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Global Kinetic Analysis of Alfvén Eigenmode in Toroidal Plasmas

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- **Motivation**
- **Alfvén Eigenmodes in Toroidal Plasmas**
- **Analysis of Alfvén Eigenmodes by TASK/WM**
- **Summary**

Motivation

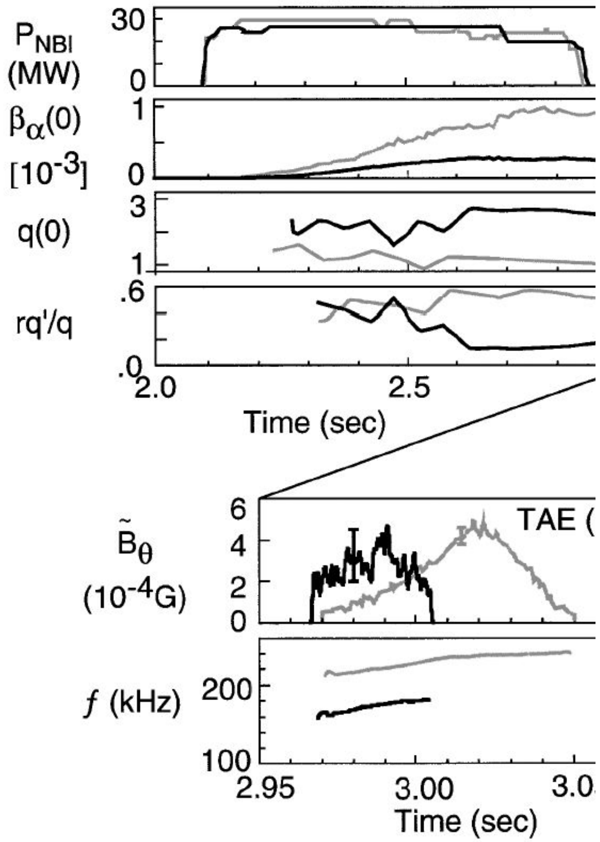
- **Existence of Energetic Ions**:
  - **ICRF heating** generates energetic ions: High energy tail in $f(v_\perp)$.
  - Negative-ion-based **neutral beam injection** produces fast ions.
  - **Fusion reaction** creates energetic alpha particles.
- **Destabilization by Energetic Particles**
  - The **least stable mode** is destabilized by energetic ions.
  - The stability depends on the radial profile of **fast ion pressure**.
  - The stability is also sensitive to the q **profile**. profile.
- **Nonlinear Interaction of Wave and Energetic Ions**
  - Radial **diffusion** and instantaneous **loss** of energetic ions
  - **Reduction** of heating power and fusion reaction rate
  - Localized **damage** of first wall

Alfvén Eigenmode Excited by Alpha Particles

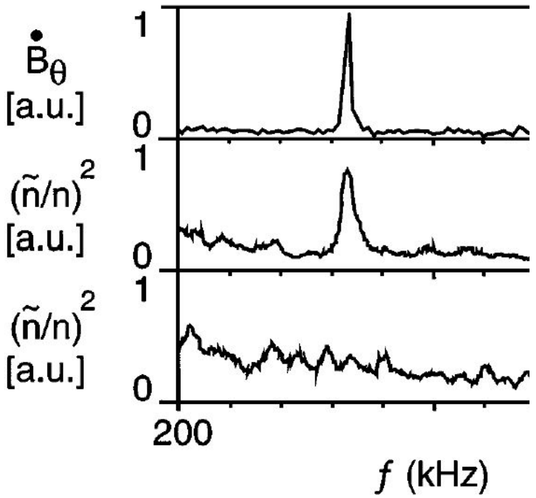
- **DT Burning experiment on TFTR**

(**Nazikian et al., PRL 78 (1997) 2976**)

