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Modeling of Transport Barrier Based on Drift Alfvén Ballooning Mode Transport Model

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Motivation

- **Development of Robust Transport Model**
  - **L-mode confinement time scaling**
  - **Large transport near the plasma edge in L-mode**
  - **H-mode confinement time scaling for given edge temperature**
  - **Formation of internal transport barrier**
  - **Profile database (ITPA)**
  - **Behavior of fluctuation**

- **Purpose of the present model**
  - **To describe both**
    - **Electrostatic ITG mode**
      - enhanced transport for large ion temperature gradient
    - **Electromagnetic Ballooning mode (CDBM)**
      - transport reduction for negative $s - \alpha$: ITB formation

$$\text{magnetic shear: } s = \frac{r}{q}\frac{\mathrm{d}q}{\mathrm{d}r}$$

$$\text{pressure gradient (Shafranov shift): } \alpha = -q^2 R \frac{\mathrm{d}\beta}{\mathrm{d}r}$$

Turbulent Transport Model

CDBM
Reduced MHD equation
Electromagnetic
Incompressible

Toroidal ITG[h
Ion Fluid Equation
Electrostatic
Boltzmann Distribution of Electron

Drift Alfvén Ballooning Mode
Reduced Two-Fluid Equation
Electromagnetic
Compressible

- **Small pressure gradient: Electrostatic ITG**
- **Large pressure gradient: Electromagnetic BM**
- **Without drift motion, it reduces to CDBM**
- **Ion parallel viscosity and compressibility destabilizes the mode**
- $s - \alpha$ **dependence similar to the CDBM mode**

Reduced Two-Fluid Equation (Slab Plasma)

- **Equation of Vorticity**

$$\left[\frac{n_{\rm i}\Lambda_0}{\Omega_{\rm i}B_0} - \frac{\epsilon_0}{e} - \frac{n_{\rm e}\Lambda_{0\rm e}}{\Omega_{\rm e}B}\right]\frac{\partial\nabla_\perp^2\phi_1}{\partial t} + \frac{n_{\rm i}}{\Omega_{\rm i}B}\frac{\partial}{\partial t}\left(\frac{\nabla_\perp^2 p_{1\rm i}}{q_{\rm i}n_{\rm i}}\right) - \frac{n_{\rm e}}{\Omega_{\rm e}B}\frac{\partial}{\partial t}\left(\frac{\nabla_\perp^2 p_{1\rm e}}{q_{\rm e}n_{\rm e}}\right)$$

$$-\nabla_\parallel(n_{\rm i}v_{1\parallel\rm i} - n_{\rm e}v_{1\parallel\rm e}) + \left(\frac{{\rm i}en_{\rm i}\Lambda_{0\rm i}\omega_{*\rm i}}{T_{\rm i}} + \frac{{\rm i}en_{\rm e}\Lambda_{0\rm e}\omega_{*\rm e}}{T_{\rm e}}\right)\phi_1$$

$$= \frac{1}{eB}\left(\boldsymbol{b}\times\boldsymbol{\kappa} + \boldsymbol{b}\times\frac{\nabla B}{B}\right)\cdot(\nabla p_{1\rm i} + \nabla p_{1\rm e})$$

- **Parallel Equation of Motion** $(j = {\rm e}, {\rm i})$

$$m_j n_{0j}\frac{\partial v_{1j\parallel}}{\partial t} + \nabla_\parallel p_{1j} - q_j n_{0j}E_{1\parallel} = 0$$

- **Equation of State** $(j = {\rm e}, {\rm i})$

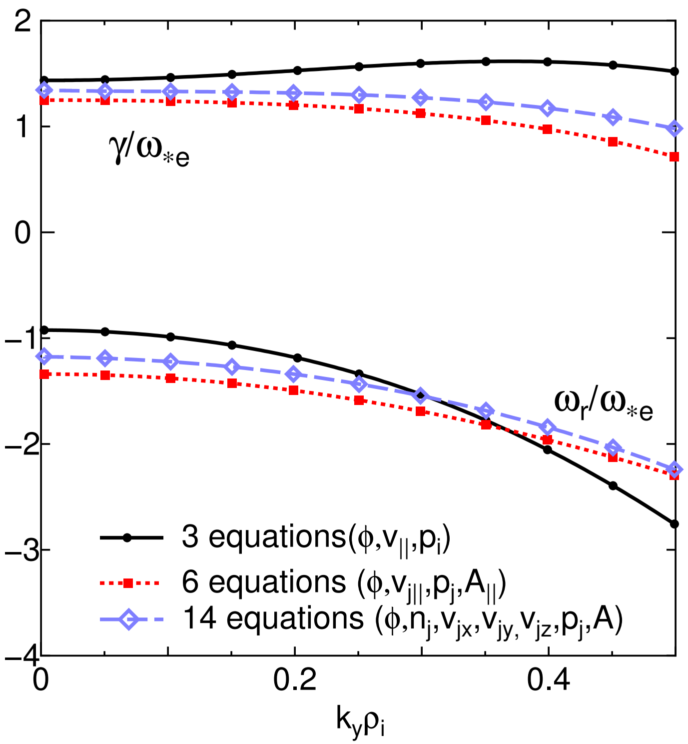
$$\frac{\partial p_{j1}}{\partial t} + \boldsymbol{v}_{E1}\cdot\nabla p_{j0} + \Gamma_j p_{j0}\nabla_\parallel v_{\parallel j1} = 0$$

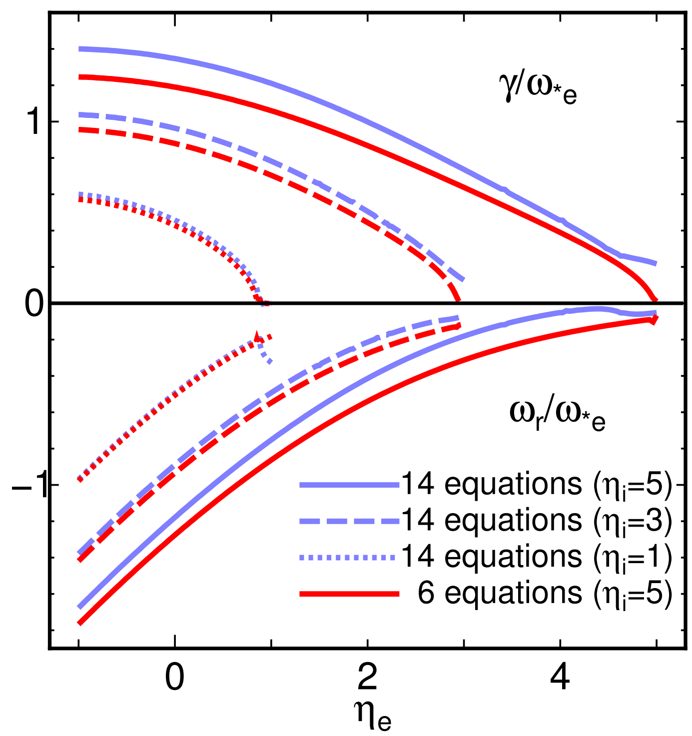
- **Ampere's Law**

$$\nabla_\perp^2 A_{1\parallel} = -\mu_0\sum_j(n_0 q_j v_{1j\parallel})$$

Linear Analysis: Slab Plasma

- **Slab ITG mode** (Comparison of three models)
  - **Ion Fluid Model** (3 eqs.)
  - **Reduced Two-Fluid Model** (6 eqs.)
  - **Full Two-Fluid Model** (14 eqs.)





