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Onset of Transport Barrier Formation

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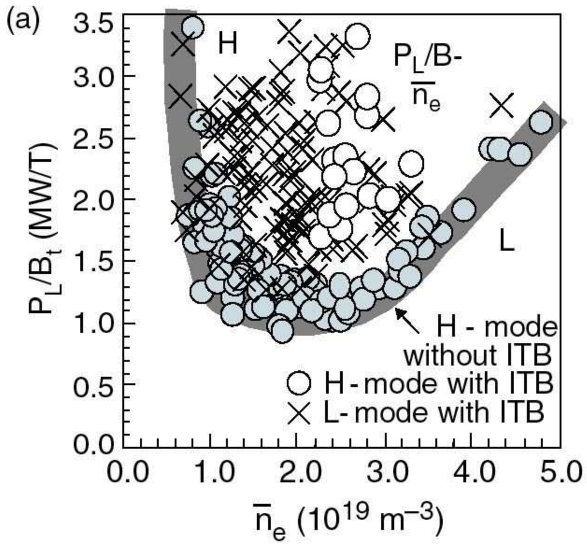
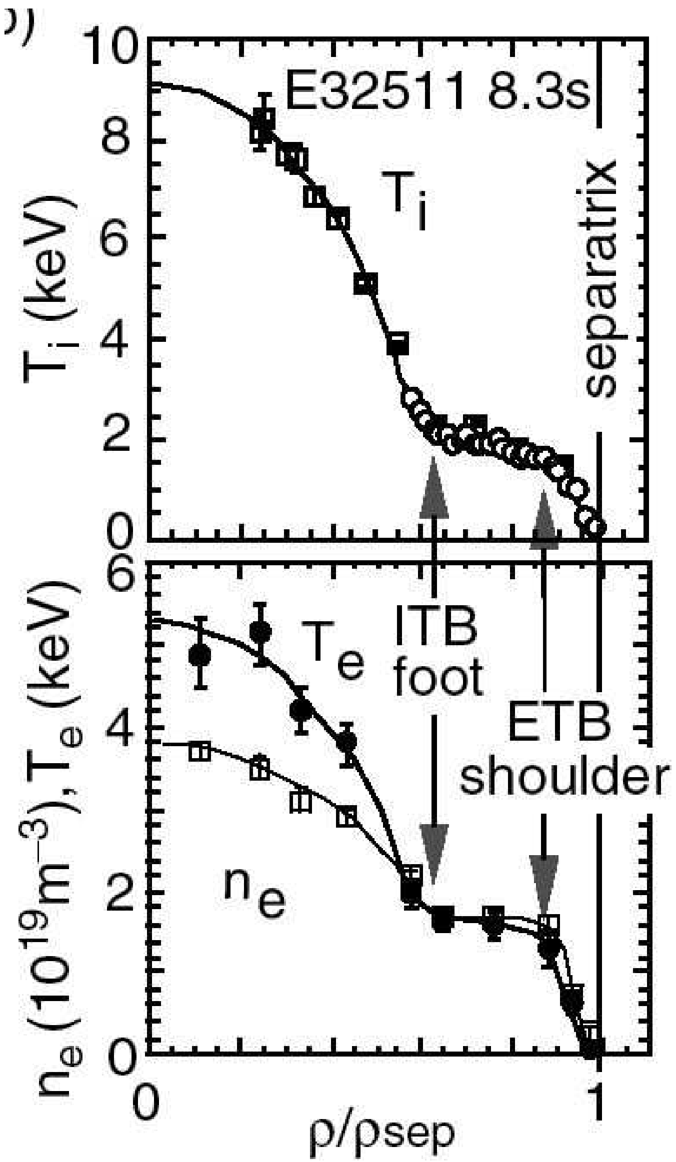
- Transport Barrier in Tokamaks
- Transport Models and Simulation Results
- Bifurcation in the Gradient-Flux Relation
- Summary

Transport Barrier in Tokamaks

- **Transport Barrier**:
  - A layer of reduced transport
  — **ETB**: Edge Transport Barrier
  — **ITB**: Internal Transport Barrier
  - Steep gradient of $n(r)$ and $T(r)$
  - Reduction of fluctuation amplitude
  - Enhancement of negative E_r
  - Power threshold





ETB: Edge Transport Barrier

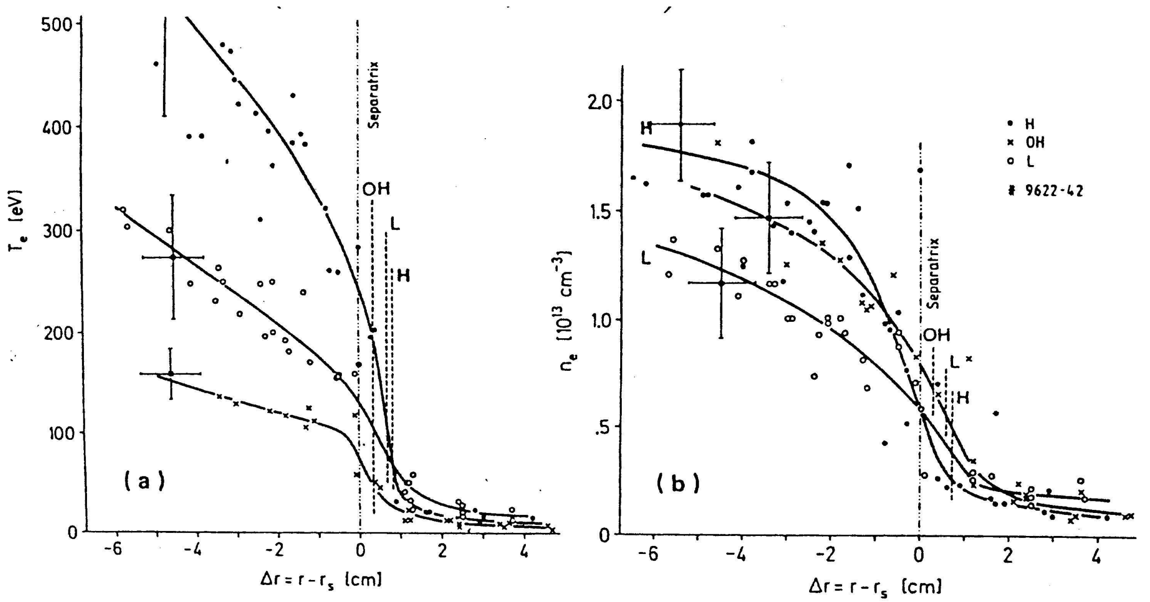
- Discovery of H mode (ASDEX, 1982)
- On the separatrix (plasma surface)
- When heating power exceeds a threshold

Density Temperature

H mode

L mode

OH



OH: only ohmic heating

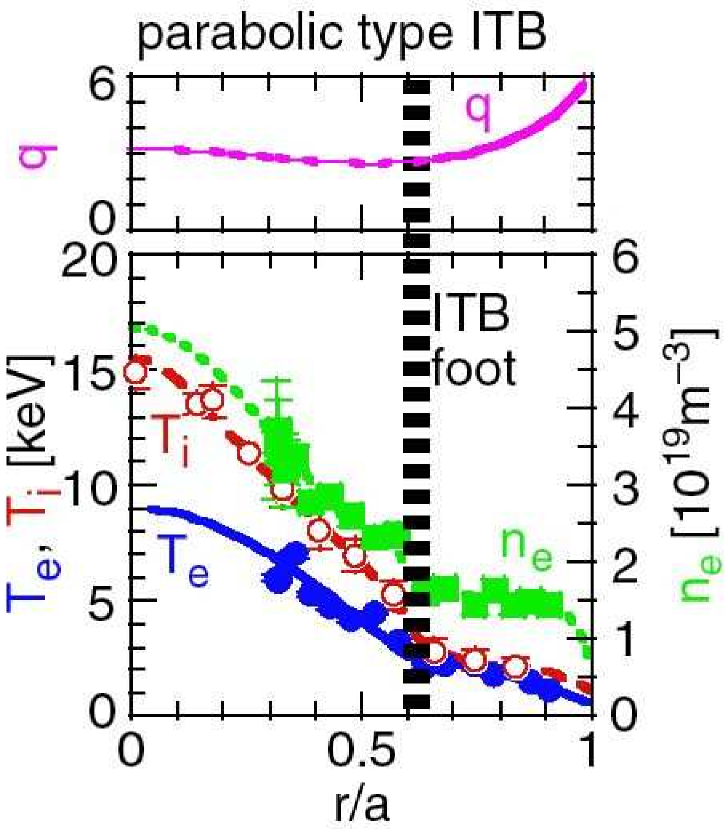
L mode: low confinement mode with additional heating