Decay phase after NBI



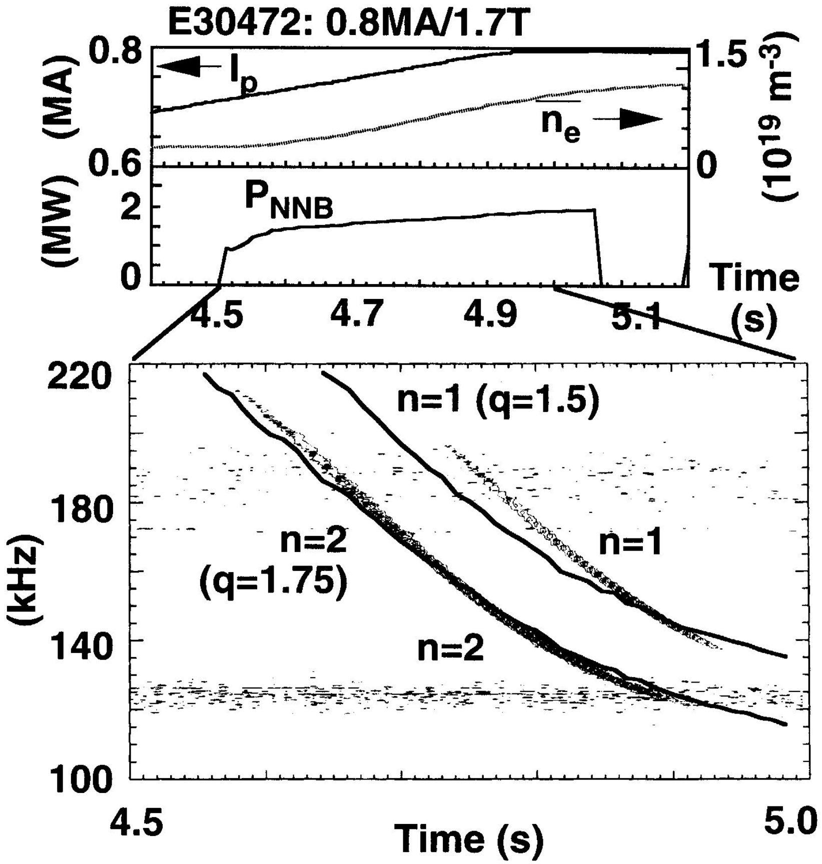
Excitation of AE



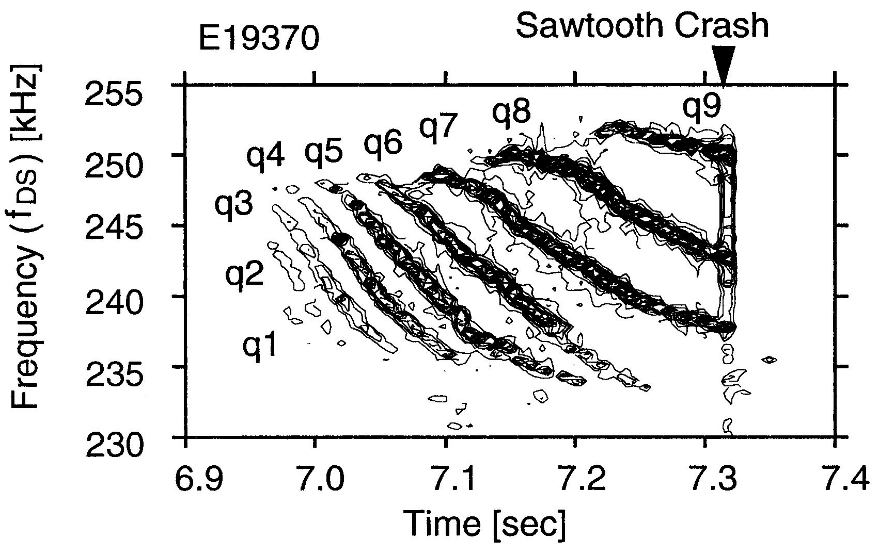
Alfvén Eigenmode excited by ICRF and NBI

- **NNBI experiment on JT-60U** (**Kusama et al., NF 39 (1999) 1837**)
- **ICRF experiment on JT-60U** (**Kimura et al., JPFR 71 (1993) 1147**)