- **Reduced two-fluid model is very close to the full two-fluid model.**

Reduced Two-Fluid Equation (Toroidal Plasma)

- **Ballooning transformation**: ξ
- **Equation of Vorticity**

$$\frac{-\mathrm{i}\omega}{\Omega_\mathrm{i} B_0}\frac{m^2}{r^2}f^2\left(n_{0\mathrm{i}}\Lambda_{0\mathrm{i}}\phi_1+\frac{p_{1\mathrm{i}}}{q_\mathrm{i}}\right)-\frac{-\mathrm{i}\omega}{\Omega_\mathrm{e} B_0}\frac{m^2}{r^2}f^2\left(n_{0\mathrm{e}}\Lambda_{0\mathrm{e}}\phi_1+\frac{p_{1\mathrm{e}}}{q_\mathrm{e}}\right)$$

$$-\frac{-\mathrm{i}\omega e m^2 f^2\phi_1}{\epsilon_0 r^2}+\frac{B_\theta}{rB_0}\frac{\partial}{\partial\xi}(n_{0\mathrm{i}}v_{1j\parallel}-n_{0\mathrm{e}}v_{1\mathrm{e}\parallel})-\frac{\mathrm{i}mB_\varphi}{erR_0B_0^2}H(\xi)(p_{1\mathrm{i}}+p_{1\mathrm{e}})=0$$

- **Parallel Equation of Motion** $(j=\mathrm{e},\mathrm{i})$

$$-\mathrm{i}\omega m_j n_{0j}v_{1j\parallel}+\frac{B_\theta}{rB_0}\frac{\partial p_1}{\partial\xi}+q_j n_{0j}\left(\frac{B_\theta\Lambda_{0j}}{rB_0}\frac{\partial\phi}{\partial\xi}-\mathrm{i}\omega_{*aj}A_\parallel\right)=0$$

- **Equation of State** $(j=\mathrm{e},\mathrm{i})$

$$-\mathrm{i}\omega p_{1j}-\mathrm{i}q_j n_{0j}\Lambda_{0j}\omega_{*j}(1+\eta_j)\phi+\frac{\Gamma_j p_{0j}B_\theta}{rB_0}\frac{\partial v_{1j\parallel}}{\partial\xi}=0$$

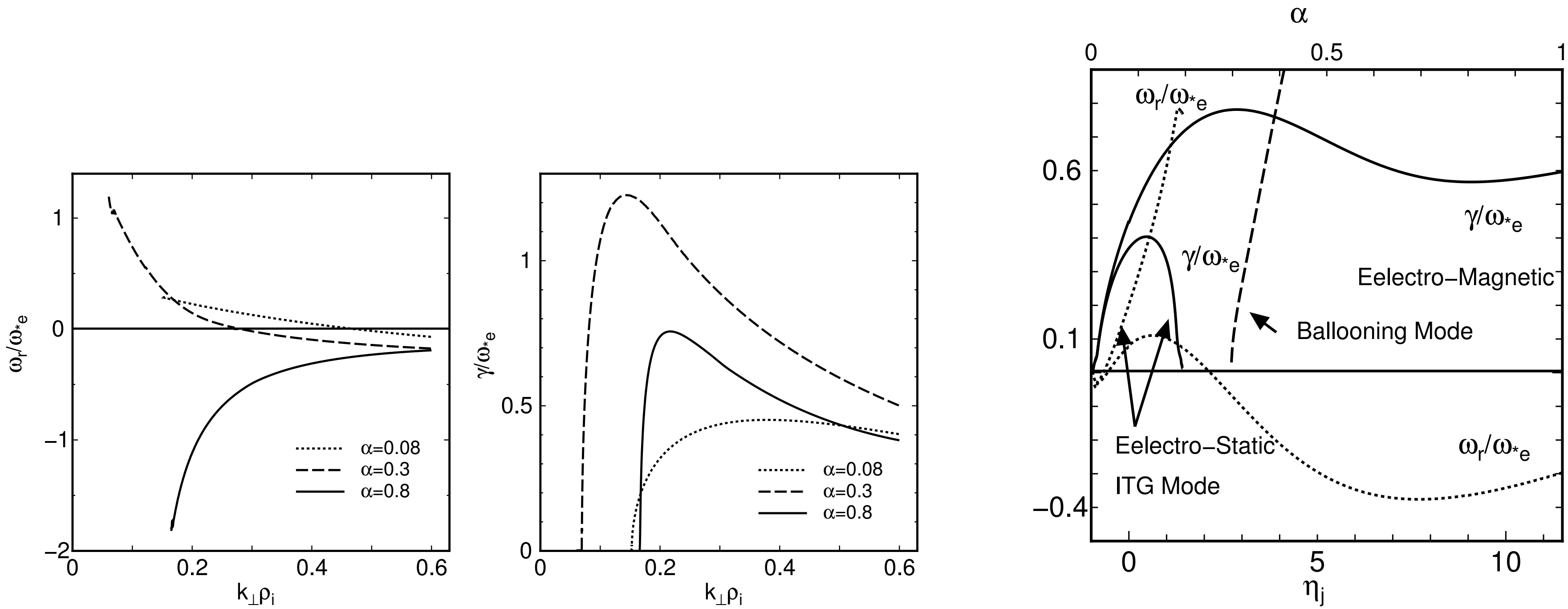
- **Ampere's law**

$$-\frac{m^2}{r^2}f^2A_\parallel=-\mu_0 e(n_{0\mathrm{i}}v_{1\mathrm{i}\parallel}-n_{0\mathrm{e}}v_{1\mathrm{e}\parallel})$$

- $H(\xi)\equiv\kappa_0+\cos\xi+(s\xi-\alpha\sin\xi)\sin\xi,\ f^2(\xi)=1+(s\xi-\alpha\sin\xi)^2$

Linear Analysis: Toroidal Plasma

- **Ballooning mode** (Transition from electrostatic to electromagnetic)
  - **Electrostatic Toroidal ITG Mode**
  - **Electromagnetic Ballooning Mode**



- **Electromagnetic effect becomes dominant for large η_{i} or α**

Nonlinear Reduced Two-Fluid Equation (Toroidal Plasma)

- **Turbulent transport coefficients are included.**

$$\frac{\mathrm{d}}{\mathrm{d}t}X_j \longrightarrow -\mathrm{i}\omega X_j - \chi_j \nabla_\perp^2 X_j, \qquad \chi_j = \frac{\langle \phi_1^2 \rangle}{\gamma_{nj}} \sim \sqrt{\langle \phi_1^2 \rangle}$$

- **Equation of Vorticity**:

$$\left[(-\mathrm{i}\omega + \mu_\mathrm{i}\frac{m^2 f^2}{r^2})\frac{n_{0\mathrm{i}}}{\Omega_\mathrm{i} B_0} - \frac{-\mathrm{i}\omega e}{\epsilon_0} - (-\mathrm{i}\omega + \mu_\mathrm{e}\frac{m^2 f^2}{r^2})\frac{n_{0\mathrm{e}}}{\Omega_\mathrm{e} B_0}\right]\frac{m^2 f^2}{r^2}\phi_1$$

$$+\left(-\mathrm{i}\omega + \chi_\mathrm{i}\frac{m^2 f^2}{r^2}\right)\frac{1}{\Omega_\mathrm{i} B_0}\frac{m^2}{r^2}f^2\frac{p_{1\mathrm{i}}}{q_\mathrm{i}} - \left(-\mathrm{i}\omega + \chi_\mathrm{e}\frac{m^2 f^2}{r^2}\right)\frac{1}{\Omega_\mathrm{e} B_0}\frac{m^2}{r^2}f^2\frac{p_{1\mathrm{e}}}{q_\mathrm{e}}$$

$$+\frac{B_\theta}{rB_0}\frac{\partial}{\partial\xi}(n_{0\mathrm{i}}v_{1j\parallel} - n_{0\mathrm{e}}v_{1\mathrm{e}\parallel}) - \frac{\mathrm{i}mB_\varphi}{erR_0B_0^2}H(\xi)(p_{1\mathrm{i}} + p_{1\mathrm{e}}) = 0$$

- **Parallel Equation of Motion**

$$m_j n_{0j}\left(-\mathrm{i}\omega + \mu_j\frac{m^2 f^2}{r^2}\right)v_{1j\parallel} + \frac{B_\theta}{rB_0}\frac{\partial p_1}{\partial\xi} + q_j n_{0j}\left(\frac{B_\theta\Lambda_{0j}}{rB_0}\frac{\partial\phi}{\partial\xi} - \mathrm{i}\omega_{*aj}A_\parallel\right) = 0$$

