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Analysis of EC Wave Propagation by Beam Tracing Method

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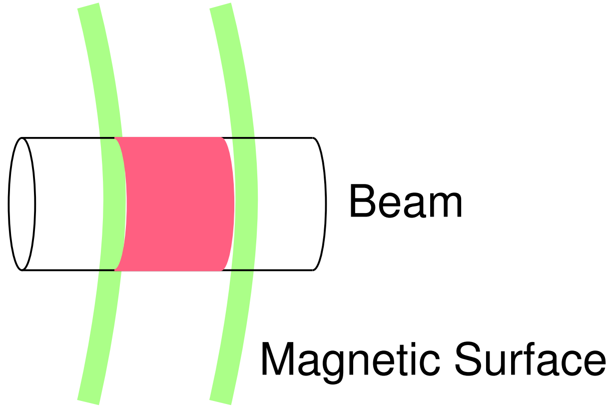
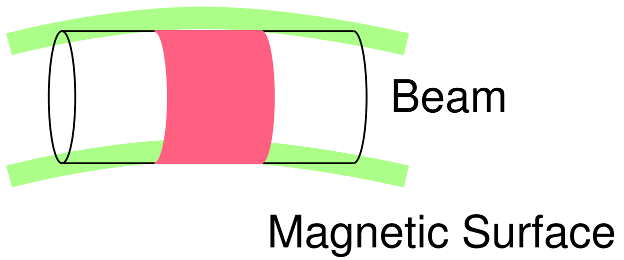
Contents

Motivation

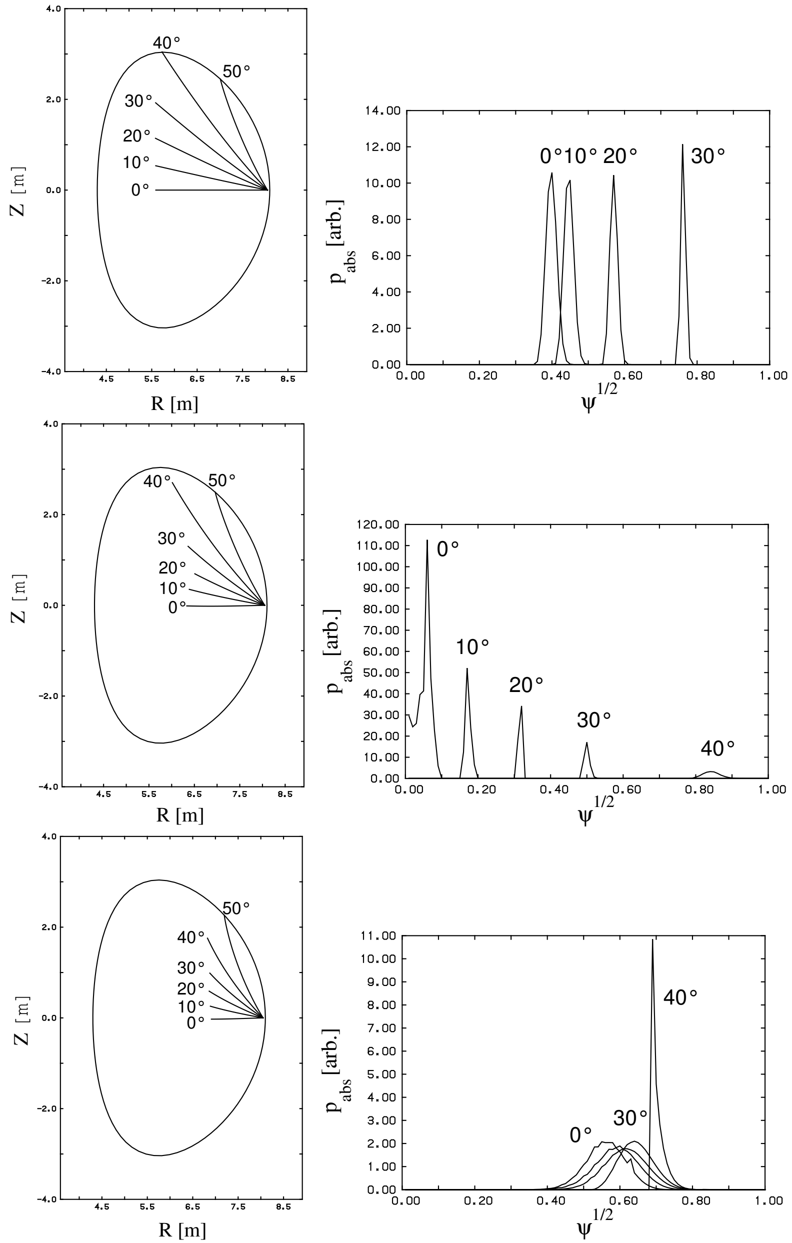
- **Current profile control by EC waves**
  - Localized profile of driven current
  - Position control by injection angle
- **Control of MHD instability**
  - Suppression of island growth due to tearing instability
  - Localized current profile is required
- **Evaluation of the current profile width**
  - Doppler broadening, decay length
  - Finite beam size, focusing
  - Defraction
- **Limitation of ray tracing**
  - Defraction effect cannot be included