H mode: high confinement mode with additional heating

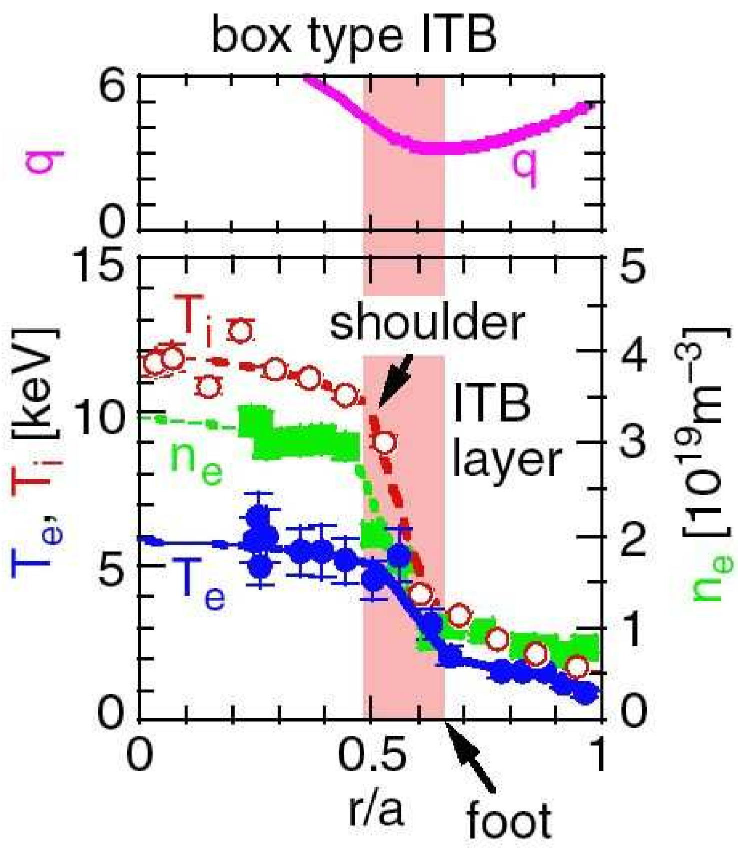
ITB: Internal Transport Barrier

- Between center and edge
- Weak or negative magnetic shear, pellet injection, rational q, ...

Parabolic



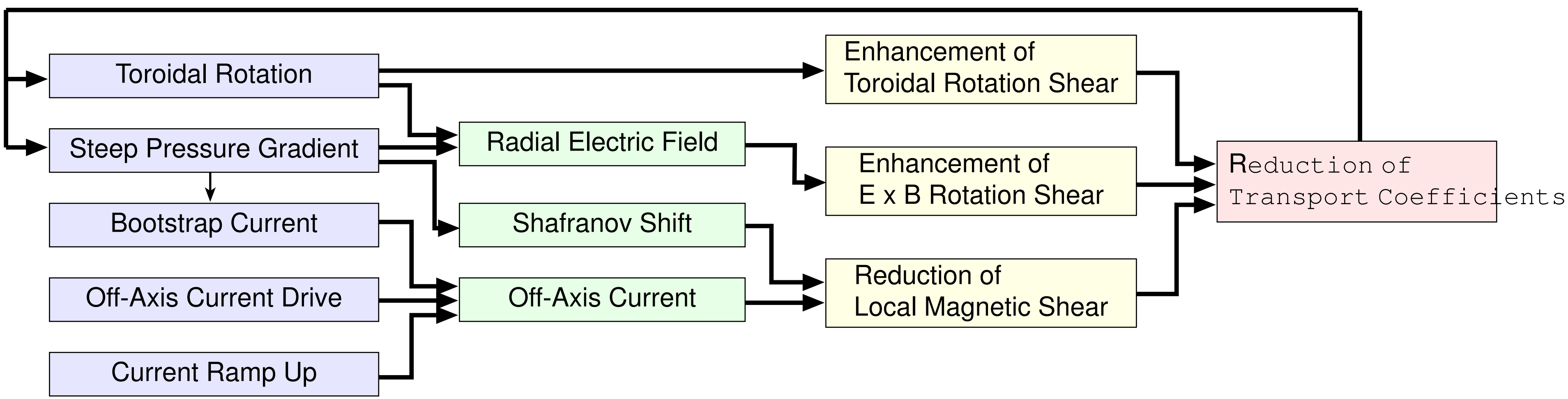
Box-shape



Physical Mechanism of ITB

- Reduction of Transport Coefficients Due to
  - Weak or negative magnetic shear
  - Large shift of magnetic axis
  - Large rotation velocity shear

- Positive feedback loop to enhance pressure gradient



Mechanism of Turbulence Suppression

- $E \times B$ **rotation shear**
  - E_r **generation through radial force balance**

$$E_r = -u_\theta B_\phi + u_\phi B_\theta + \frac{1}{e_s}\frac{\mathrm{d}}{\mathrm{d}r}P$$

  - $E \times B$ **shearing rate (Hahm and Burrel)**

$$\omega_E = \frac{RB_\theta}{B}\frac{\mathrm{d}}{\mathrm{d}r}\left(\frac{E_r}{RB_\theta}\right)\frac{k_\theta}{k_r}$$

  - **Criteria for suppression**

$$\omega_E > \gamma_{\mathrm{Lin}}$$

- **Magnetic shear** s **and normalized pressure gradient** α
  - **Bollooning mode and toroidal ITG mode localized outside of torus**
  - **Thermal diffusivity** χ **as a function of** $s - \alpha$

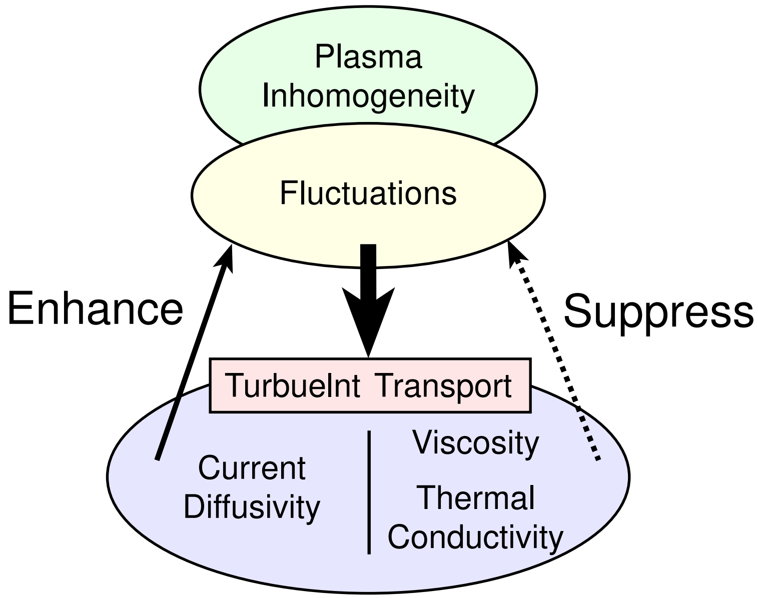
  — **Magnetic shear:** $s \equiv (r/q)(\mathrm{d}dq/\mathrm{d}r)$

  — **Normalized pressure gradient:** $\alpha \equiv -q^2R(\mathrm{d}\beta/\mathrm{d}r)$

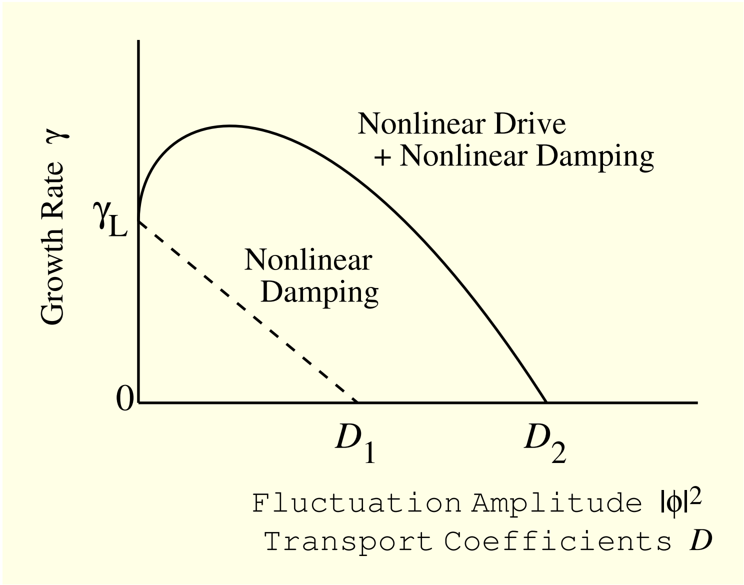
- **Positive feedback of pressure gradient increase**

Self-Sustained Turbulence

- **Self-Sustained Turbulence**
  (K. Itoh et al., 1992)
  - Turbulence sustained by the enhancement of transport coefficients due to the turbulence itself



  – Weakly unstable in linear stage
  – NL dissipation destabilizes the mode.
  – Saturation due to the balance between NL drive and NL damping



CDBM Transport Model

- **Current-Diffusive Ballooning Mode**
  - **Ballooning mode: MHD mode localized in bad curvature region**
    - — **Ideal ballooning mode (second stability)**
    - — **Resistive ballooning mode (plasma near edge)**
    - — **Current-diffusive ballooning mode (core plasma)**
- **Reduced MHD Equation (One fluid model including transport)**

  Vorticity equation
  $$\frac{\partial}{\partial t}\frac{n_0 m_{\rm i}}{B_0}\nabla_\perp^2\phi = B_0\nabla_\parallel j_\parallel + \nabla p \times \nabla\frac{2r\cos\theta}{R_0}\cdot\hat{z} + \mu\frac{n_0 m_{\rm i}}{B_0}\nabla_\perp^4\phi$$

  Ohm's law
  $$\frac{\partial}{\partial t}A = -\nabla_\parallel\phi - \eta j_\parallel + \lambda\nabla_\perp^2 j_\parallel \quad \textbf{where} \quad j_\parallel = -\frac{1}{\mu_0}\nabla_\perp^2 A$$