- **Equation of State**

$$\left(-\mathrm{i}\omega + \chi_j\frac{m^2 f^2}{r^2}\right)p_{1j} - \mathrm{i}q_j n_{0j}\Lambda_{0j}\omega_{*j}(1+\eta_j)\phi + \frac{\Gamma_j p_{0j} B_\theta}{rB_0}\frac{\partial v_{1j\parallel}}{\partial\xi} = 0$$

- **Ampere's Law**

$$-\frac{m^2}{r^2} f^2 A_{\parallel} = -\mu_0 e(n_{0\mathrm{i}} v_{1\mathrm{i}\parallel} - n_{0\mathrm{e}} v_{1\mathrm{e}\parallel})$$

- **CDBM Eigenmode Equation**

$$\frac{\partial}{\partial \xi} \frac{\gamma f}{\gamma + \eta_{\mathrm{r}} m^2 f + \lambda m^4 f^2} \frac{\partial \phi_1}{\partial \xi} - (\gamma^2 f + \gamma \mu m^2 f)\phi_1 + \frac{\alpha \gamma}{\gamma + \chi m^2 f} H(\xi)\phi = 0$$

Marginal Stability Condition ($\gamma = 0$)

$$\frac{1}{\lambda} \frac{\partial^2 \phi}{\partial \xi^2} - \mu m^6 f^3 \phi + \frac{\alpha m^2 f}{\chi} H(\xi)\phi = 0$$

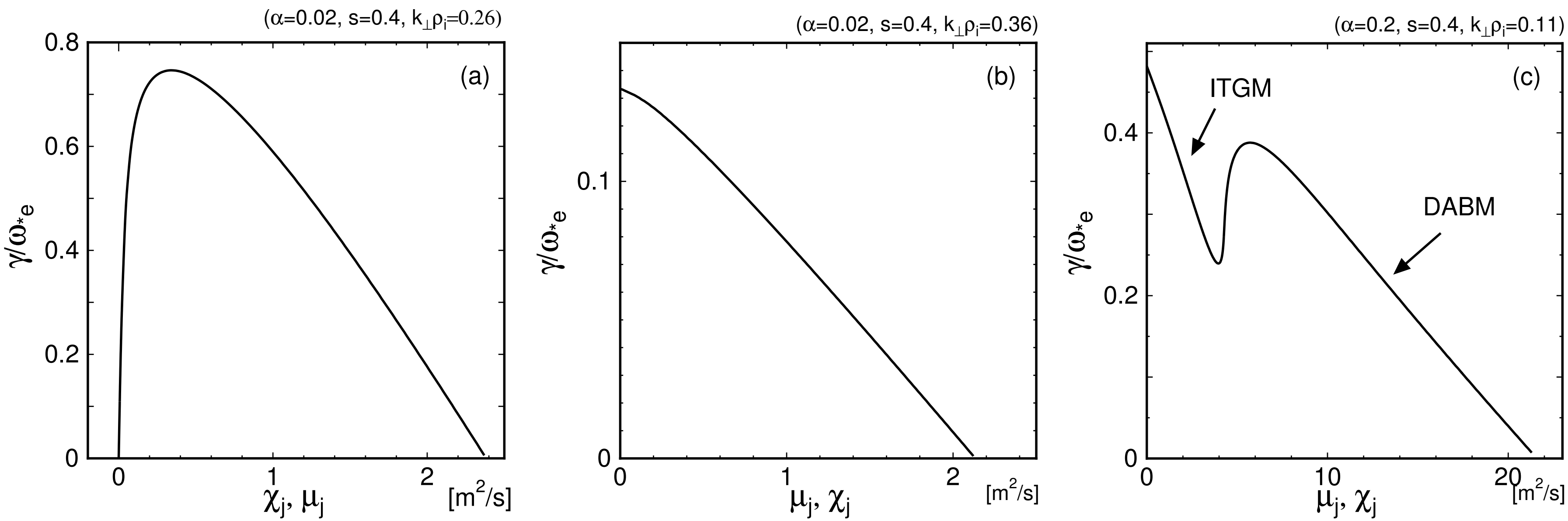
- **Low β DABM Eigenmode Equation**: Marginal Stability Condition

$$\frac{\partial^2 \phi}{\partial \xi^2} - \mu_{\mathrm{i}} \chi_{\mathrm{i}} \frac{\tau_{AP}^2 c^2 m^6 f^3}{r^6 \omega_{p\mathrm{i}}^2} \phi + \frac{m^2 c^2 \alpha_{\mathrm{i}}}{2 r^2 \omega_{p\mathrm{i}}^2} H(\xi)\phi = 0$$

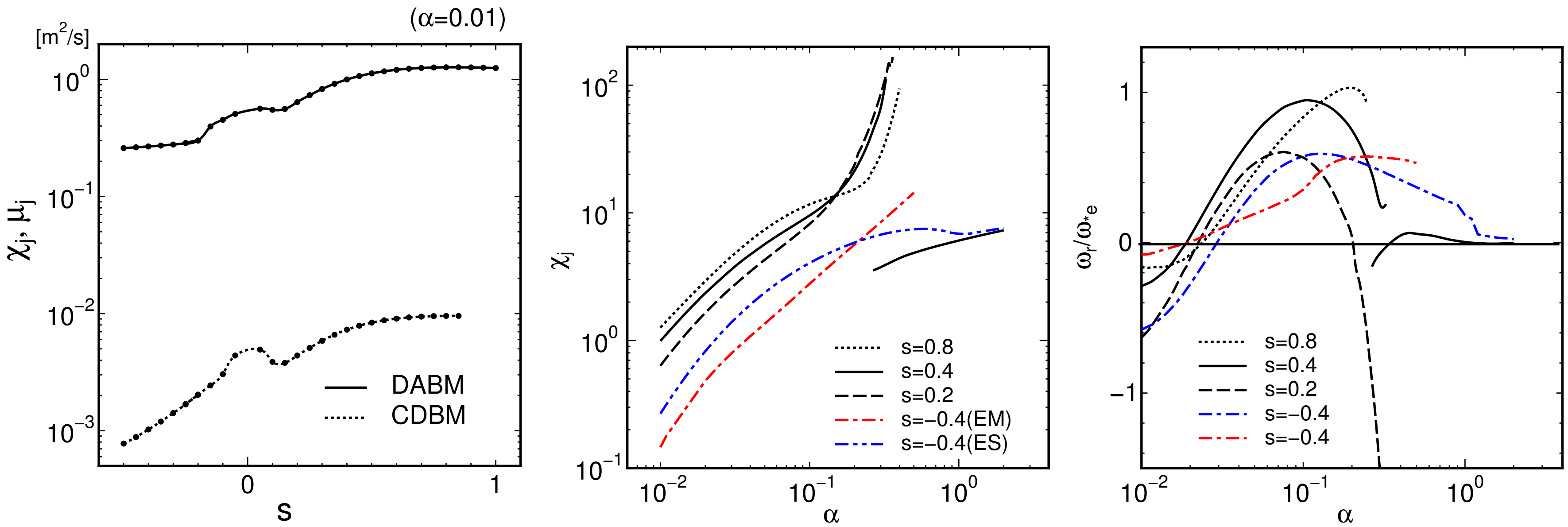
- **Eigenmode equations for CDBM and DABM are similar.**

Nonlinear Analysis (Toroidal Plasma)

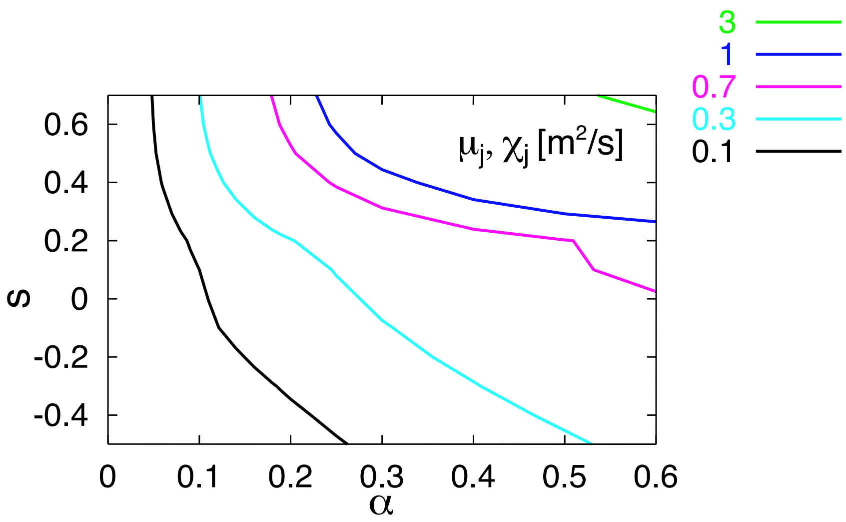
- **Amplitude Dependence of the Growth Rate**
  - Linear growth rate ($\chi_j = 0$) is sensitive to $k_\perp \rho_\mathrm{i}$.
  - For large α, electromagnetic effect becomes important.
  - Saturation level can be estimated from the marginal condition.