NNBI



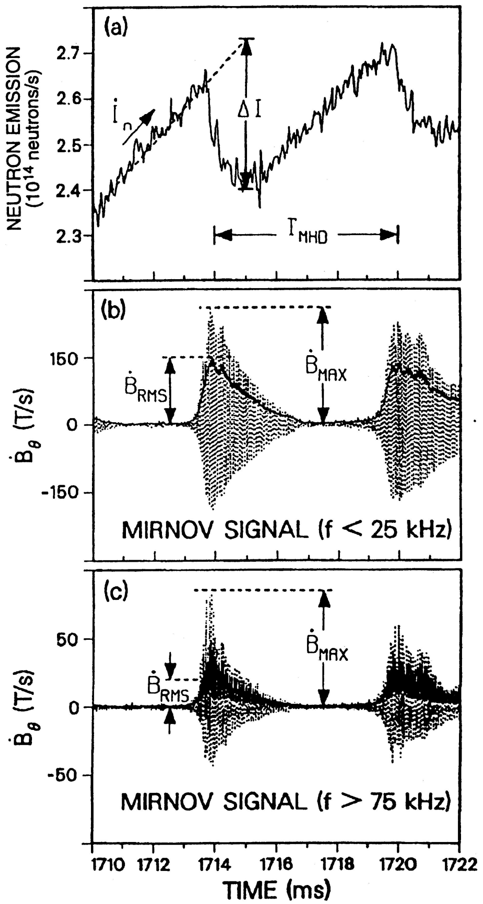
ICRF



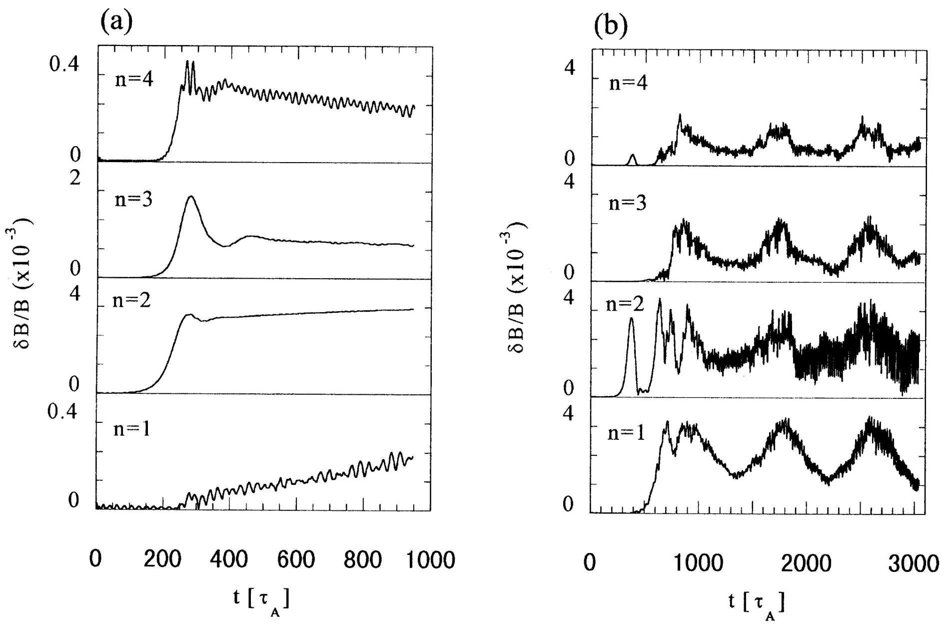
Burst Excitation of Alfvén Eigenmode

- **NBI experiment on DIII-D** (**Duong et al., NF 33 (1993) 749**)
- **Nonlinear Simulation of TAE** (**Todo, JPFR 75 (1999) 567**)