  Energy equation
  $$\frac{\partial}{\partial t}p + \frac{1}{B_0}\nabla\phi\times\nabla p_0\cdot\hat{z} = \chi\nabla^2 p$$

- **Transport Coefficients:**

  μ : **Ion viscosity,** η : **Resistivity,**

  λ : **Current diffusivity,** χ : **Thermal diffusivity**

- **Ballooning transformation to 1D eigenvalue problem**
  - **Marginal stability**

CDBM Turbulence

- **Marginal Stability Condition** $(\gamma = 0)$

$$\chi_{\rm TB} = F(s,\alpha,\kappa,\omega_{\rm E1})\,\alpha^{3/2}\,\frac{c^2}{\omega_{\rm pe}^2}\,\frac{v_{\rm A}}{qR}$$

Magnetic shear $\quad s \equiv \frac{r}{q}\frac{{\rm d}q}{{\rm d}r}$

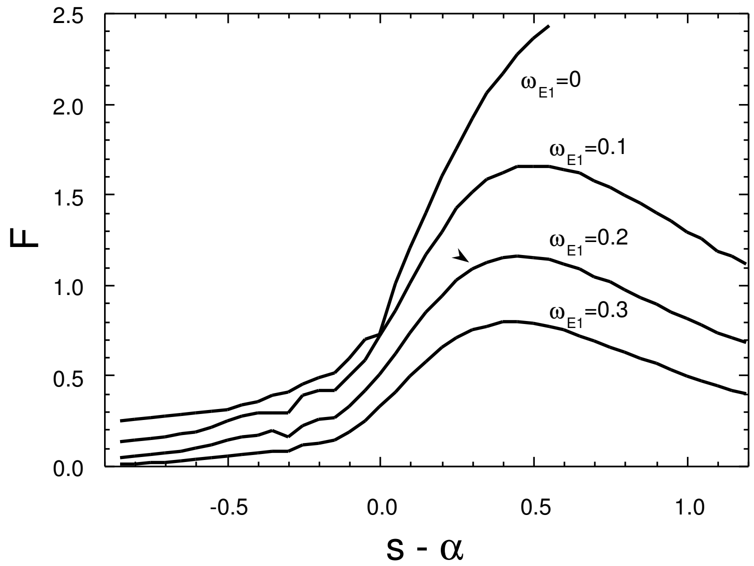
Pressure gradient $\quad \alpha \equiv -q^2 R \frac{{\rm d}\beta}{{\rm d}r}$

Magnetic curvature $\quad \kappa \equiv -\frac{r}{R}\left(1-\frac{1}{q^2}\right)$

$E \times B$ **rotation shear** $\quad \omega_{\rm E1} \equiv \frac{r^2}{s v_{\rm A}}\frac{\rm d}{{\rm d}r}\frac{E}{rB}$

- **Weak and negative magnetic shear,**
  Shafranov shift and
  $E \times B$ **rotation shear**
  reduce thermal diffusivity.

$s-\alpha$ **dependence of** $F(s,\alpha,\kappa,\omega_{\rm E1})$



Fitting Formula

$$F = \begin{cases} \dfrac{1}{1+G_1\omega_{\rm E1}^2}\dfrac{1}{\sqrt{2(1-2s')(1-2s'+3s'^2)}} & \textbf{for} \quad s' = s-\alpha < 0 \\[2ex] \dfrac{1}{1+G_1\omega_{\rm E1}^2}\dfrac{1+9\sqrt{2}s'^{5/2}}{\sqrt{2}(1-2s'+3s'^2+2s'^3)} & \textbf{for} \quad s' = s-\alpha > 0 \end{cases}$$

Heat Transport Simulation

- **Simple One-Dimensional Analysis**
  - **No impurity, No neutral, No sawtooth**
  - **Fixed density profile:** $n_e(r) \propto (1 - r^2/a^2)^{1/2}$
  - **Thermal diffusivity (adjustable parameter** $C = 12$**)**

$$\chi_e = C\chi_{TB} + \chi_{NC,e}$$
$$\chi_i = C\chi_{TB} + \chi_{NC,i}$$

- **Transport Equation**

$$\frac{\partial}{\partial t}\frac{3}{2}n_e T_e = -\frac{1}{r}\frac{\partial}{\partial r} r\, n_e \chi_e \frac{\partial T_e}{\partial r} + P_{OH} + P_{ie} + P_{He}$$

$$\frac{\partial}{\partial t}\frac{3}{2}n_i T_i = -\frac{1}{r}\frac{\partial}{\partial r} r\, n_i \chi_i \frac{\partial T_i}{\partial r} - P_{ie} + P_{Hi}$$

$$\frac{\partial}{\partial t}B_\theta = \frac{\partial}{\partial r}\eta_{NC}\left[\frac{1}{\mu_0}\frac{1}{r}\frac{\partial}{\partial r}rB_\theta - J_{BS} - J_{LH}\right]$$

- **Standard Plasma Parameter**

$R = 3\,\mathrm{m}$ $B_t = 3\,\mathrm{T}$ **Elongation** $= 1.5$

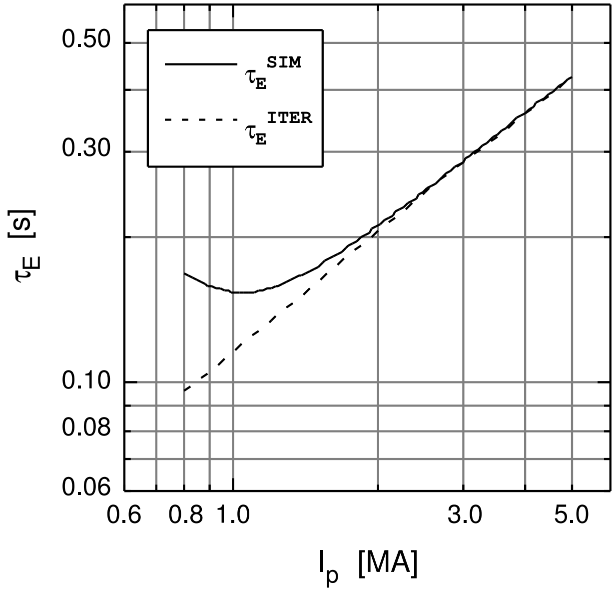
$a = 1.2\,\mathrm{m}$ $I_p = 3\,\mathrm{MA}$ $n_{e0} = 5 \times 10^{19}\,\mathrm{m}^{-3}$

Simulation of L-mode and Improved Confinement

- **Zero-Dimensional Analysis with fixed $F(s, \alpha, \kappa)$ (gyro-Bohm scaling)**

$$\tau_\mathrm{E} \propto F^{-0.4} A_\mathrm{i}^{0.2} I_\mathrm{P}^{0.8} n^{0.6} B^{0} a^{1.0} R_0^{1.2} P^{-0.6}$$

- **Deviation from L-mode scaling at low I_p**
  - Increase of P_in
  - Increase of pressure gradient
    $\longrightarrow$ Increase of α
  - Increase of bootstrap current
    $\longrightarrow$ Decrease of s
  - Reduction of χ

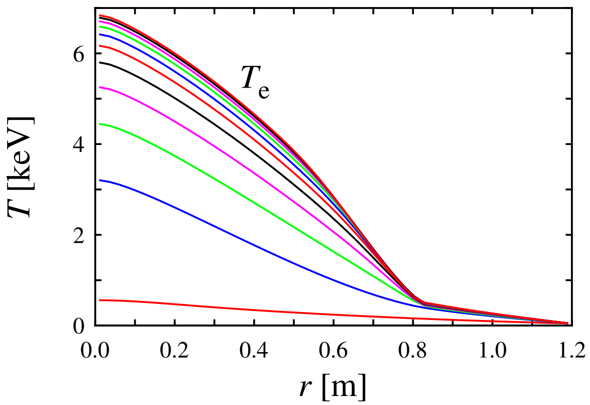


- **Various improved core confinement modes have been reproduced**
  - High-β_p mode
  - PEP (Pellet Enhanced Performance) mode
  - LHEP (Lower Hybrid Enhanced Performance) mode
  - Negative Magnetic Shear mode

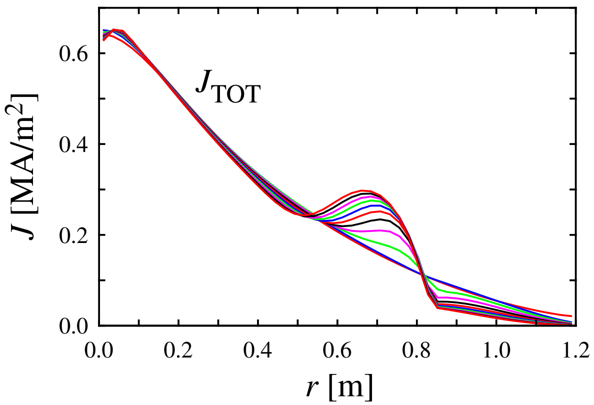
High β_p mode (1)

- $R = 3\,\mathrm{m}$, $a = 1.2\,\mathrm{m}$, $\kappa = 1.5$, $B_0 = 3\,\mathrm{T}$, $I_p = 1\,\mathrm{MA}$
- **Time evolution during the first one second after heating switched on**