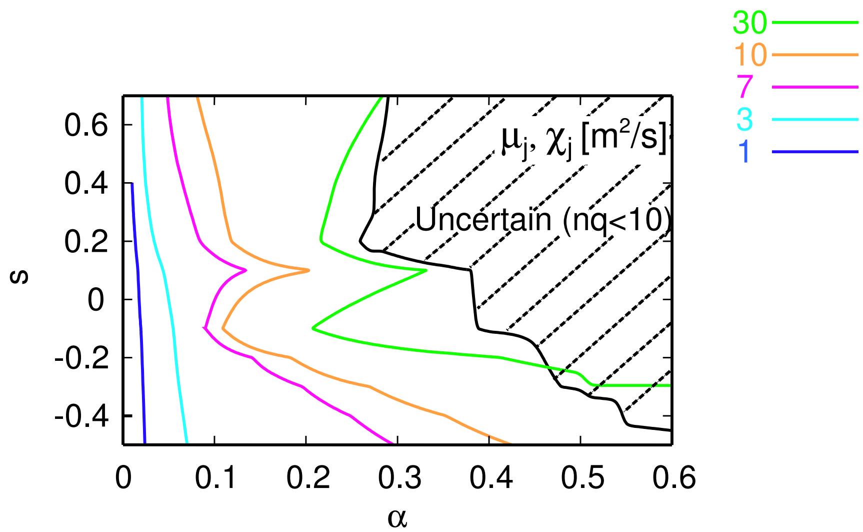
- **Dependence on s and α**
  - **Transport of DABM is much larger than that of CDBM.**
  - **Negative magnetic shear reduces the transport.**
  - **χ is proportional to $\alpha^{3/2}$ for small α.**
  - **There exists critical α above which transport is strongly enhanced.**



- **Contour of χ on s-α plane**



CDBM



DABM

DABM Turbulence Model

- **Low β DABM Eigenmode Equation**: $(\gamma = 0)$

$$\frac{\partial^2 \phi}{\partial \xi^2} - \mu_\mathrm{i}\chi_\mathrm{i}\frac{\tau_{AP}^2 c^2 m^6 f^3}{r^6 \omega_{p\mathrm{i}}^2}\phi + \frac{m^2 c^2 \alpha_\mathrm{i}}{2r^2\omega_{p\mathrm{i}}^2}H(\xi)\phi = 0$$

- **Marginal Stability Condition**

$$\chi_\mathrm{DABM} = F(s,\alpha,\kappa)\,\alpha\,\frac{c^2}{\omega_\mathrm{pe}^2}\,\frac{v_\mathrm{Te}}{qR}$$

Magnetic shear $\quad s \equiv \dfrac{r}{q}\dfrac{\mathrm{d}q}{\mathrm{d}r}$

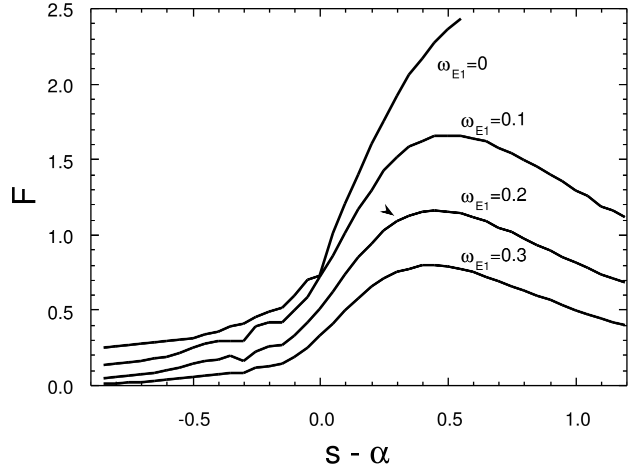
Pressure gradient $\quad \alpha \equiv -q^2 R\dfrac{\mathrm{d}\beta}{\mathrm{d}r}$

Magnetic curvature $\quad \kappa \equiv -\dfrac{r}{R}\left(1-\dfrac{1}{q^2}\right)$

  - Different gradient dependence with CDBM

- **Weak and negative magnetic shear and Shafranov shift reduce χ.**

$s-\alpha$ dependence of $F(s,\alpha,\kappa,\omega_\mathrm{E1})$



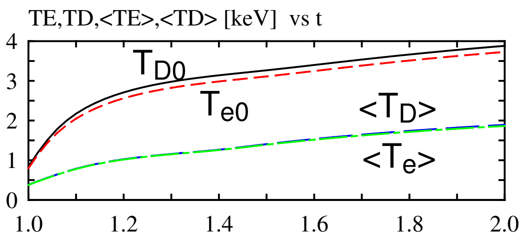
Fitting Formula

$$F = \begin{cases} \dfrac{1}{1+G_1\omega_\mathrm{E1}^2}\dfrac{1}{\sqrt{2(1-2s')(1-2s'+3s'^2)}} & \text{for } s' = s-\alpha < 0 \\[2ex] \dfrac{1}{1+G_1\omega_\mathrm{E1}^2}\dfrac{1+9\sqrt{2}s'^{5/2}}{\sqrt{2}(1-2s'+3s'^2+2s'^3)} & \text{for } s' = s-\alpha > 0 \end{cases}$$

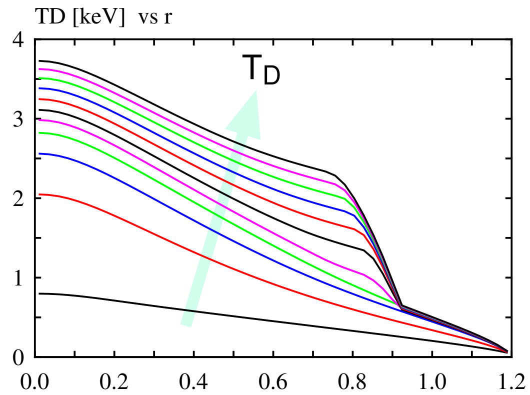
High β_p mode

- Empirical factor of 3 was chosen to reproduce the L-mode scaling.
- $R = 3\,\mathrm{m}$, $a = 1.2\,\mathrm{m}$, $\kappa = 1.5$, $B_0 = 3\,\mathrm{T}$, $I_\mathrm{p} = 1.5\,\mathrm{MA}$
- Time evolution during the first one second after 10 MW heating switched on.