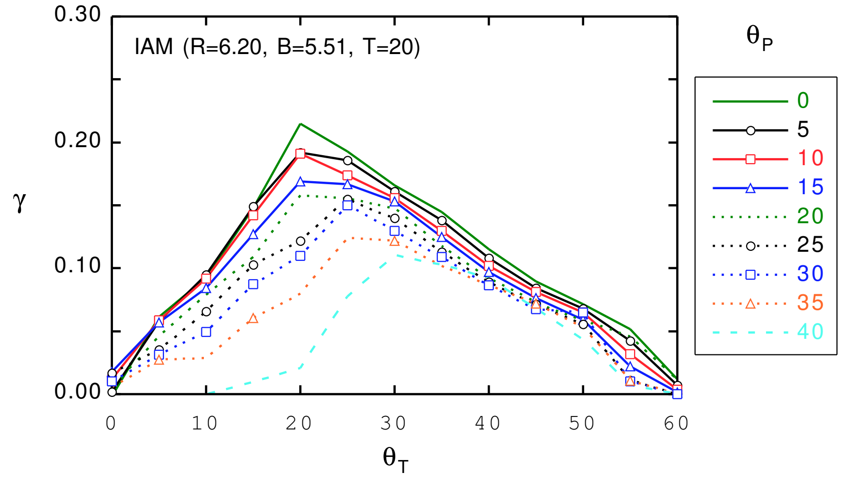


Analysis by Ray Tracing for ITER-FEAT

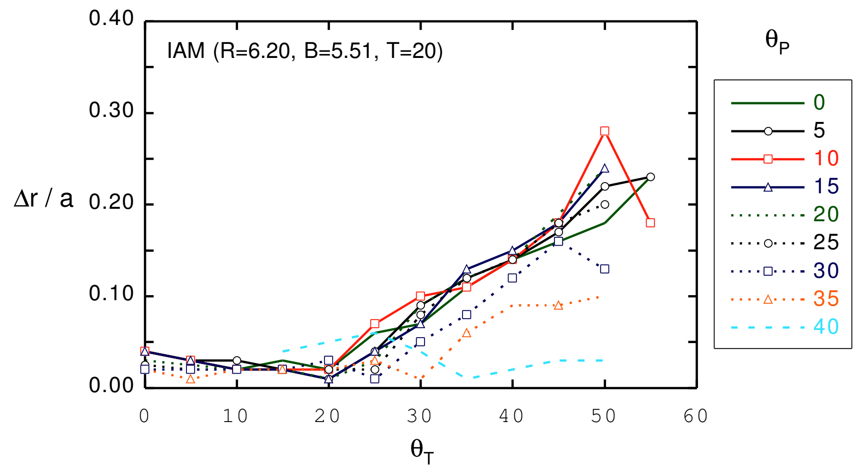
- **Injection angle dependence**



- **Current drive efficiency**



- **Absorption width (single ray)**



Propagation of Short-Wavelength Waves

- **Ray Tracing** (Geometrical Optics)
  - Wave length $\lambda \ll$ Characteristic scale length L of the medium
  - Plane wave: Beam size d is sufficiently large
    — **Fresnel condition**: $L \ll d^2/\lambda$
  - Beam : Diffraction effect determines the beam size d
- **Beam Tracing**
  - Propagation of beam with finite size
    — Spatial evolution of beam size
  - References
    — G. V. Pereverzev, in *Reviews of Plasma Physics*, Vol. 19, p. 1.
    — A. G. Peeters, Phys. Plasmas **3** (1996) 4386.
    — G. V. Pereverzev, Phys. Plasmas **4** (1998) 3529.

Ray Tracing Method

- **Maxwell equation** for wave electric field $\boldsymbol{E}\,\mathrm{e}^{-\mathrm{i}\omega t}$

$$\nabla \times \nabla \times \boldsymbol{E} - \frac{\omega^2}{c^2} \overleftrightarrow{\epsilon} \cdot \boldsymbol{E} = 0$$

(Dielectric tensor $\overleftrightarrow{\epsilon}$: Hermite part $\overleftrightarrow{\epsilon}_{\mathrm{H}} \gg$ Anti-Hermite part $\overleftrightarrow{\epsilon}_{\mathrm{A}}$)

- **Dispersion relation** in a homogeneous plasma:
  Plane wave : constant wave vector $\boldsymbol{k}$

$$K = \det\left[\frac{c^2}{\omega^2}\left(-k^2 \overleftrightarrow{I} + \boldsymbol{k}\boldsymbol{k}\right) + \overleftrightarrow{\epsilon}_{\mathrm{H}}\right] = 0$$

- Expansion parameter

$$\delta = \sqrt{\frac{c}{\omega L}} \ll 1$$

- **Eikonal expression** of wave electric field

$$\boldsymbol{E}(\boldsymbol{r}) = \mathrm{Re}\left[v_{\mathrm{a}}(\delta^2 \boldsymbol{r})\,\mathrm{e}^{\,\mathrm{i}\,s(\boldsymbol{r})}\right]$$