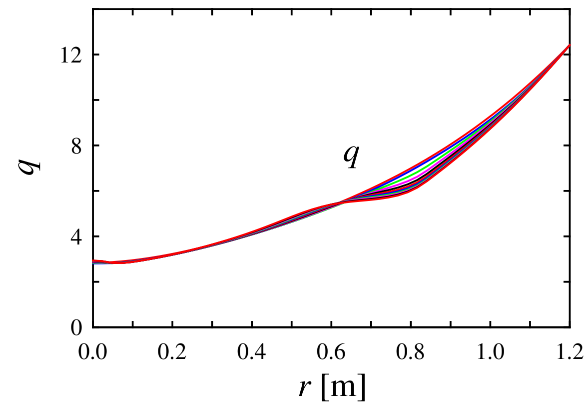
Temperater



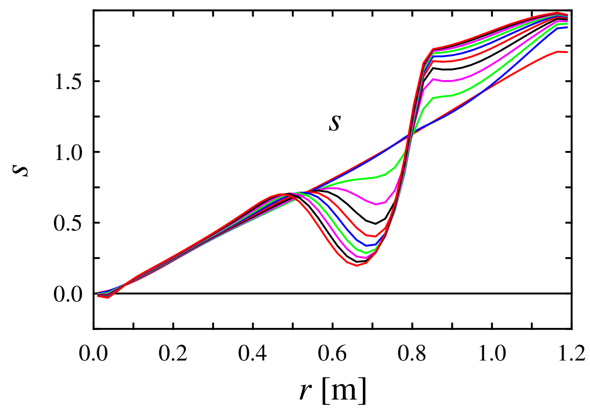
Current



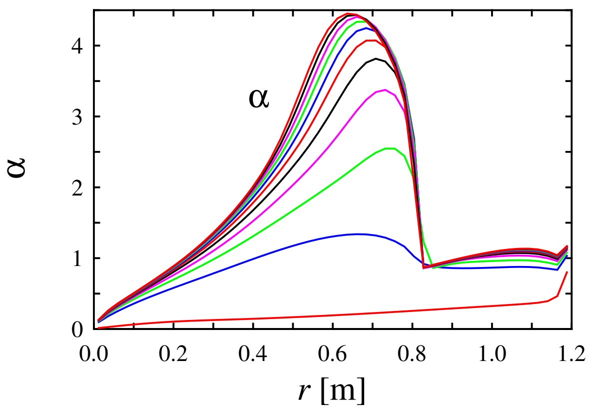
Safety factor



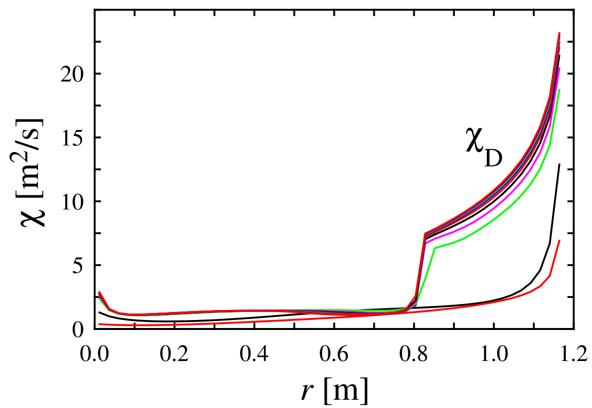
Shear



Normalized Pressure



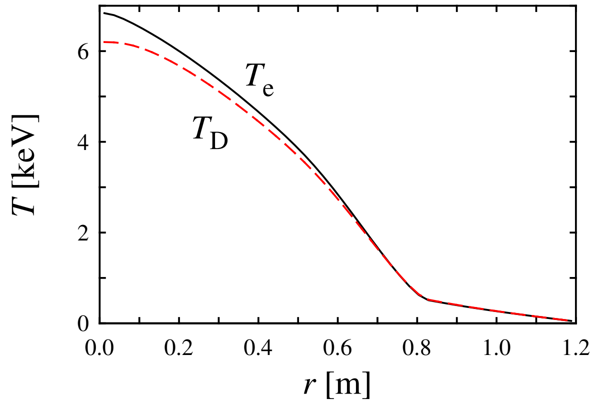
Thermal diffusivity



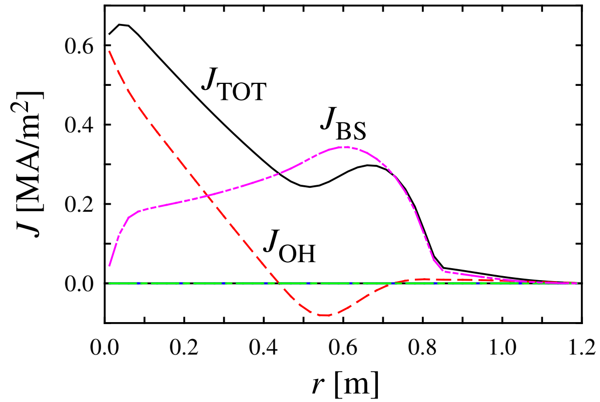
High β_p mode (2)

- **One second after heating power of $P_H = 20\,\mathrm{MW}$ was switched on**

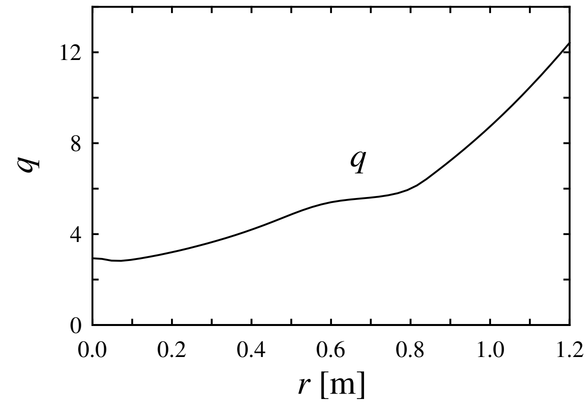
Temperater profile



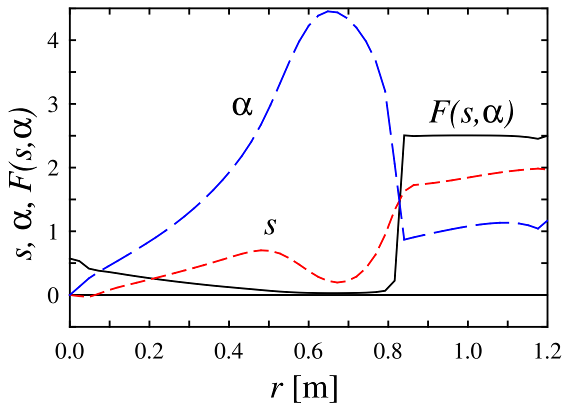
Current profile



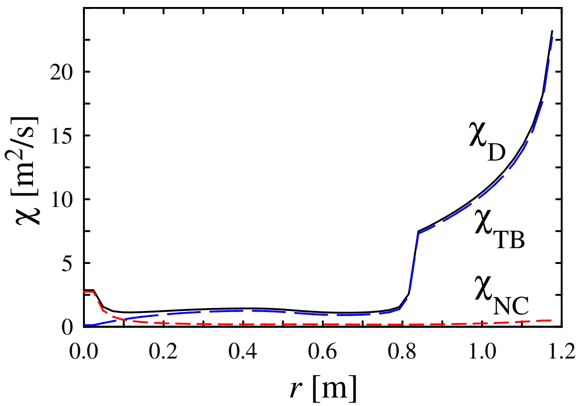
Safety factor



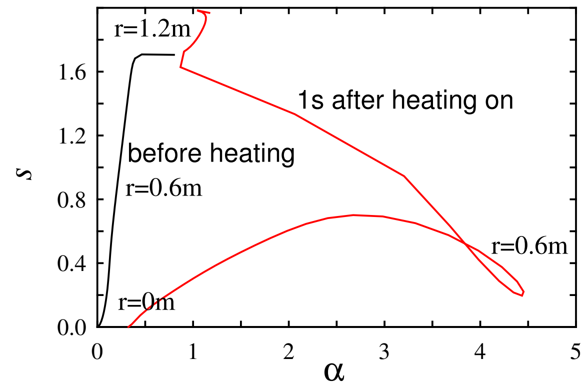
Shear and pressure



Thermal diffusivity



$s - \alpha$ diagram



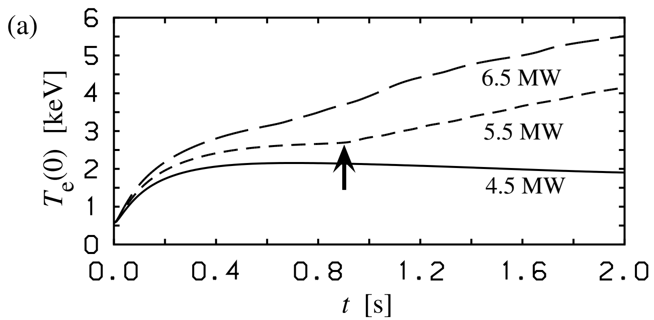
Effect of $E \times B$ Rotation Shear

- **Reduction of transport** due to small s and large α: $F(s, \alpha, \kappa)$

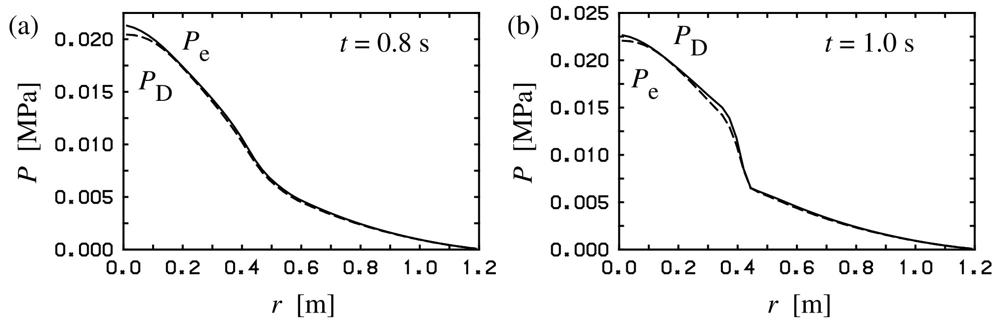
  $\Longrightarrow$ **Rapid increase of rotation shear:** $1/[1 + G(s, \alpha)\omega_{E1}^2]$