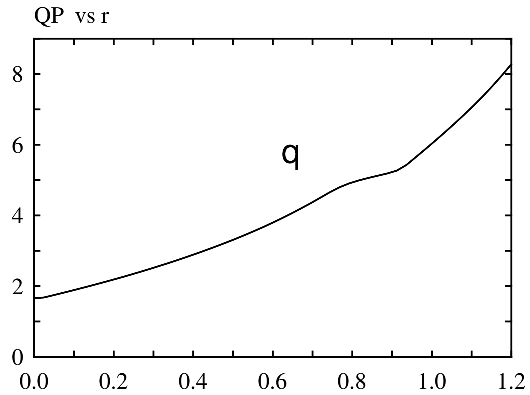
Temperature evolution



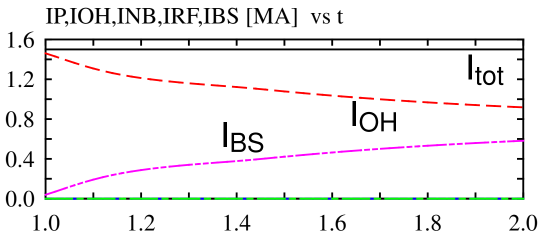
Temperature



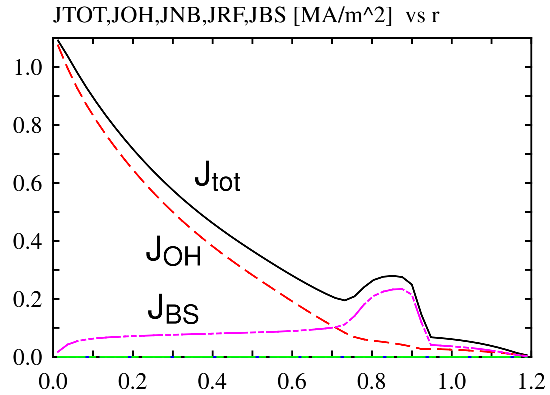
Safety factor



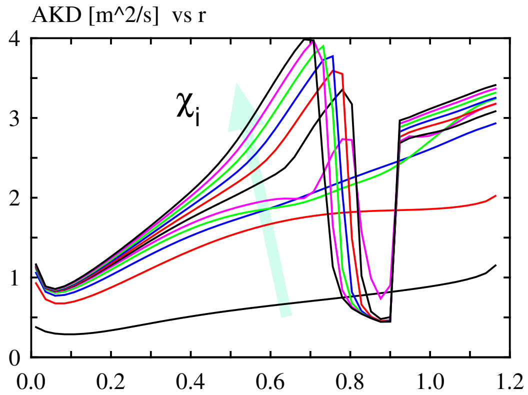
Current evolution



Current density



Thermal diffusivity



Summary

- **In order to describe both the electrostatic ion temperature gradient (ITG) mode and the electromagnetic current diffusive ballooning mode (CDBM), we have derived a set of reduced two-fluid equations in both slab and toroidal configurations and numerically solved them as an eigenvalue problem.**

- **Linear analysis in a toroidal configuration describes a ballooning mode, which we call DABM (Drift Alfvén Ballooning Mode).**
  - **Small pressure gradient: Toroidal ITG mode**
  - **Large pressure gradient: Ballooning mode with stabilizing ω_***

- **Based on the theory of self-sustained turbulence, we have numerically calculated the transport coefficients from the marginal stability condition.**
  - **When α is small, χ is approximately proportional to $\alpha^{3/2}$.**
  - **χ is an increasing function of $s - \alpha$; similar to CDBM model which successfully reproduces the ITB formation.**

- ◦ **When α exceeds a critical value, χ starts to increase strongly with α, which may suggest the stiffness of the profile**

- **Using an electrostatic approximation, we have derived a formula of thermal diffusivity slightly different from the CDBM model.**

- **Preliminary transport simulation reproduces the formation of ITB. The barrier locates in outer region and the gradient is steeper than the CDBM model.**

- **More general expression is required for high-β electromagnetic region.**

CDBM Turbulence Model

- **Marginal Stability Condition** $(\gamma = 0)$

$$\chi_{\text{CDBM}} = F(s, \alpha, \kappa, \omega_{\text{E1}})\, \alpha^{3/2} \frac{c^2}{\omega_{\text{pe}}^2} \frac{v_{\text{A}}}{qR}$$

Magnetic shear $\quad s \equiv \dfrac{r}{q}\dfrac{\mathrm{d}q}{\mathrm{d}r}$

Pressure gradient $\quad \alpha \equiv -q^2 R \dfrac{\mathrm{d}\beta}{\mathrm{d}r}$

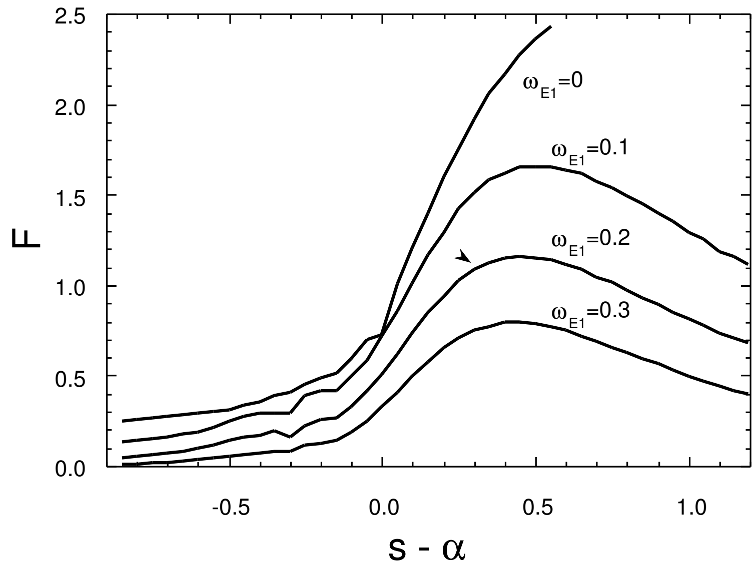
Magnetic curvature $\quad \kappa \equiv -\dfrac{r}{R}\left(1 - \dfrac{1}{q^2}\right)$

$E \times B$ **rotation shear** $\quad \omega_{\text{E1}} \equiv \dfrac{r^2}{s v_{\text{A}}} \dfrac{\mathrm{d}}{\mathrm{d}r} \dfrac{E}{rB}$

- **Weak and negative magnetic shear,**
Shafranov shift and
$E \times B$ **rotation shear**

reduce thermal diffusivity.

$s - \alpha$ dependence of $F(s, \alpha, \kappa, \omega_{\text{E1}})$



Fitting Formula

$$F = \begin{cases} \dfrac{1}{1 + G_1 \omega_{\text{E1}}^2} \dfrac{1}{\sqrt{2(1-2s')(1-2s'+3s'^2)}} & \text{for} \quad s' = s - \alpha < 0 \\[2ex] \dfrac{1}{1 + G_1 \omega_{\text{E1}}^2} \dfrac{1 + 9\sqrt{2} s'^{5/2}}{\sqrt{2}(1 - 2s' + 3s'^2 + 2s'^3)} & \text{for} \quad s' = s - \alpha > 0 \end{cases}$$

Heat Transport Simulation

- **Simple One-Dimensional Analysis**
  - No impurity, No neutral, No sawtooth
  - Fixed density profile: $n_\mathrm{e}(r) \propto (1 - r^2/a^2)^{1/2}$
  - Thermal diffusivity (adjustable parameter $C = 12$)

$$\chi_\mathrm{e} = C\chi_\mathrm{TB} + \chi_\mathrm{NC,e}$$
$$\chi_\mathrm{i} = C\chi_\mathrm{TB} + \chi_\mathrm{NC,i}$$

- **Transport Equation**

$$\frac{\partial}{\partial t}\frac{3}{2}n_\mathrm{e}T_\mathrm{e} = -\frac{1}{r}\frac{\partial}{\partial r} r\, n_\mathrm{e}\chi_\mathrm{e}\frac{\partial T_\mathrm{e}}{\partial r} + P_\mathrm{OH} + P_\mathrm{ie} + P_\mathrm{He}$$

$$\frac{\partial}{\partial t}\frac{3}{2}n_\mathrm{i}T_\mathrm{i} = -\frac{1}{r}\frac{\partial}{\partial r} r\, n_\mathrm{i}\chi_\mathrm{i}\frac{\partial T_\mathrm{i}}{\partial r} - P_\mathrm{ie} + P_\mathrm{Hi}$$

$$\frac{\partial}{\partial t}B_\theta = \frac{\partial}{\partial r}\eta_\mathrm{NC}\left[\frac{1}{\mu_0}\frac{1}{r}\frac{\partial}{\partial r}rB_\theta - J_\mathrm{BS} - J_\mathrm{LH}\right]$$