- Solvable condition of Maxwell's equation with eikonal expression

$$\frac{\mathrm{d}\boldsymbol{r}}{\mathrm{d}\tau} = \frac{\partial K}{\partial \boldsymbol{k}}, \qquad \frac{\mathrm{d}\boldsymbol{k}}{\mathrm{d}\tau} = -\frac{\partial K}{\partial \boldsymbol{r}}$$

Beam Tracing Method

- Beam size perpendicular to the beam direction: first order in δ
- **Beam shape** : Weber function Hermite polynomial: H_n)

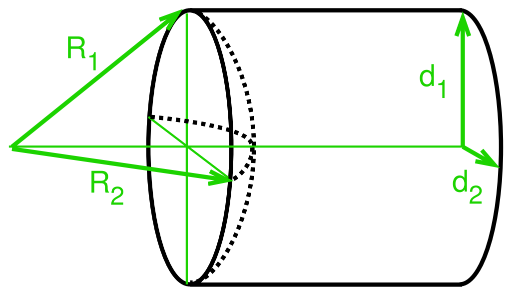
$$\boldsymbol{E}(\boldsymbol{r}) = \mathrm{Re}\left[\sum_{mn} C_{mn}(\delta^2\boldsymbol{r})v_{\mathrm{e}}(\delta^2\boldsymbol{r})H_m(\delta\xi_1)H_n(\delta\xi_2)\,\mathrm{e}^{\,\mathrm{i}\,s(\boldsymbol{r})-\phi(\boldsymbol{r})}\right]$$

  - Amplitude : C_{mn}, Polarization : v_{e}, Phase : $s(\boldsymbol{r}) + \mathrm{i}\,\phi(\boldsymbol{r})$

$$s(\boldsymbol{r}) = s_0(\tau) + k^0_\alpha(\tau)[r^\alpha - r^\alpha_0(\tau)] + \frac{1}{2}s_{\alpha\beta}[r^\alpha - r^\alpha_0(\tau)][r^\beta - r^\beta_0(\tau)]$$

$$\phi(\tau) = \frac{1}{2}\phi_{\alpha\beta}[r^\alpha - r^\alpha_0(\tau)][r^\beta - r^\beta_0(\tau)]$$

  - Position of beam axis : $\boldsymbol{r}_0$, Wave number on beam axis: $\boldsymbol{k}^0$
  - **Curvature radius** of equi-phase surface: $R_\alpha = \dfrac{1}{\lambda s_{\alpha\alpha}}$
  - **Beam radius** $d_\alpha = \sqrt{\dfrac{2}{\phi_{\alpha\alpha}}}$



- Gaussian beam : case with $m = 0$, $n = 0$

Beam Propagation Equation

- Solvable condition for Maxwell's equation with beam field

$$\frac{\mathrm{d}r_0^\alpha}{\mathrm{d}\tau} = \frac{\partial K}{\partial k_\alpha}$$

$$\frac{\mathrm{d}k_\alpha^0}{\mathrm{d}\tau} = -\frac{\partial K}{\partial r^\alpha}$$

$$\frac{\mathrm{d}s_{\alpha\beta}}{\mathrm{d}\tau} = -\frac{\partial^2 K}{\partial r^\alpha \partial r^\beta} - \frac{\partial^2 K}{\partial r^\beta \partial k_\gamma} s_{\alpha\gamma} - \frac{\partial^2 K}{\partial r^\alpha \partial k_\gamma} s_{\beta\gamma} - \frac{\partial^2 K}{\partial k^\gamma \partial k^\delta} s_{\alpha\gamma} s_{\beta\delta} + \frac{\partial^2 K}{\partial k^\gamma \partial k^\delta} \phi_{\alpha\gamma}\phi_{\beta\delta}$$

$$\frac{\mathrm{d}\phi_{\alpha\beta}}{\mathrm{d}\tau} = -\left(\frac{\partial^2 K}{\partial r^\alpha \partial k^\gamma} + \frac{\partial^2 K}{\partial k^\gamma \partial k_\delta} s_{\alpha\delta}\right)\phi_{\beta\gamma} - \left(\frac{\partial^2 K}{\partial r^\beta \partial k^\gamma} + \frac{\partial^2 K}{\partial k^\gamma \partial k_\delta} s_{\beta\delta}\right)\phi_{\alpha\gamma}$$

- By integrating this set of 18 ordinary differential equations, we obtain trace of the beam axis, wave number on the beam axis, curvature of equi-phase surface, and beam size.