  $\Longrightarrow$ **Transition to enhanced ITB**

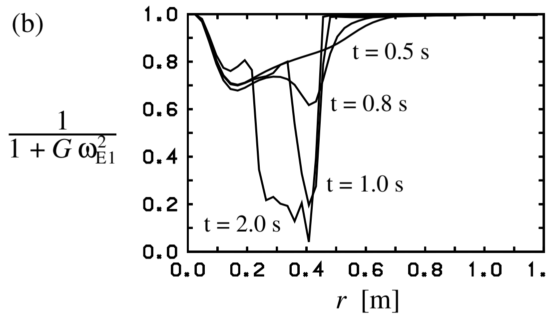
Time evolution of $T_e(0)$



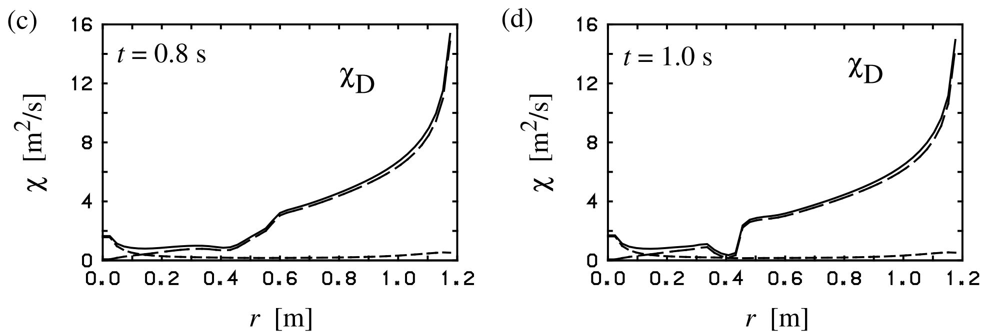
Pressure Profile Before and After Transition



Rapid Change of Rotation Shear

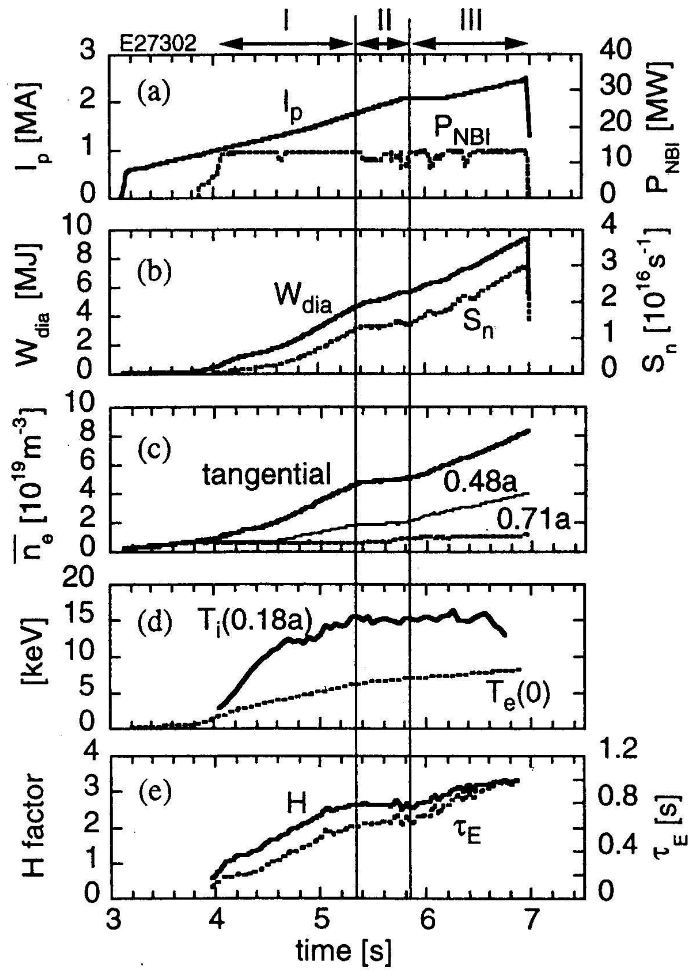


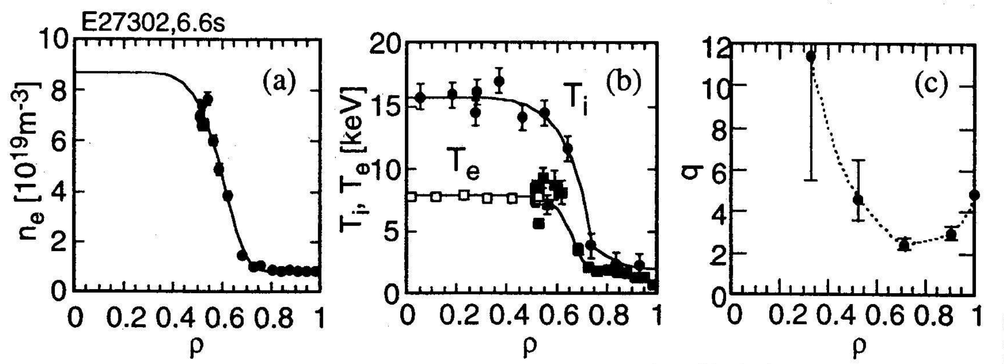
Thermal Diffusivity Before and After Transition



Reversed Magnetic Shear Configuration

- **ITB Formation by Current Ramp Up** (JT-60U)