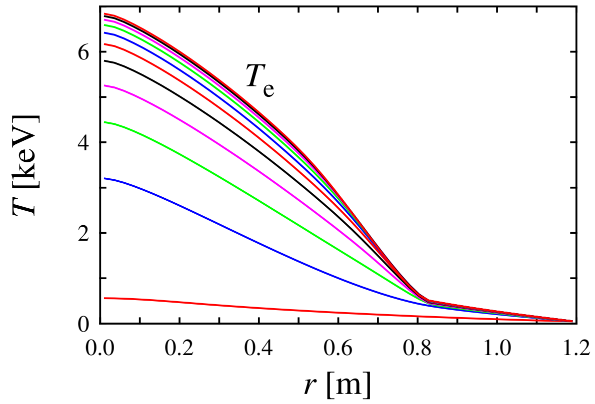
- **Standard Plasma Parameter**

$$R = 3\,\mathrm{m} \quad B_\mathrm{t} = 3\,\mathrm{T} \quad \mathrm{Elongation} = 1.5$$
$$a = 1.2\,\mathrm{m} \quad I_\mathrm{p} = 3\,\mathrm{MA} \quad n_\mathrm{e0} = 5\times10^{19}\,\mathrm{m}^{-3}$$

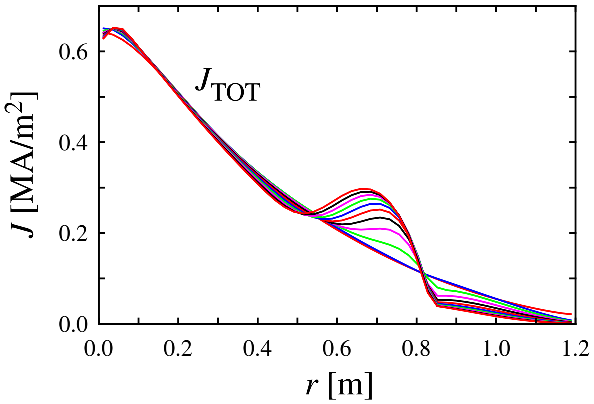
High β_p mode (1)

- $R = 3\,\mathrm{m}$, $a = 1.2\,\mathrm{m}$, $\kappa = 1.5$, $B_0 = 3\,\mathrm{T}$, $I_p = 1\,\mathrm{MA}$
- Time evolution during the first one second after heating switched on

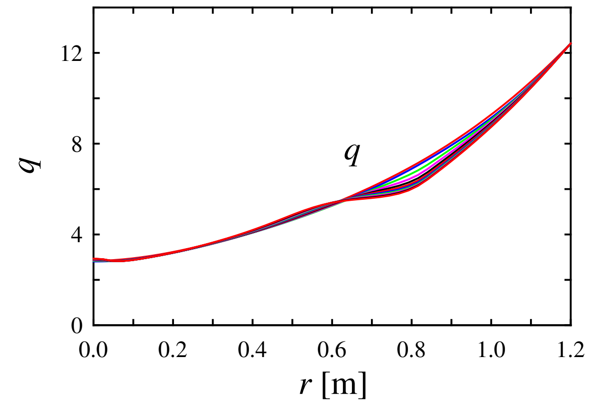
Temperater



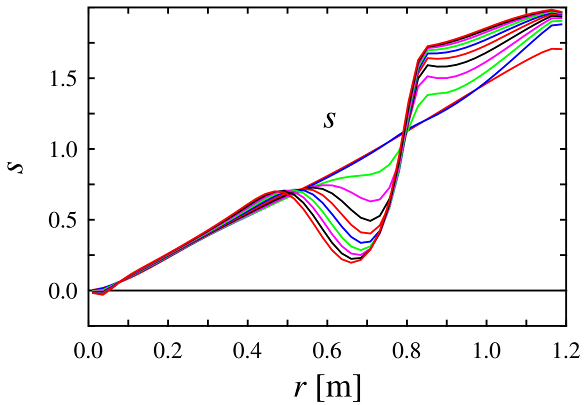
Current



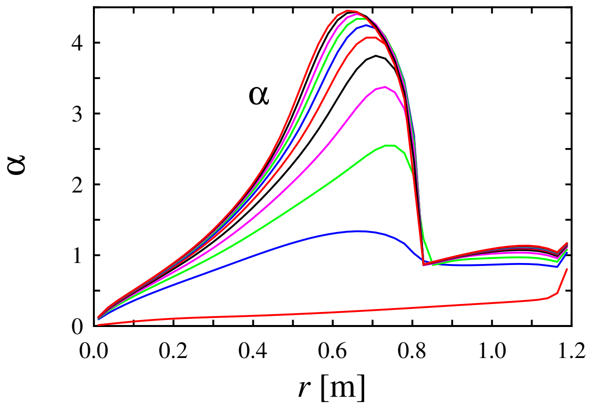
Safety factor



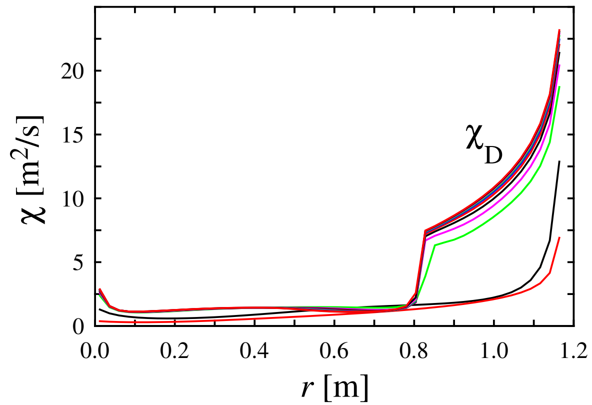
Shear



Normalized Pressure



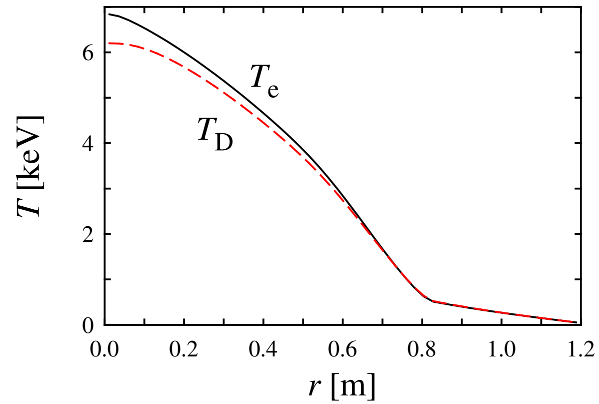
Thermal diffusivity



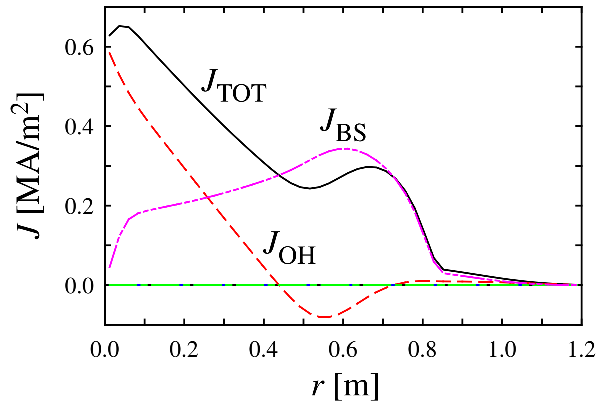
High β_p mode (2)

