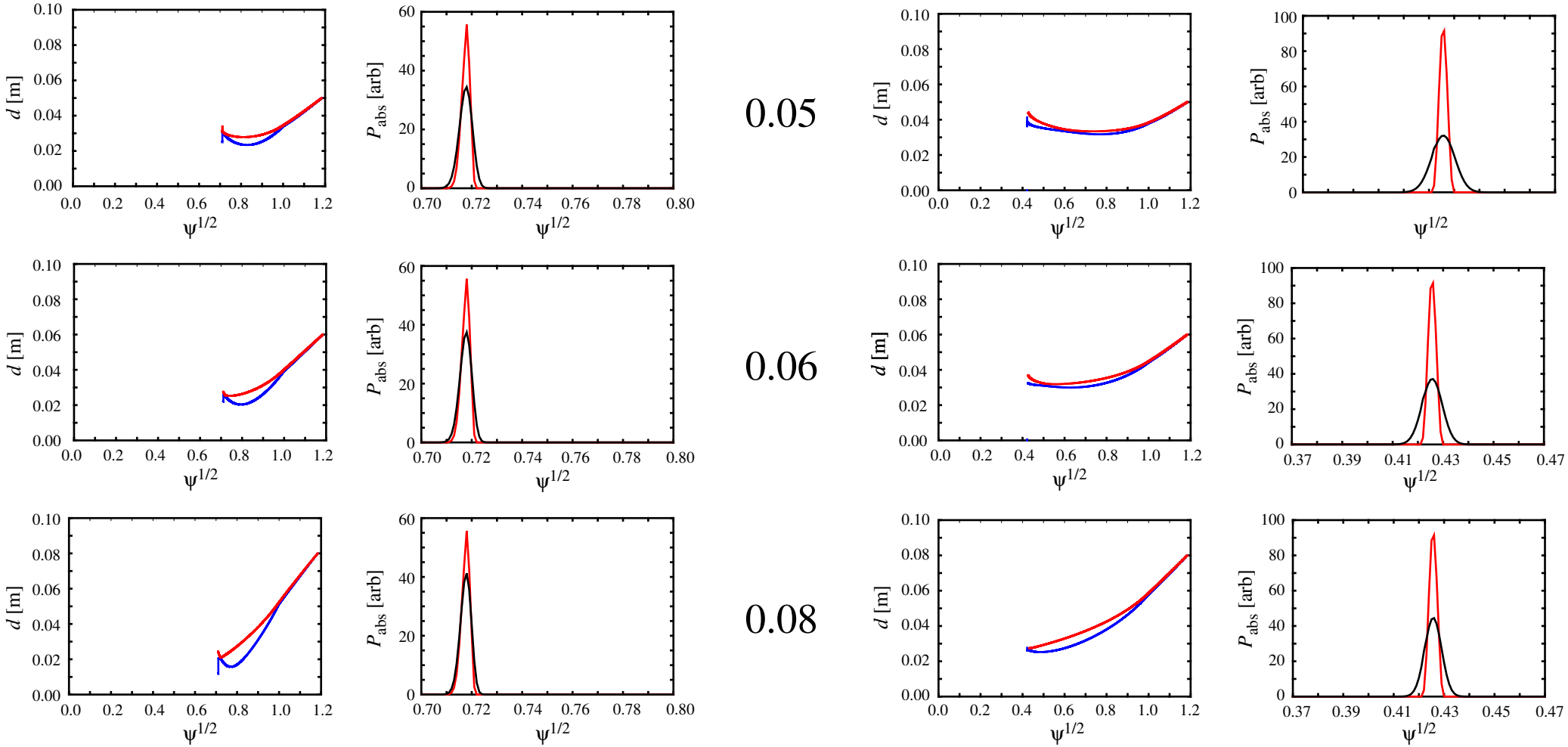
$\theta_p = 60°$

$R_c = 3\,\mathrm{m}$

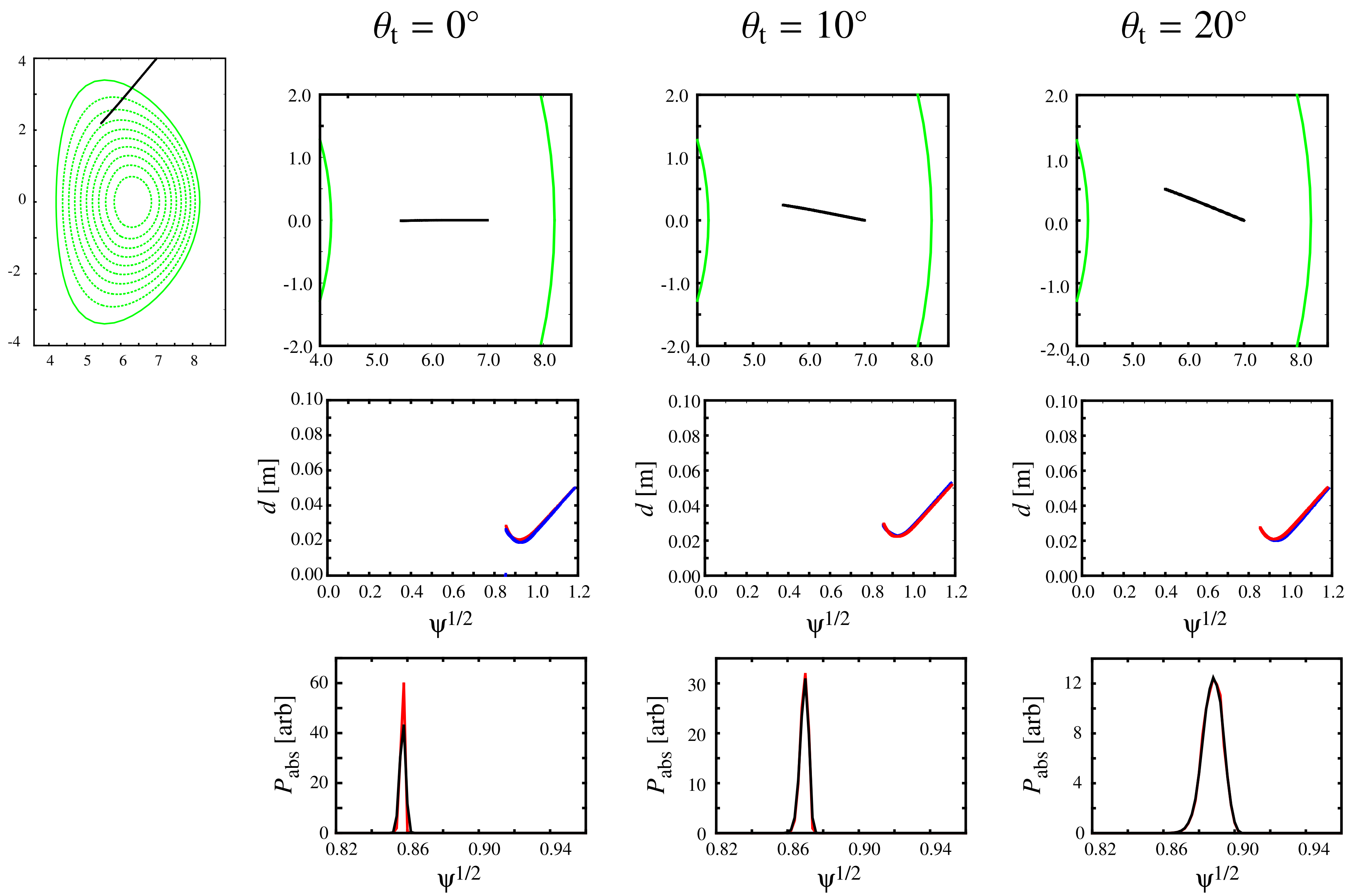
d_{ini} [m]

$\theta_p = 70°$

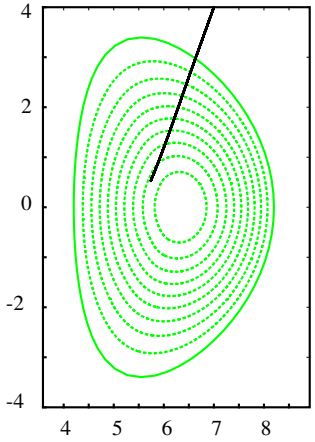
$R_c = 4\,\mathrm{m}$



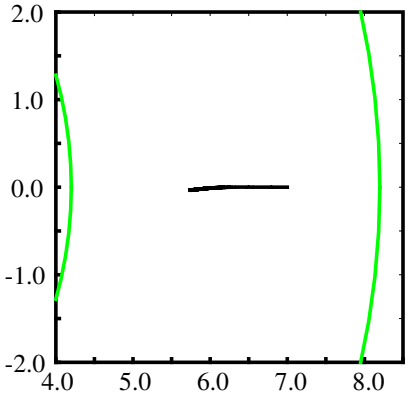
Beam Tracing in ITER-FEAT Plasma: R_c = 2 m, d_{ini} = 0.05 m