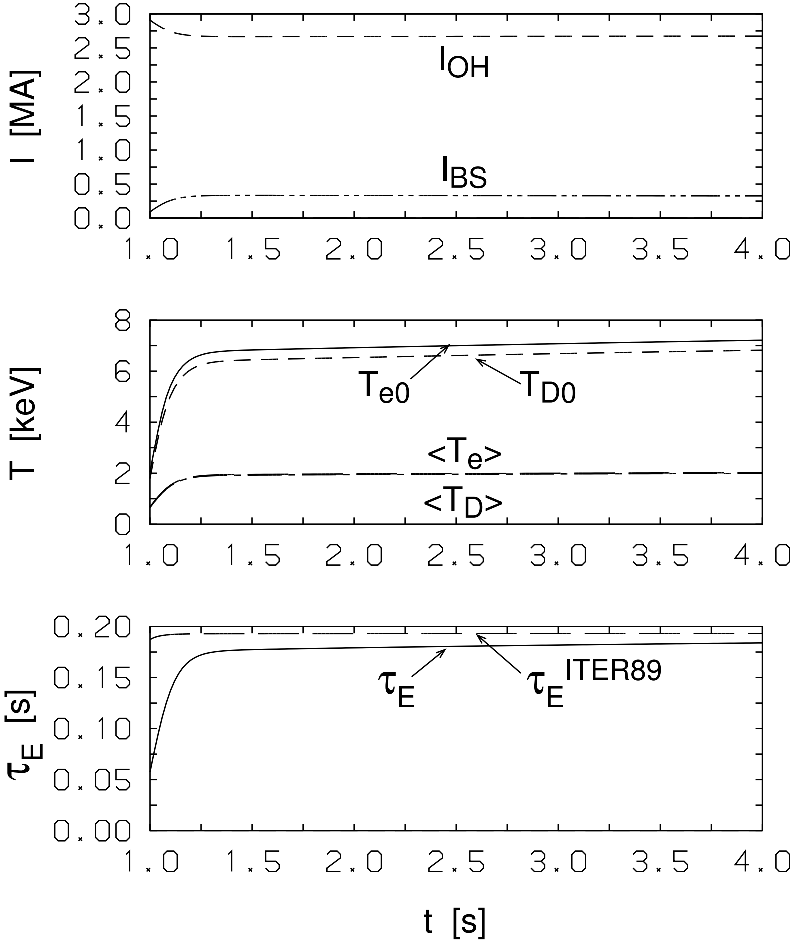


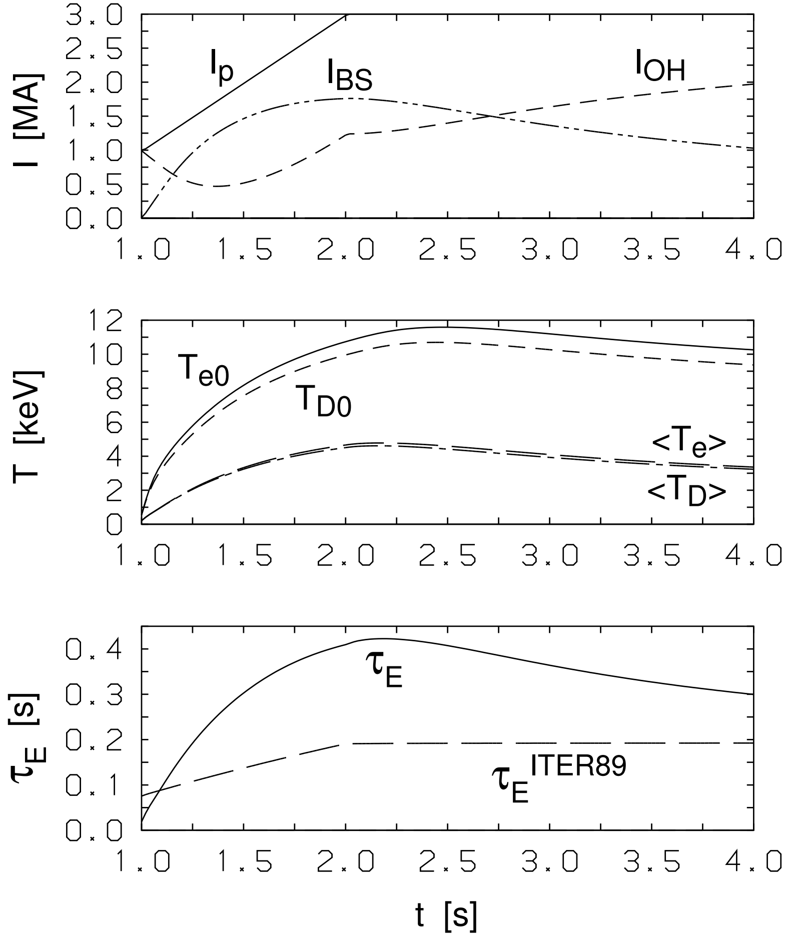


Simulation of Reversed Shear Configuration

I_p **: 3 MA constant**
Heating : 20 MW
H factor $\simeq$ 0.95

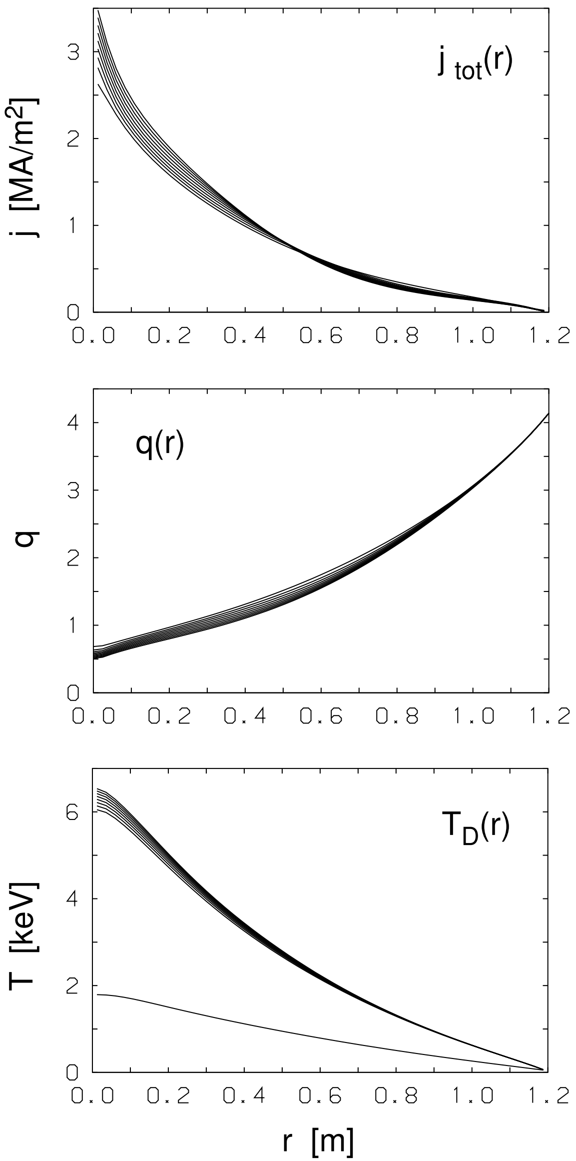
I_p **: 1 MA $\longrightarrow$ 3 MA/1 s**
Heating : 20 MW
H factor $\simeq$ 1.6



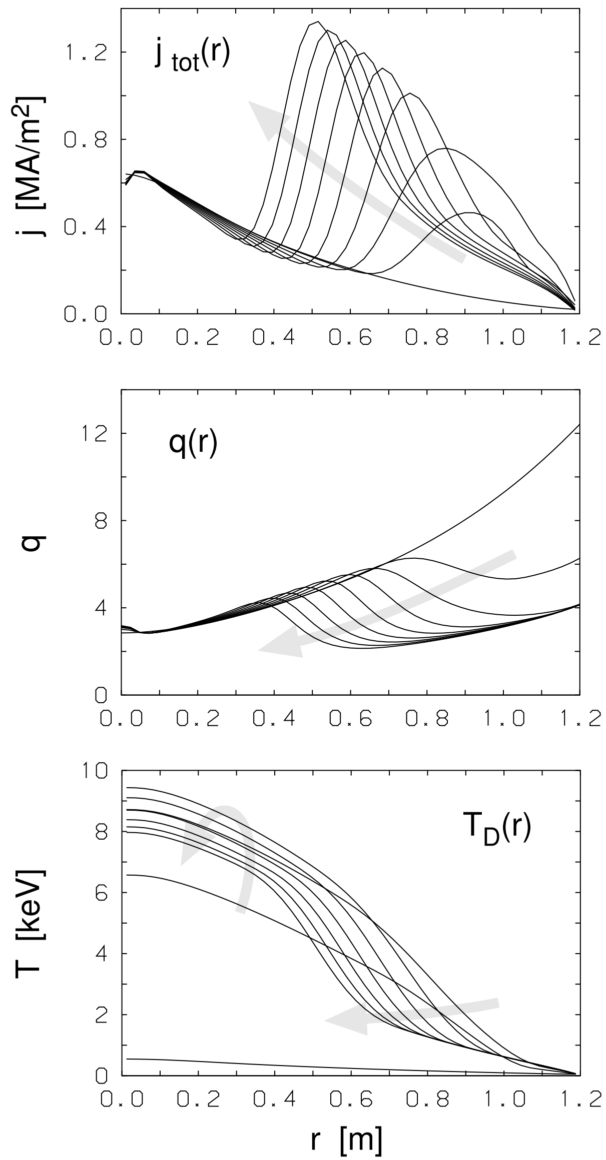


Evolution of Reversed Shear Configuration

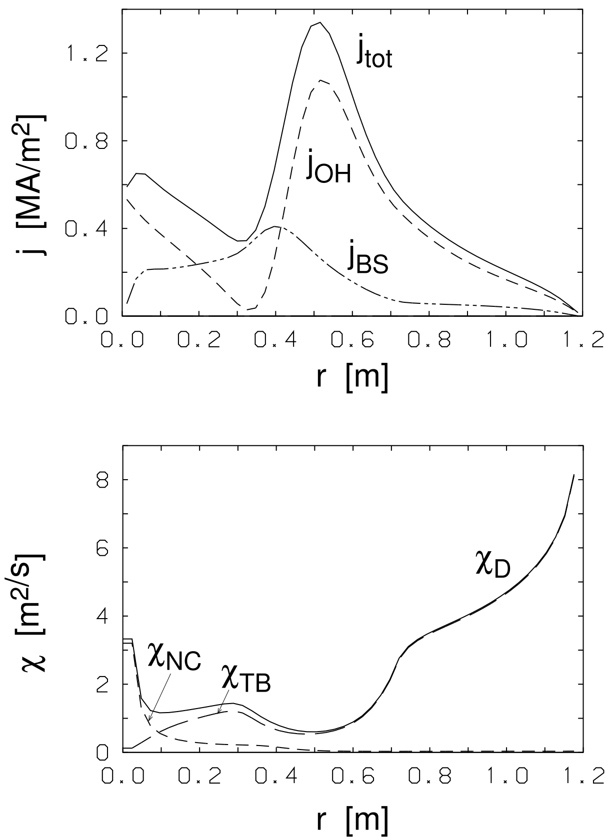
I_p : 3 MA **constant**



I_p : 1 MA ⟶ 3 MA/1 s



$t = 5\,\mathrm{s}$



Bifurcation in the Gradient-Flux Relation

- **Transition in barrier formation is soft or hard?**
  - **ETB: Fast transition of $E_r \longrightarrow$ hard transition**
  - **ITB: Experimental observation ?**

- **Analysis of ITB based on CDBM model**
  - **Constraint: Constant heating power $P_{\rm H}$ inside ITB**
  - **Heat flux:**

$$q_{\rm H} = -n\chi\frac{{\rm d}T}{{\rm d}r} = \frac{P_{\rm H}}{4\pi^2 rR}$$

  - **Pressure gradient:**

$$\alpha = -q^2 R\,\frac{{\rm d}\beta}{{\rm d}r} = nq^2 R\,\frac{2\mu_0}{B^2}\left(1+\frac{1}{\eta_{\rm T}}\right)\frac{{\rm d}T}{{\rm d}r}, \qquad \eta_{\rm T} = \frac{{\rm d}\ln T}{{\rm d}\ln n}$$