- Equation for the wave amplitude C_{mn}

$$\boldsymbol{\nabla} \cdot \left(\boldsymbol{v}_{\mathrm{g}0}|C_{mn}|^2\right) = -2\left(\gamma|C_{mn}|^2\right)$$

Group velocity: $\boldsymbol{v}_{\mathrm{g}0}$, Damping rate: $\gamma \equiv (v_\mathrm{e}^* \cdot \overleftrightarrow{\epsilon}_\mathrm{A} \cdot v_\mathrm{e})/(\partial K/\partial\omega)$

Beam Tracing in a Uniform Plasma

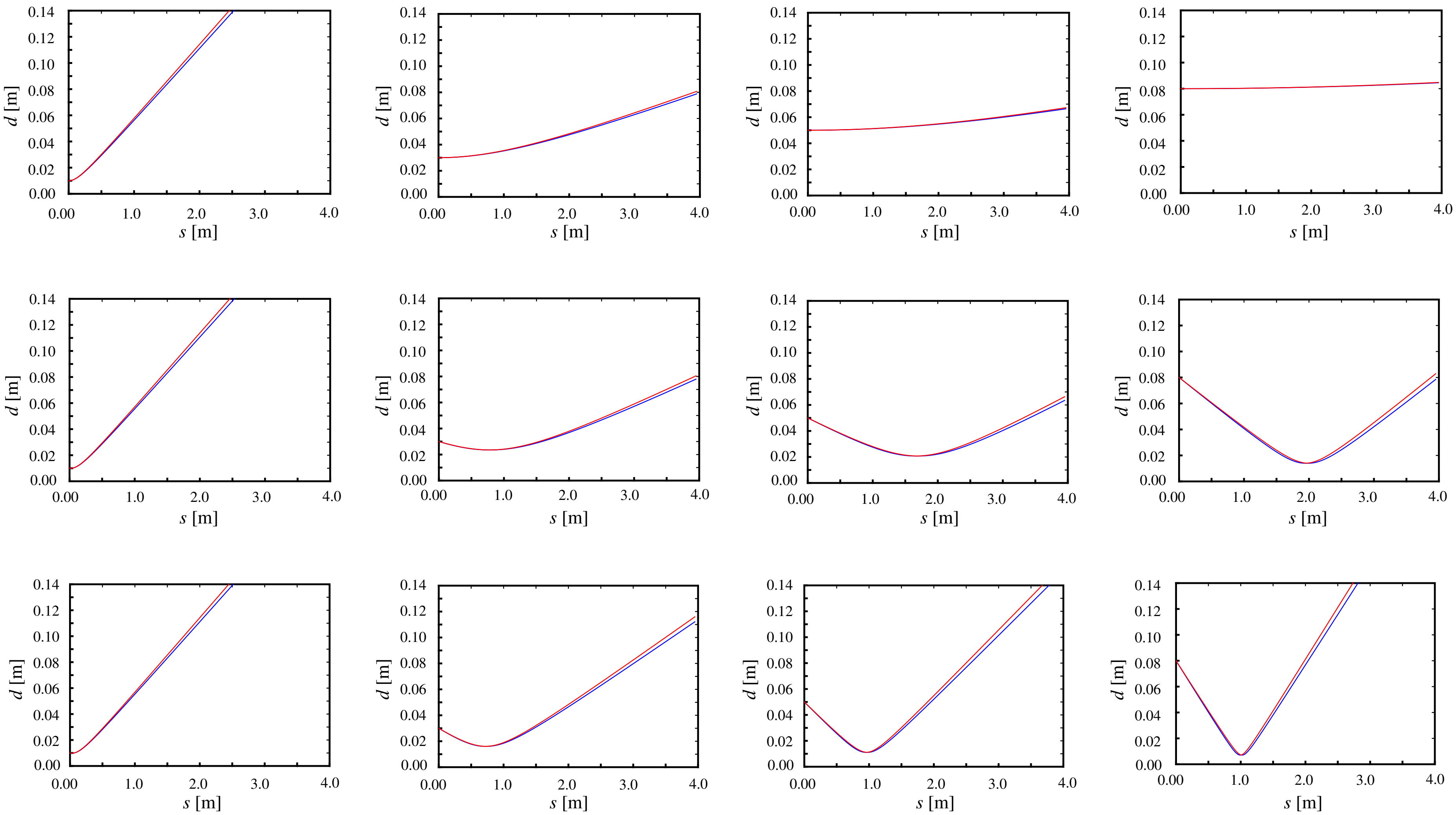
- **170 GHz, Ordinary Mode, Perpendicular Injection**

R_c | $d_{ini} = 0.01$ m | $d_{ini} = 0.03$ m | $d_{ini} = 0.05$ m | $d_{ini} = 0.08$ m