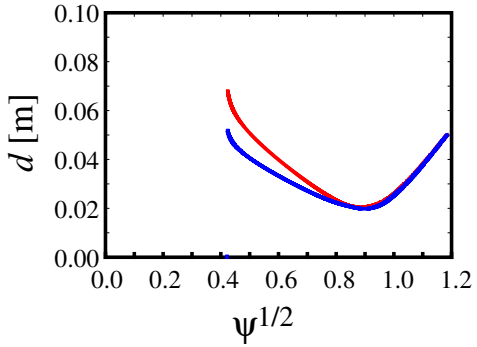


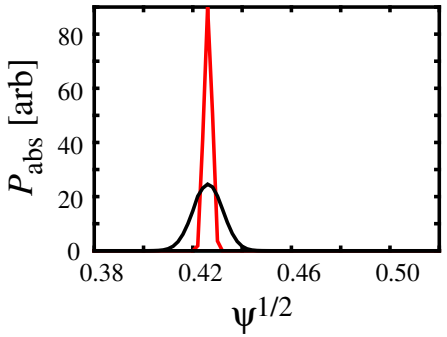
Beam Tracing in ITER-FEAT Plasma: $R_c = 2\,\mathrm{m}$, $d_{ini} = 0.05\,\mathrm{m}$



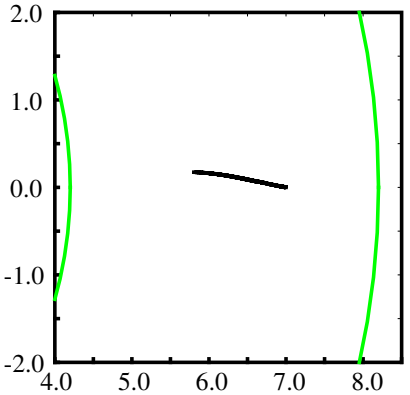
$\theta_t = 0°$

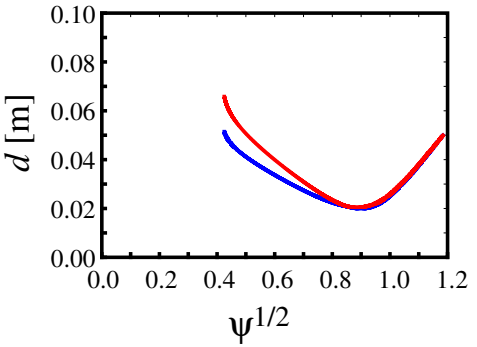


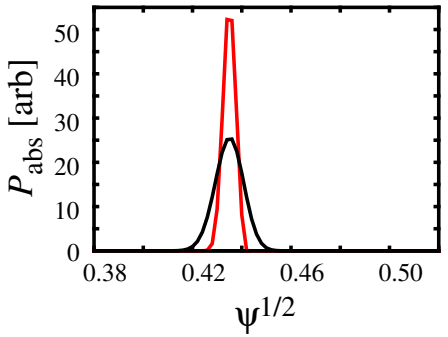




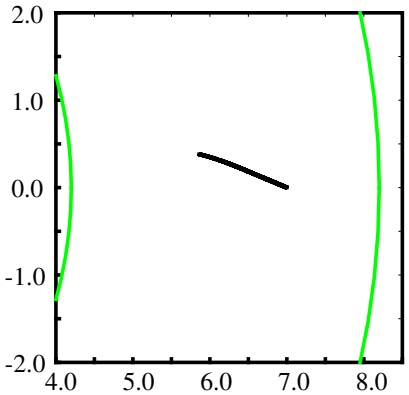
$\theta_t = 10°$

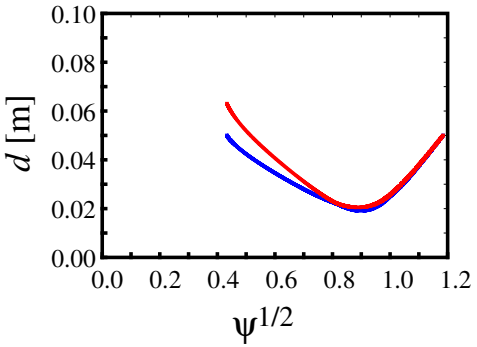


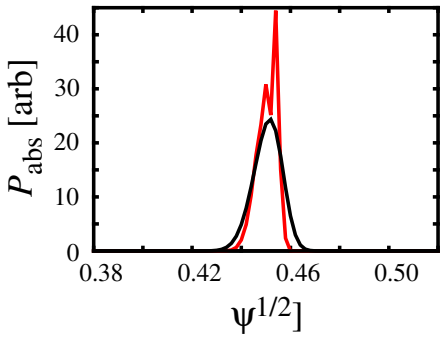




$\theta_t = 20°$







Summary

- **Based on the formulation of beam tracing, the wave propagation code TASK/WR was extended to calculate the spatial evolution of the EC beam size.**
- **We have confirmed the diffraction effect and the initial wave front curvature dependence of the beam size.**

$$d_{\min} \sim \frac{\lambda}{\pi d_{\mathrm{ini}}} R_{\mathrm{c}}$$

- **In the case of ITER-FEAT (170GHz), initial beam radius of 5cm is required to focus with beam length 3m.**
- **For toroidally oblique injection, Doppler broadening may mask the effect of diffraction.**
- **To dos:**
  - **Systematic parameter servey, comparison with ray tracing**
  - **Fokker-Planck analysis to estimate current drive efficiency**
  - **NTM stabilization**