  - **Thermal diffusivity:**

$$\chi_{\rm TB} = C\frac{F(s,\alpha)}{1+G\omega_E^2}\,\alpha^{3/2}\,\frac{c^2}{\omega_{\rm pe}^2}\,\frac{v_{\rm A}}{qR}$$

Heat Flux Relation

- **Heat flux relation can be rewritten as**

$$\hat{P}_{\mathrm{H}} = [\hat{\chi}_{\mathrm{TB}} + \hat{\chi}_{\mathrm{NC}}]\,\alpha$$

- **Normalization:** P_{H} **and** χ **are normalized by** P_{H0} **and** χ_0

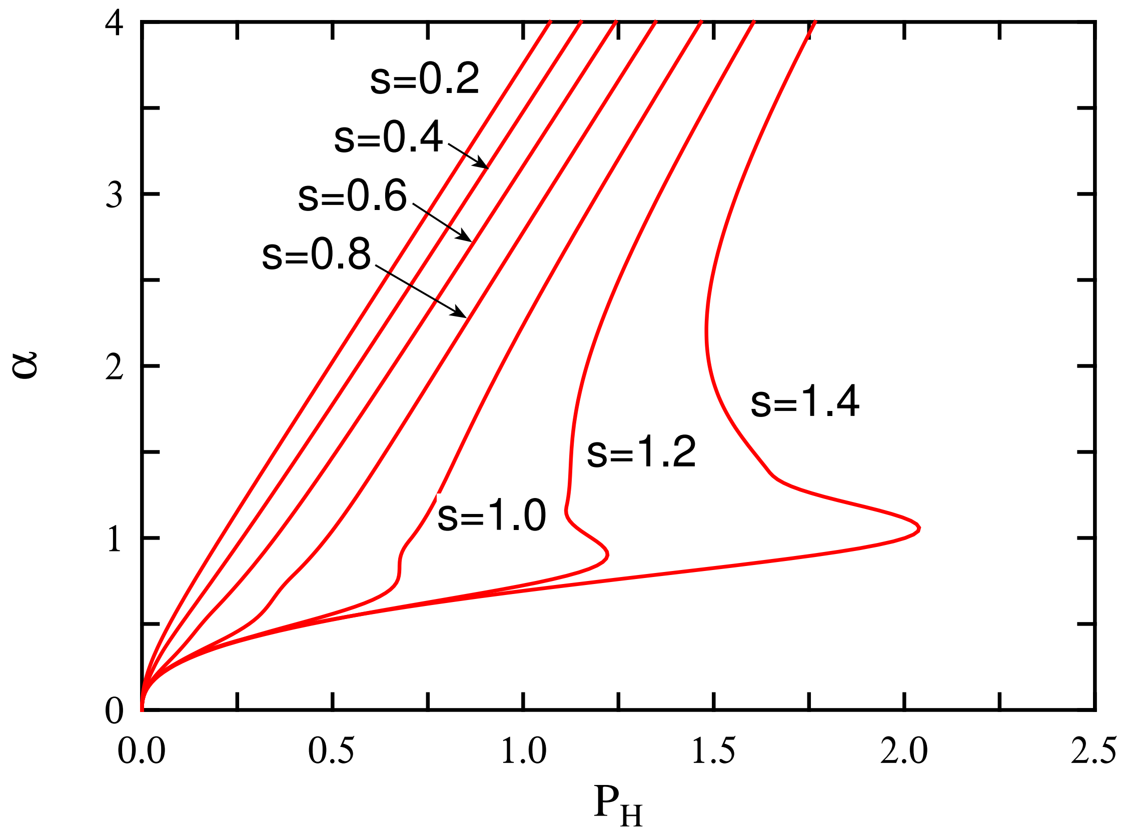
$$P_{\mathrm{H0}} = 2\pi^2 \frac{r}{qR}\,\frac{B^2}{\mu_0}\,\frac{\eta_{\mathrm{T}}}{1+\eta_{\mathrm{T}}}\,\chi_0, \qquad \chi_0 = C\,\frac{c^2}{\omega_{\mathrm{pe}}^2}\,\frac{v_{\mathrm{A}}}{qR}$$

- **Therefore**

$$\hat{P}_{\mathrm{H}} = \frac{P_{\mathrm{H}}}{P_{\mathrm{H0}}}, \qquad \hat{\chi}_{\mathrm{TB}} = \frac{\chi_{\mathrm{TB}}}{\chi_0} = \frac{F(s,\alpha)}{1+G\omega_{\mathrm{E}}^2}\,\alpha^{3/2}, \qquad \hat{\chi}_{\mathrm{NC}} = \frac{\chi_{\mathrm{NC}}}{\chi_0}$$

Condition of Bifurcation

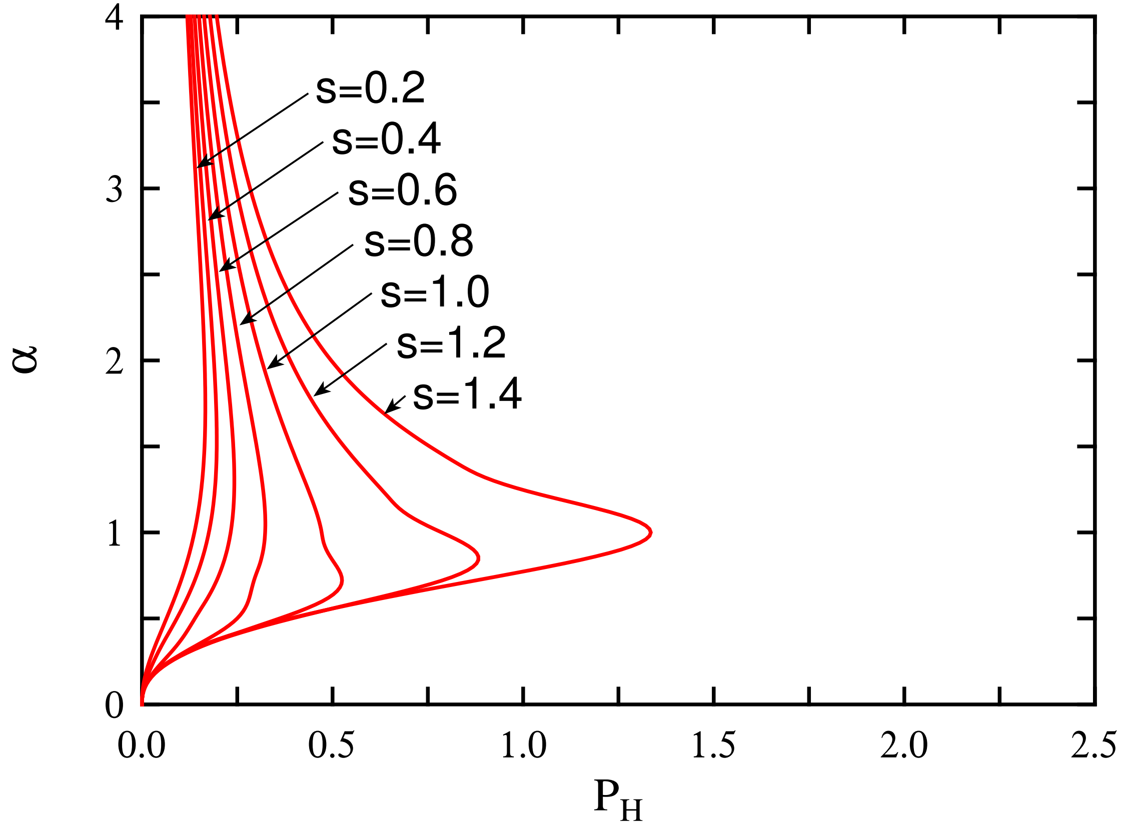
- **Effect of Shafranov shift ($G = 0$, $\hat{\chi}_{\rm NC} = 0$)**



- **For $s \gtrsim 1.0$, bifurcation may occur.**
- **Threshold power: $\hat{P}_{\rm H0} = 1.25$**