TFTR

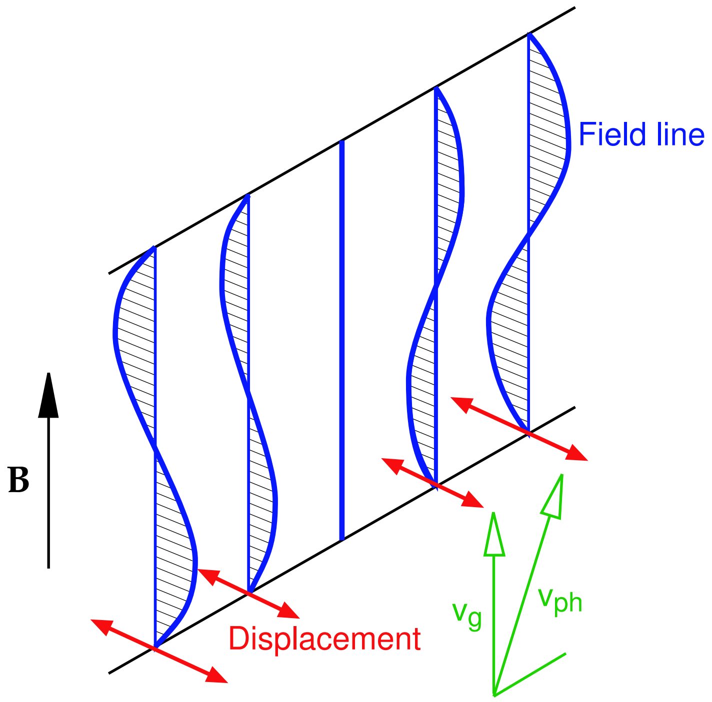


Simulation

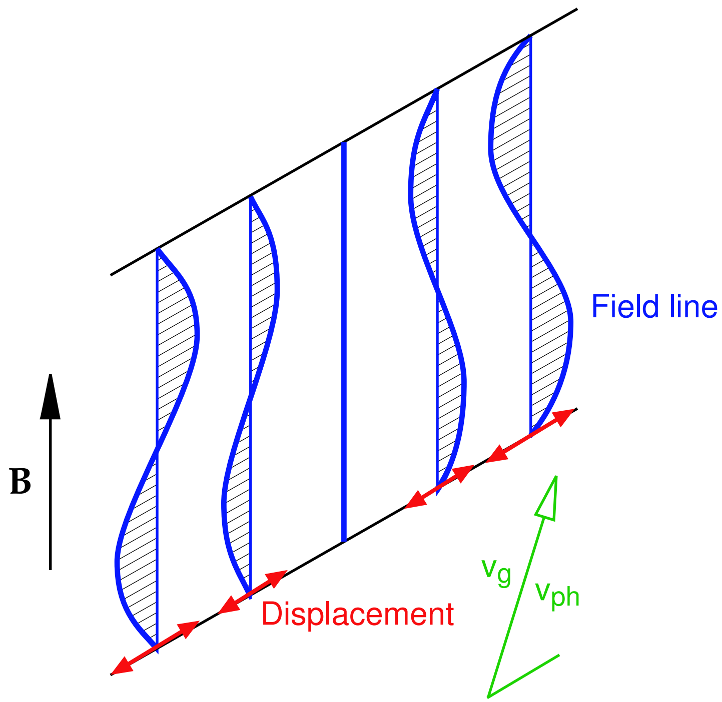


Alfvén Waves

- **Shear Alfvén Wave (SAW) and Compressional Alfvén Wave (CAW)**



(a) Shear Alfven Wave



(b) Compressional Alfven Wave

- **Shear Alfvén Wave**:
  No coupling with adjacent line of force $\Longrightarrow$ Frequency independent of $k_\perp$

$$\omega = k_\parallel v_A, \qquad \text{Alfvén Velocity} \quad v_A^2 = \frac{c^2}{1 + \omega_{pi}^2/\omega_{ci}^2}$$

Alfvén Waves in Inhomogeneous Plasmas

- **Static magnetic field** : z-axis, **Density inhomogeneity** : x-axis
- **Maxwell's Equation** : $-\nabla \times \nabla \times \boldsymbol{E} + \dfrac{\omega^2}{c^2} \epsilon \cdot \boldsymbol{E} = \boldsymbol{0}$

$$\begin{pmatrix} -k_y^2 - k_z^2 & -\mathrm{i}\, k_y \frac{\partial}{\partial x} & -\mathrm{i}\, k_z \frac{\partial}{\partial x} \\ -\mathrm{i}\, k_y \frac{\partial}{\partial x} & -k_z^2 + \frac{\partial^2}{\partial x^2} & +k_z k_y \\ -\mathrm{i}\, k_z \frac{\partial}{\partial x} & +k_z k_y & -k_y^2 + \frac{\partial^2}{\partial x^2} \end{pmatrix} \cdot \boldsymbol{E} + \frac{\omega^2}{c^2} \begin{pmatrix} S & -\mathrm{i}\, D & 0 \\ \mathrm{i}\, D & S & 0 \\ 0 & 0 & P \end{pmatrix} \cdot \boldsymbol{E} = \boldsymbol{0}$$