∞

2 m

1 m

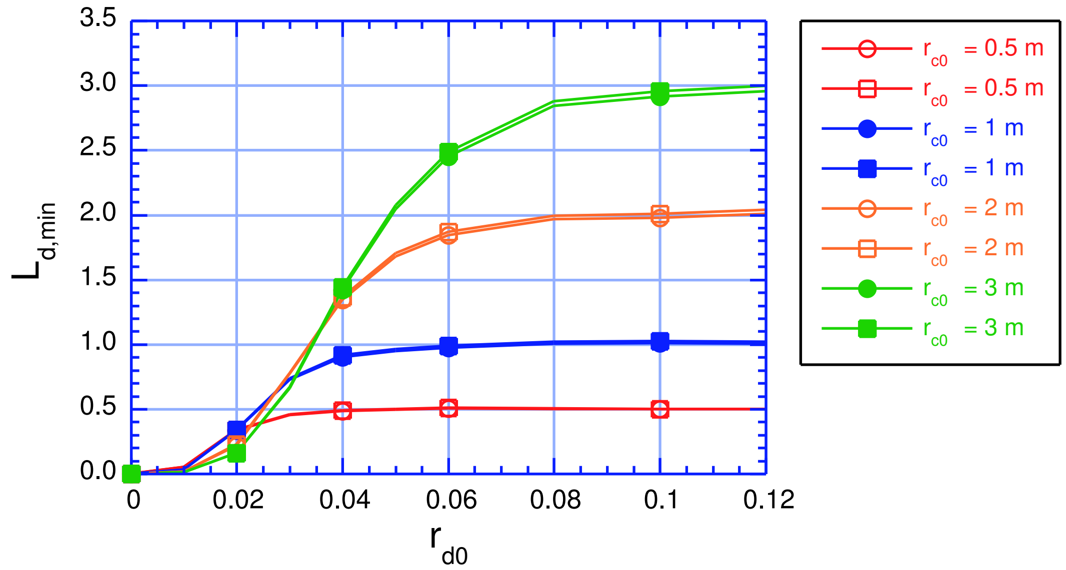


Dependence of Initial Beam Radius, d_{ini}

Beam length where $d = d_{\min}$

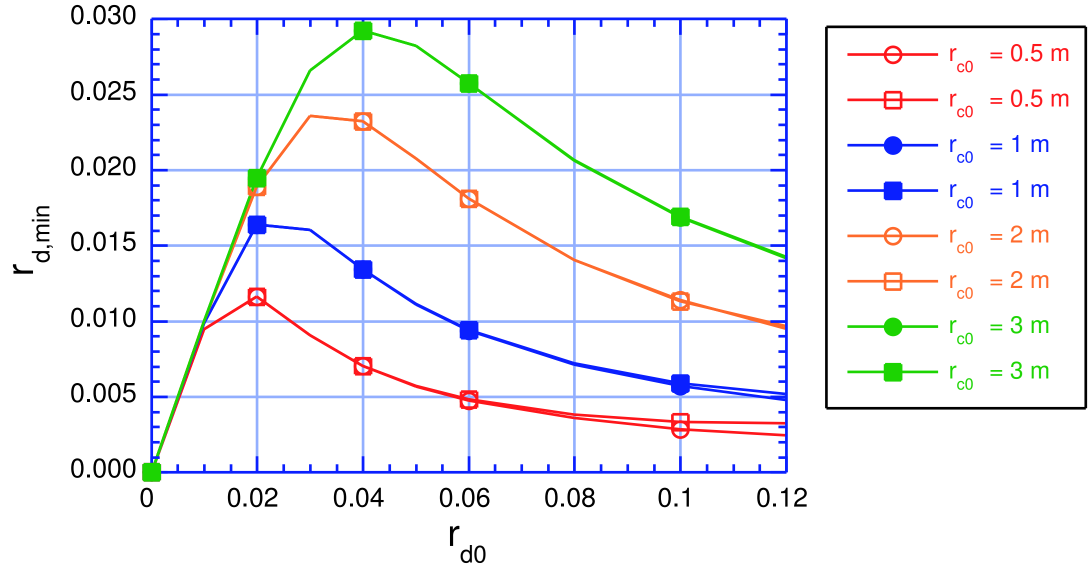
necessary condition:

$$d_{\text{ini}} > \sqrt{R_c \lambda}$$

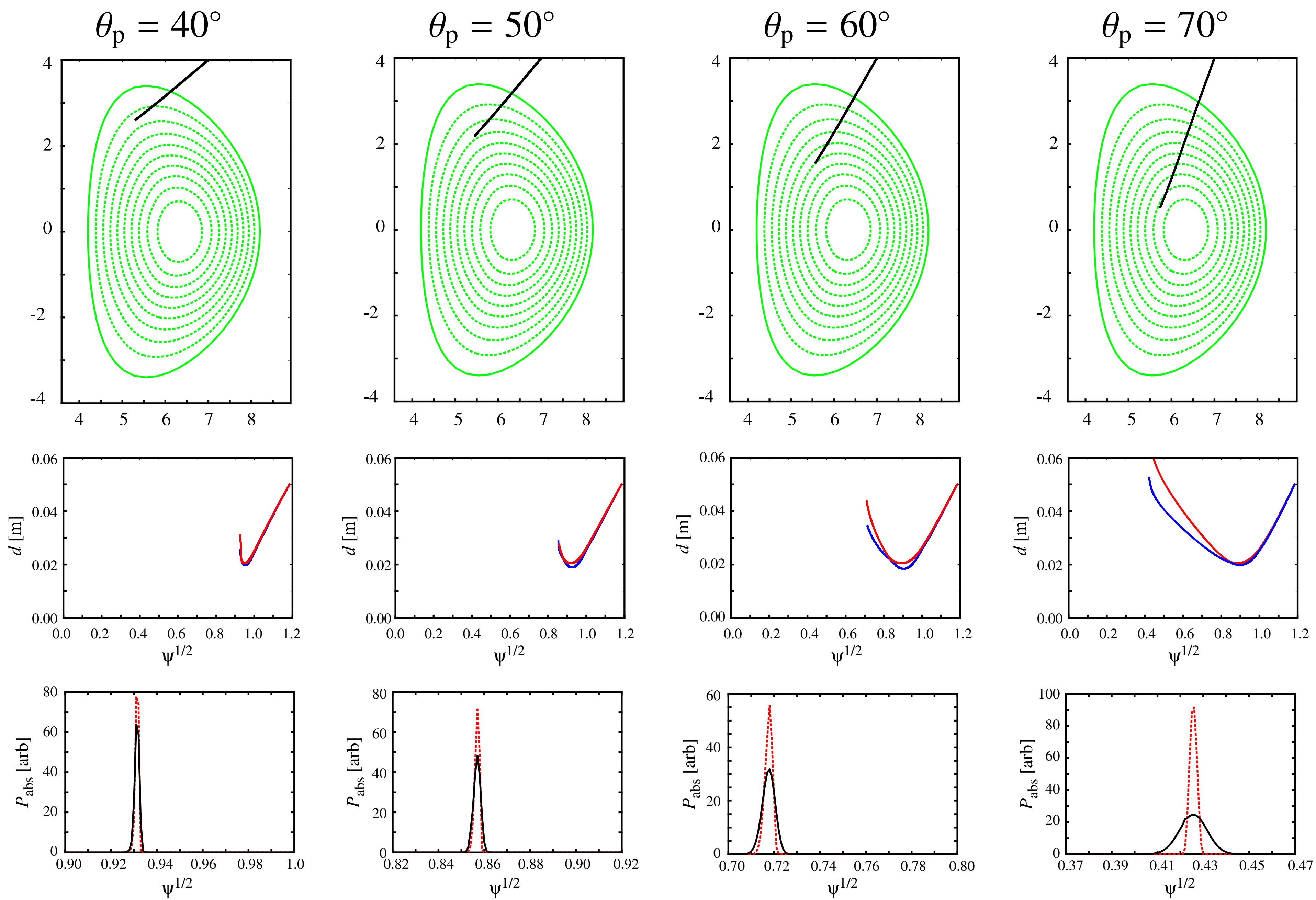


Minimum beam radius $d_{\min}$

$$d_{\min} \sim \frac{\lambda}{\pi d_{\text{ini}}} R_c$$

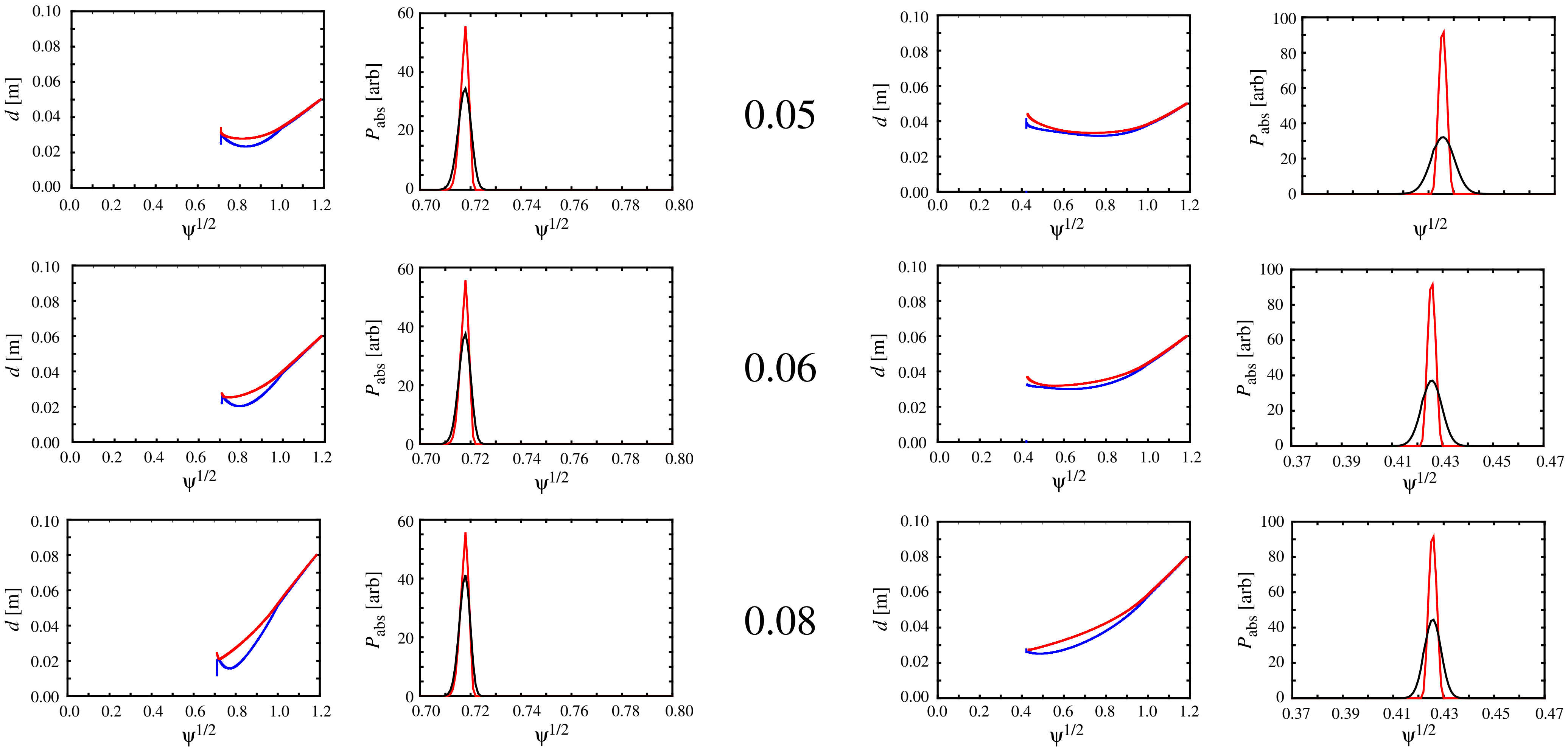


Beam Tracing in ITER-FEAT Plasma: R_c = 2 m, d_{ini} = 0.05 m

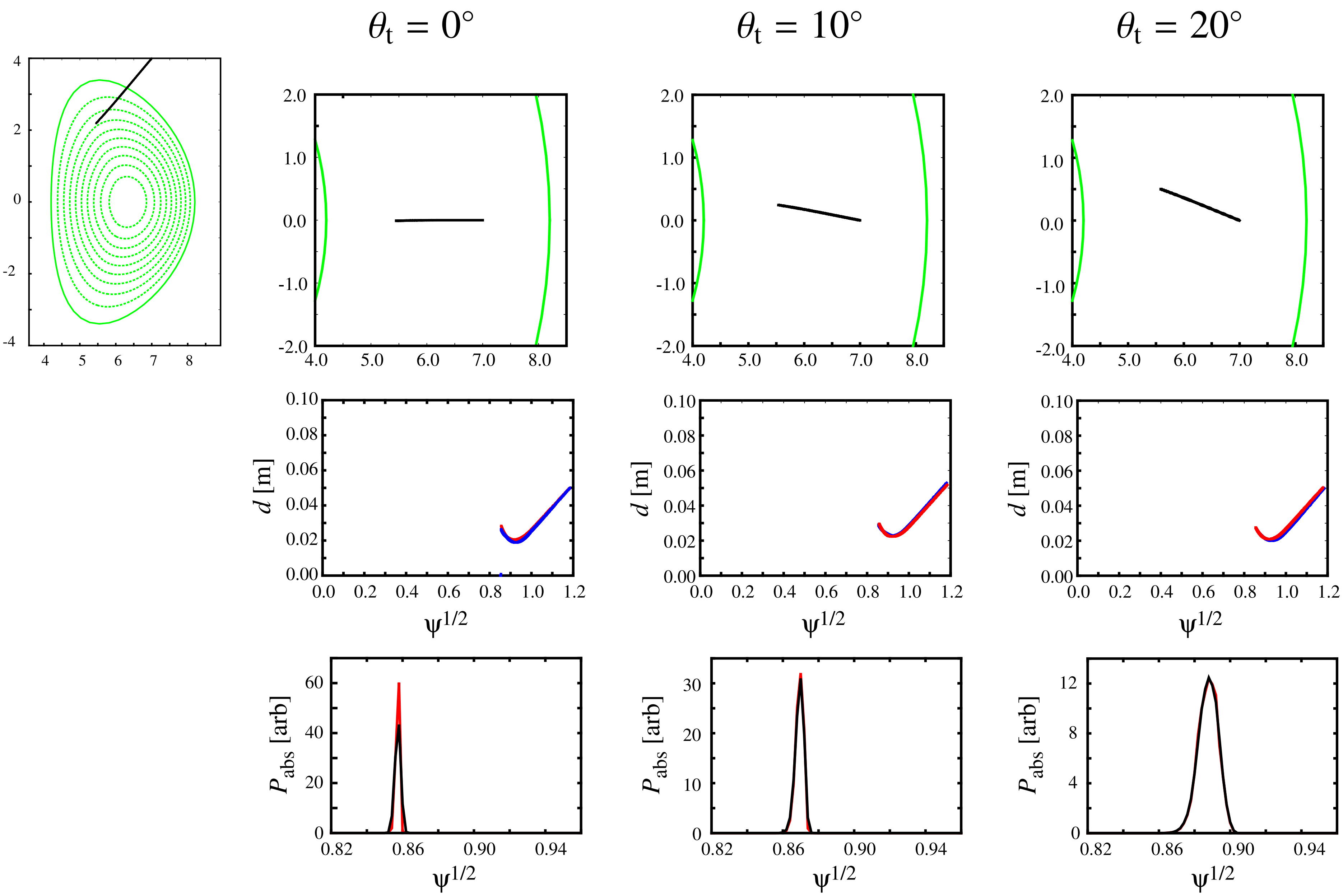


Beam Tracing in ITER-FEAT Plasma

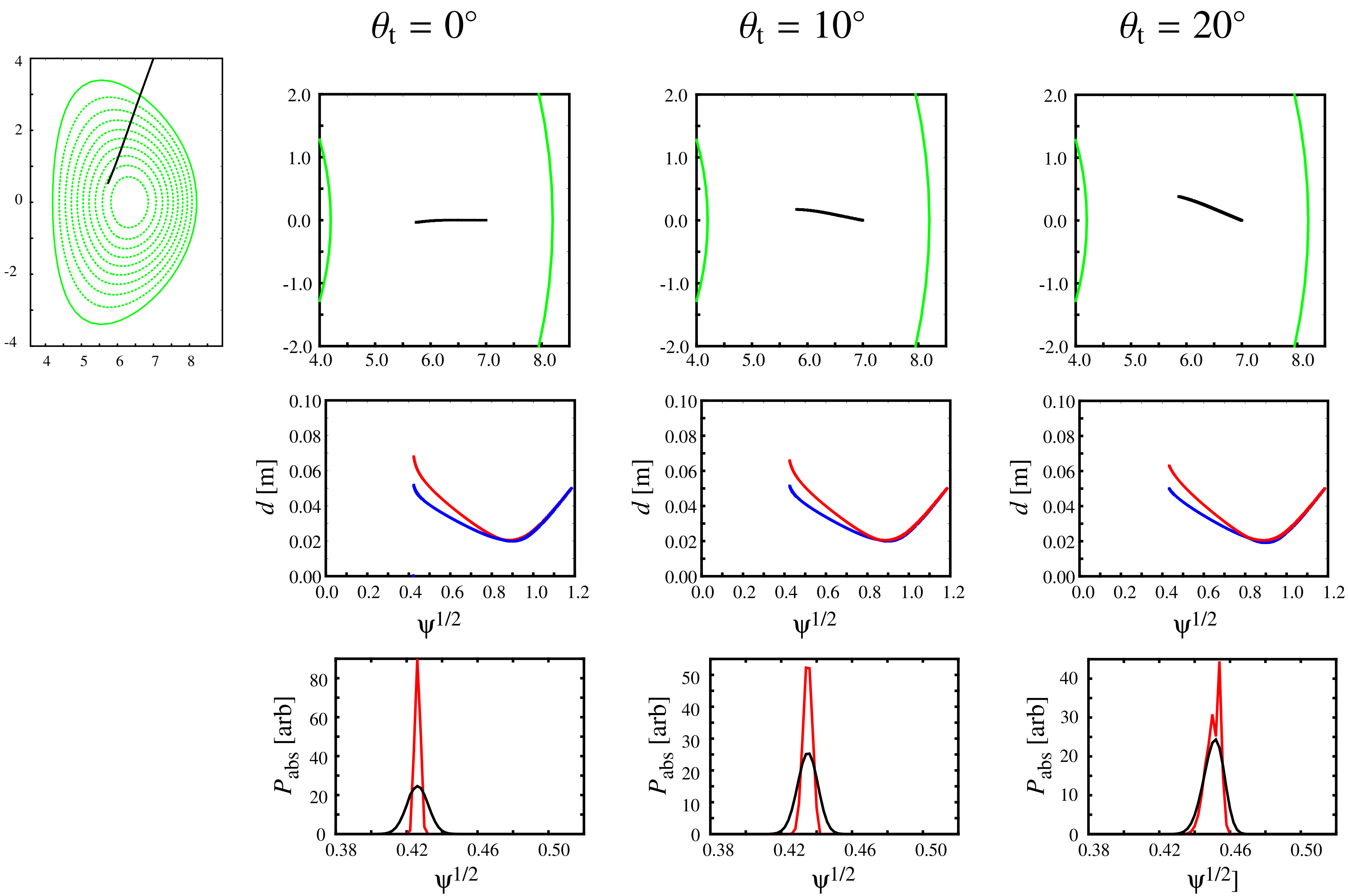
$\theta_p = 60°$ | d_{ini} [m] | $\theta_p = 70°$
$R_c = 3$ m | | $R_c = 4$ m



Beam Tracing in ITER-FEAT Plasma: R_c = 2 m, d_{ini} = 0.05 m



Beam Tracing in ITER-FEAT Plasma: $R_c = 2$ m, $d_{ini} = 0.05$ m



Summary

- **Based on the formulation of beam tracing, the wave propagation code TASK/WR was extended to calculate the spatial evolution of the EC beam size.**
- **We have confirmed the diffraction effect and the initial wave front curvature dependence of the beam size.**

$$d_{\rm min} \sim \frac{\lambda}{\pi d_{\rm ini}} R_{\rm c}$$

- **In the case of ITER-FEAT (170GHz), initial beam radius of 5cm is required to focus with beam length 3m.**
- **For toroidally oblique injection, Doppler broadening may mask the effect of diffraction.**
- **To dos:**
  - **Coupling with Fokker-Planck Analysis**
  - **NTM stabilization**