Condition of Bifurcation

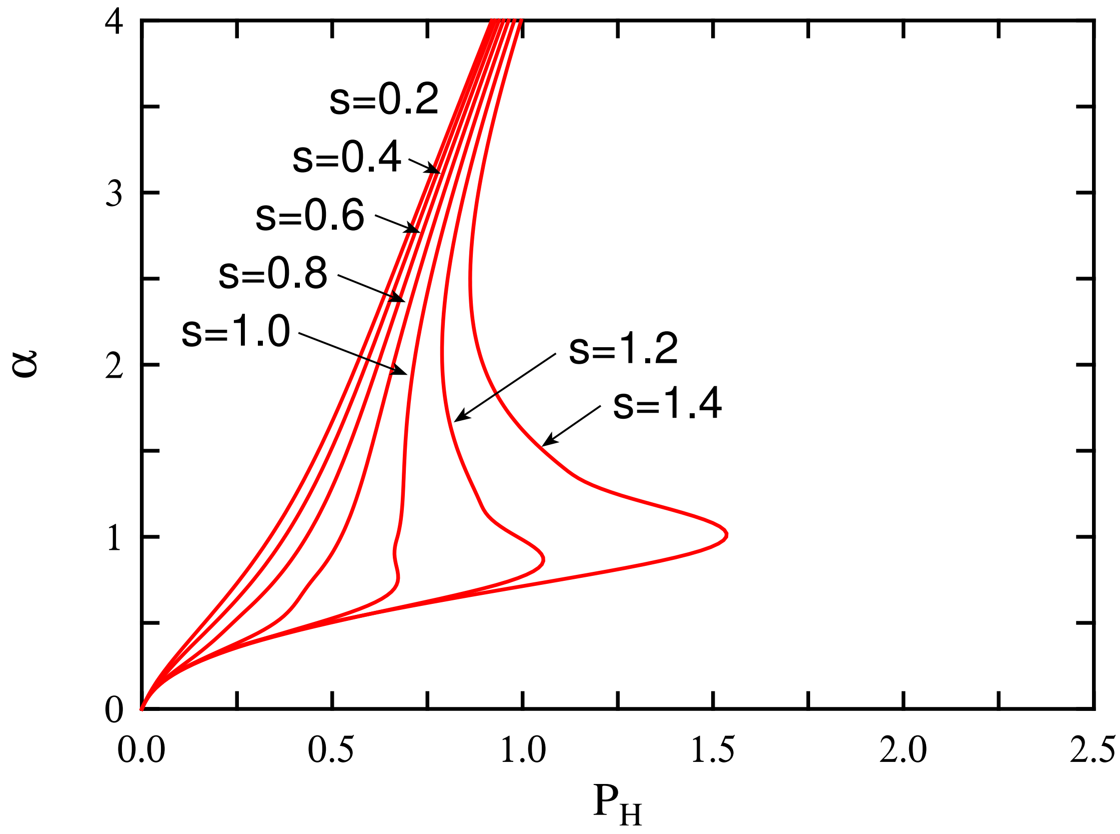
- **Effect of $E \times B$ rotation shear ($G' = 0.5$, $\hat{\chi}_{\rm NC} = 0$)**
- **Approximation:** $G\omega_{\rm E}^2 \simeq G'\alpha^2$



- **Thresholds of both s and $\hat{P}_{\rm H0}$ are reduced.**

Condition of Bifurcation

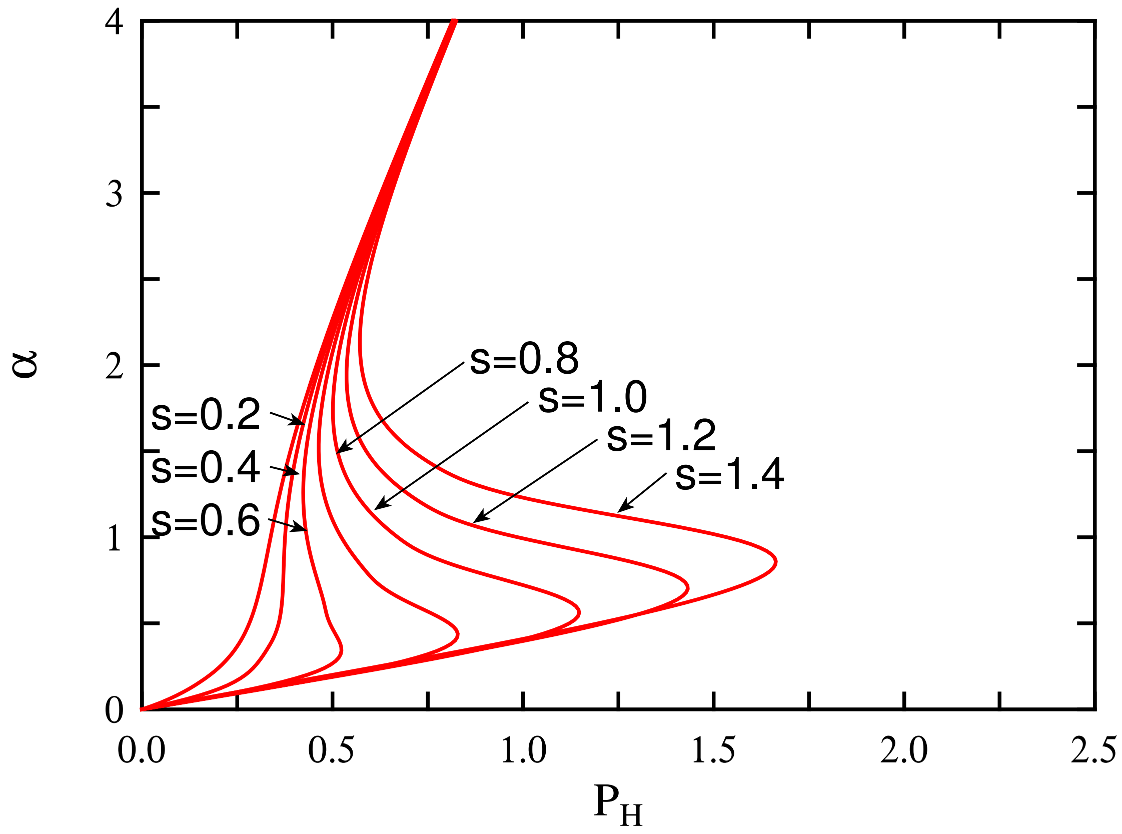
- **Effect of neoclassical transport** ($G' = 0.5$, $\hat{\chi}_{\rm NC} = 0.2$)
- **Approximation:** $G\omega_{\rm E}^2 \simeq G'\alpha^2$



- α **after transition is finite but large.**

Condition of Bifurcation

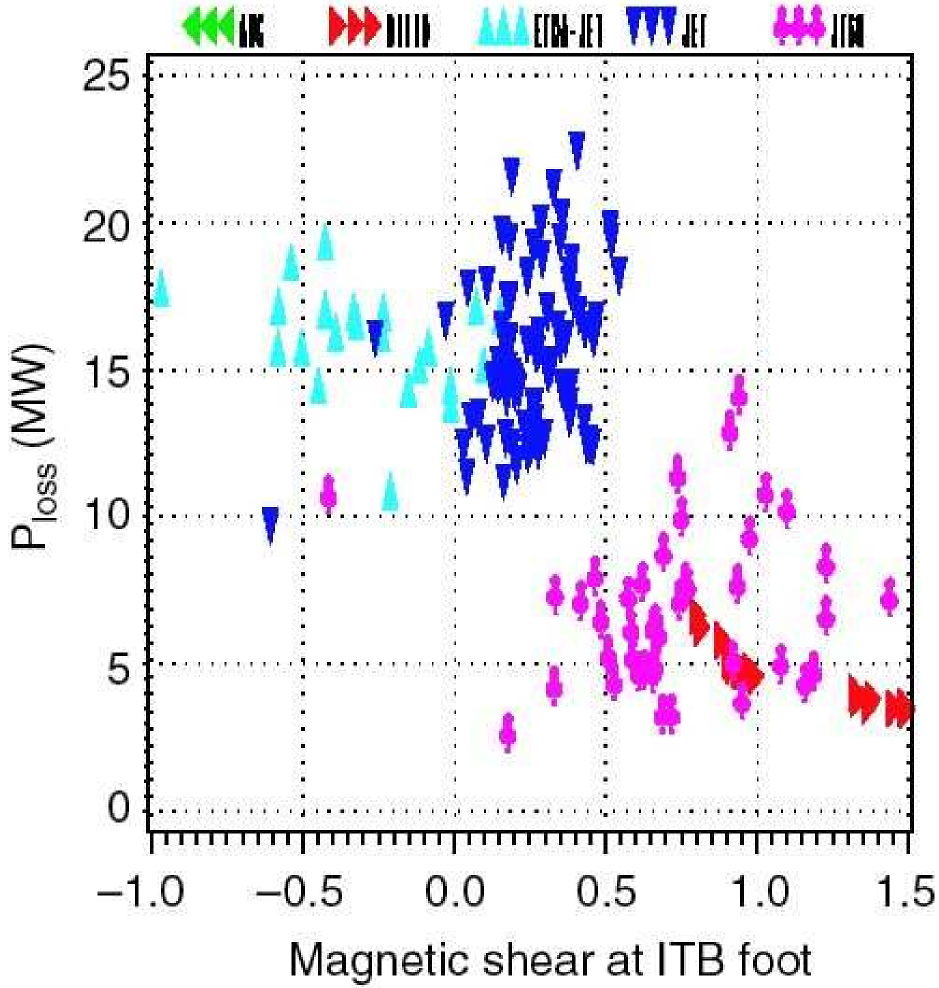
- **Without α dependence of χ ($G' = 0.5$, $\hat{\chi}_{\rm NC} = 0.2$)**
- **Approximation:** $G\omega_{\rm E}^2 \simeq G'\alpha^2$



- **Weak dependence on α leads to lower s for hard transition.**

Preliminary Experimental Observation

- ACC Sips et al., PPCF 44 (2002) A391
- s dependence of heating power to form ITB



Summary

- We have examined the possibility of bifurcation in transport barrier formation based on the ballooning type transport model.

- In the high β_p mode, hard transition may occur for $s \gtrsim 1.0$.

- The effect of $E \times B$ rotation shear reduces the threshold of both s and $\hat{P}_\mathrm{H}$. The effect of $\hat{\chi}_\mathrm{NC}$ has to be taken into account to obtain finite α solution.

- In the case of low or negative magnetic shear, soft transition is dominant.

- These behaviors are consistent with preliminary experimental observation.

- Work in progress
  - Formulation including, BM, ITG and Drift-Alfvén wave.
  - Transport simulation including plasma rotation.