- One second after heating power of $P_H = 20\,\mathrm{MW}$ was switched on

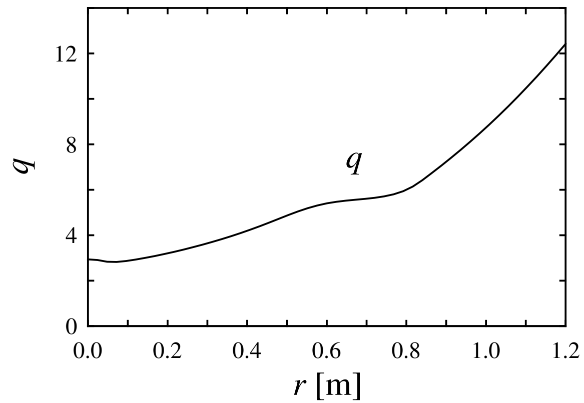
Temperater profile



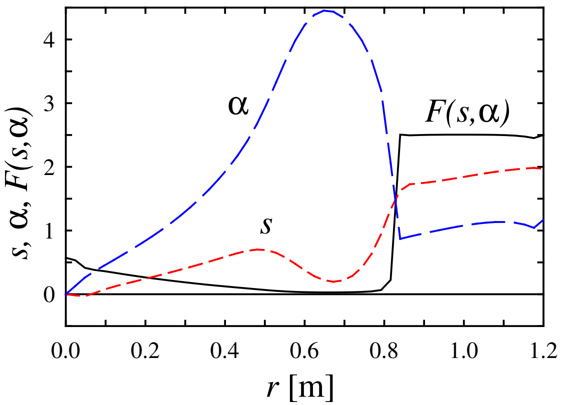
Current profile



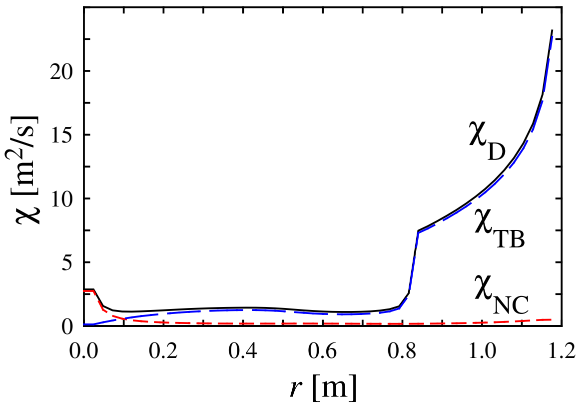
Safety factor



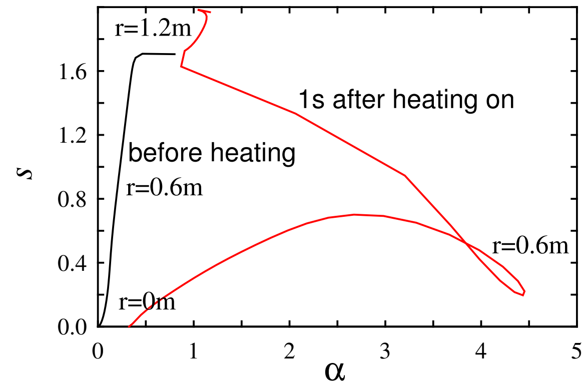
Shear and pressure



Thermal diffusivity

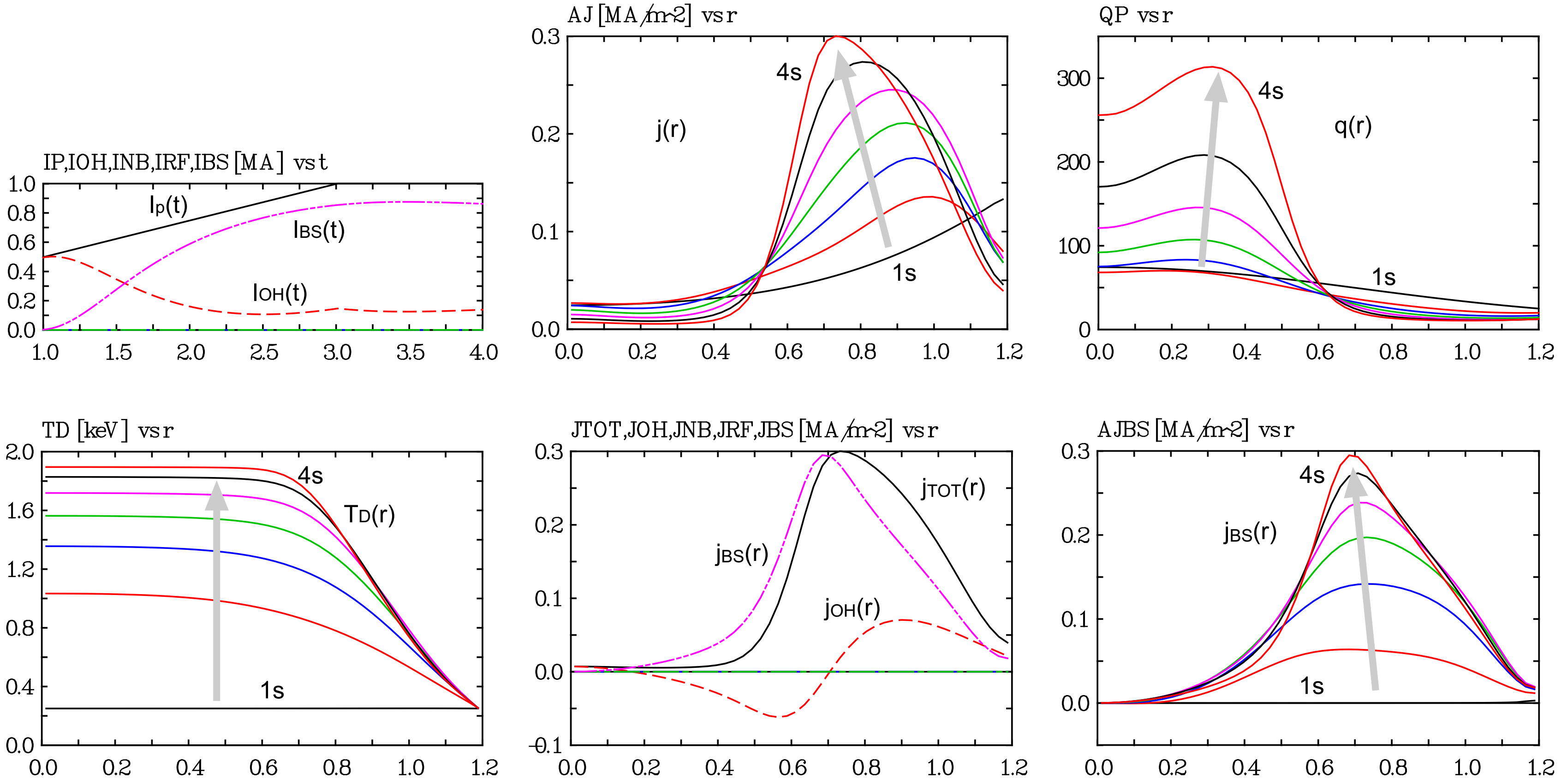


$s - \alpha$ diagram



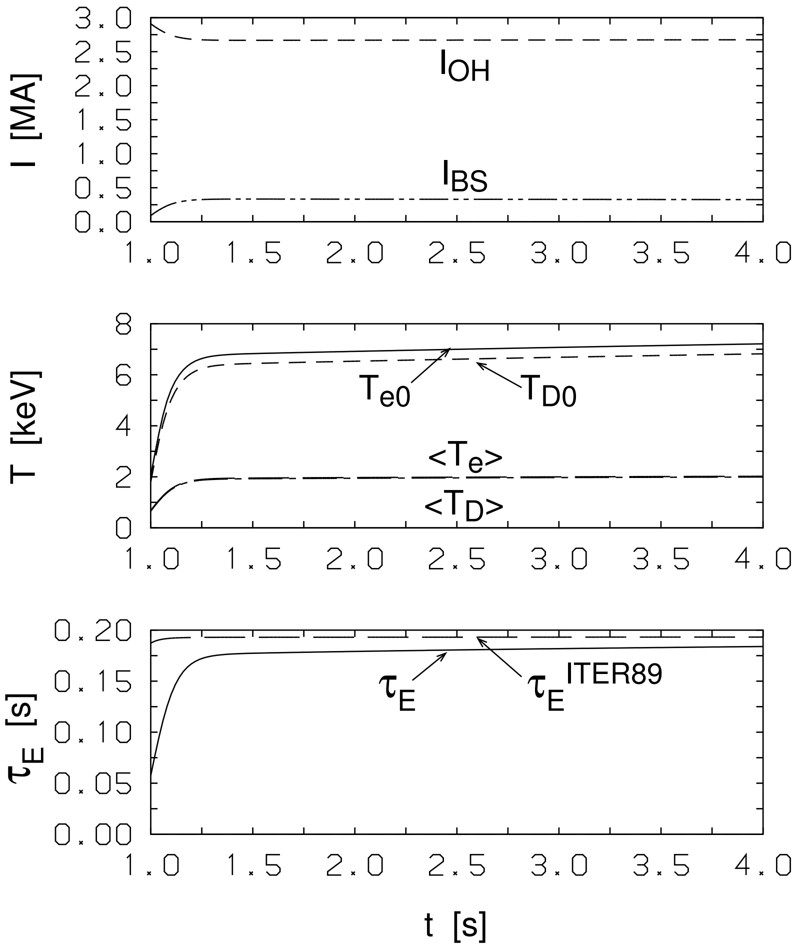
Simulation of Current Hole Formation

- Current ramp up: $I_\mathrm{p} = 0.5 \longrightarrow 1.0\,\mathrm{MA}$
- Moderate heating: $P_\mathrm{H} = 5\,\mathrm{MW}$
- **Current hole** is formed.
- The formation is sensitive to the edge temperature.

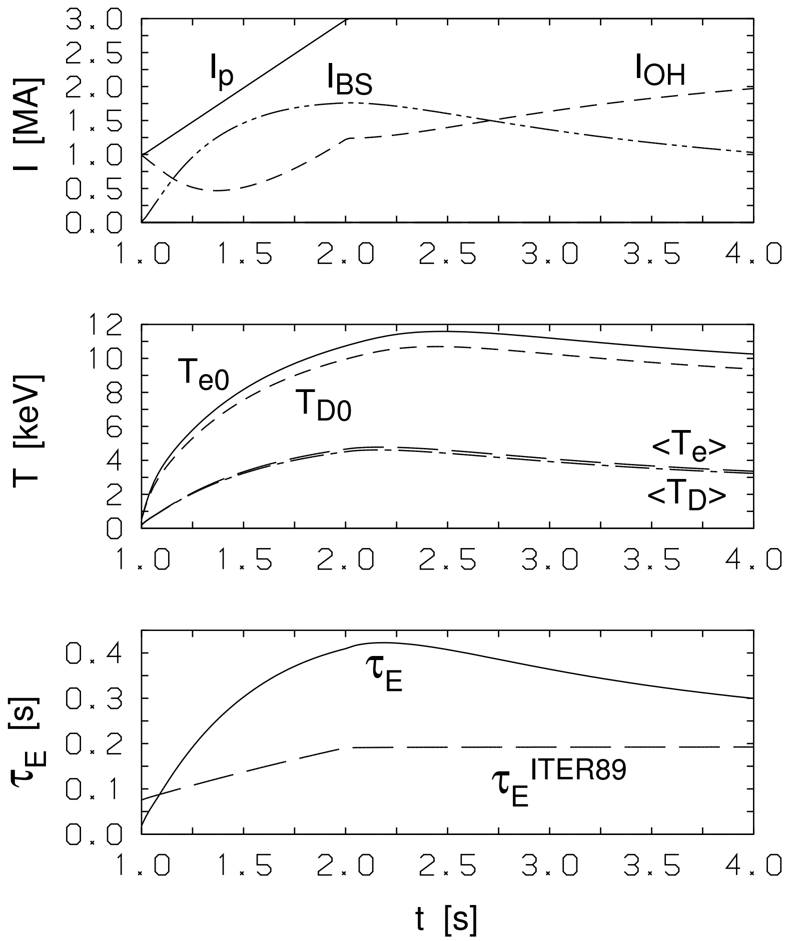


Simulation of Reversed Shear Configuration

I_p : 3 MA constant
Heating : 20 MW
H factor $\simeq$ 0.95

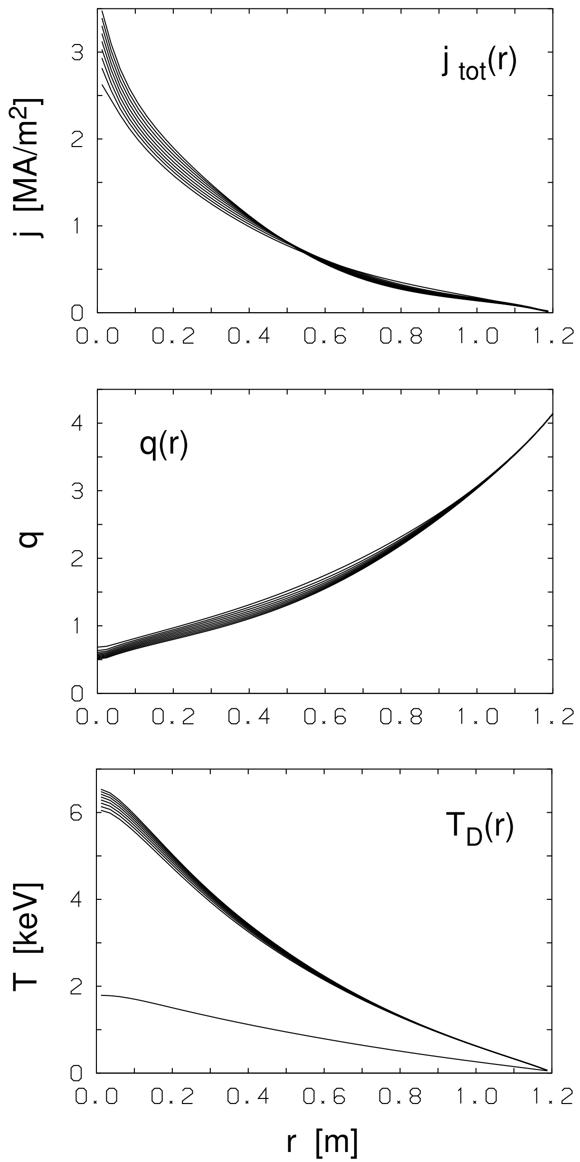


I_p : 1 MA $\longrightarrow$ 3 MA/1 s
Heating : 20 MW
H factor $\simeq$ 1.6

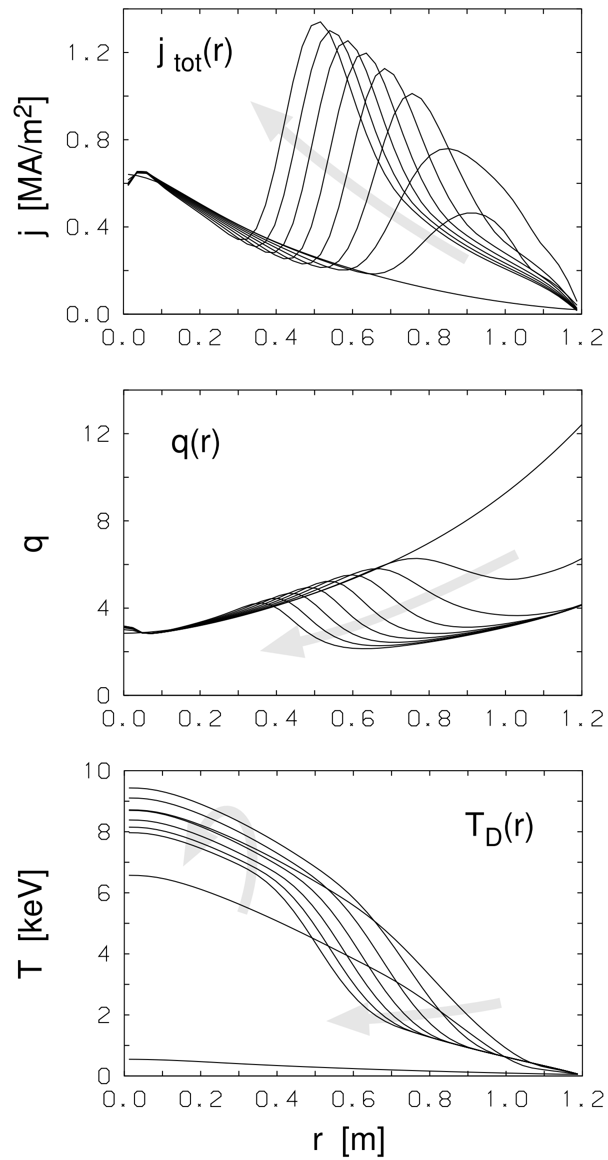


Evolution of Reversed Shear Configuration

I_p : 3 MA **constant**



I_p : 1 MA $\longrightarrow$ 3 MA/1 s



$t = 5\,\mathrm{s}$