- **Dielectric tensor** : ϵ
  - **Local model**

$$S \simeq 1 + \frac{\omega_{\mathrm{pi}}^2}{\omega_{\mathrm{ci}}^2}, \qquad D \simeq \frac{\omega_{\mathrm{pi}}^2}{\omega_{\mathrm{ci}}^2} \frac{\omega}{\omega_{\mathrm{ci}}}, \qquad P \simeq \begin{cases} -\dfrac{\omega_{\mathrm{pe}}^2}{\omega^2} & \text{(Cold plasma)} \\ +\dfrac{\omega_{\mathrm{pe}}^2}{k_\parallel^2 v_{\mathrm{Te}}^2} & \text{(Hot plasma)} \end{cases}$$

  - **Differential operator model** : Finite Larmor radius effect ($k_\perp \rho \ll 1$)
  - **Integral operator model** : Finite orbit width effect (Arbitrary $k_\perp \rho$)

MHD model

- **Ideal MHD approximation** : $(S \simeq \omega_{pi}^2/\omega_{ci}^2, D = 0, P = \infty)$
- **Equation describing wave electric field** :

(normalized by c/ω)

$$\frac{\partial}{\partial x}\frac{S - k_z^2}{S - k_z^2 - k_x^2}\frac{\partial}{\partial x}E_y + (S - k_z^2)E_y = 0$$

- **Shear Alfvén Resonance** :

$$S - k_z^2 \sim S'_r(x - x_0) + \mathrm{i}\,S_i, \quad \delta = S_i/S'_r$$

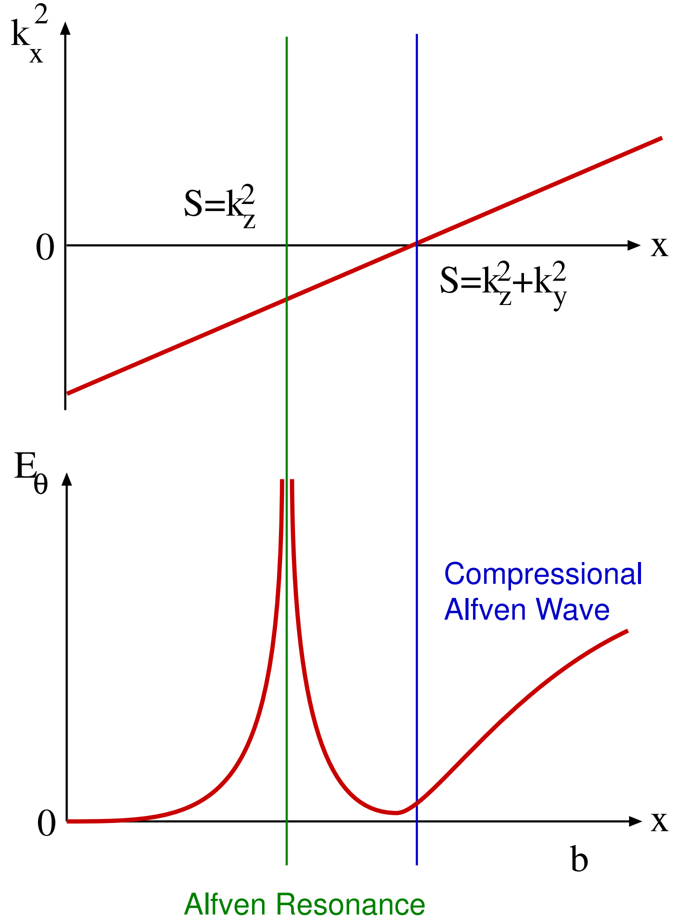
- **Logarithmic singularity** : $E_y = C\ln(x - x_0 + \mathrm{i}\,\delta)$

$$\frac{\partial^2 E_y}{\partial x^2} + \frac{1}{x - x_0 + \mathrm{i}\,\delta}\frac{\partial E_y}{\partial x} - k_y^2 E_y = 0$$

- **Power absorbed at the singularity** :

$$P_{abs} = \frac{\omega}{2}\frac{\pi|C|^2}{\mu_0}\frac{S'_r}{k_y^2}$$

Increase of density



Propagation of Shear Alfvén Wave

- **Effect of finite frequency** :

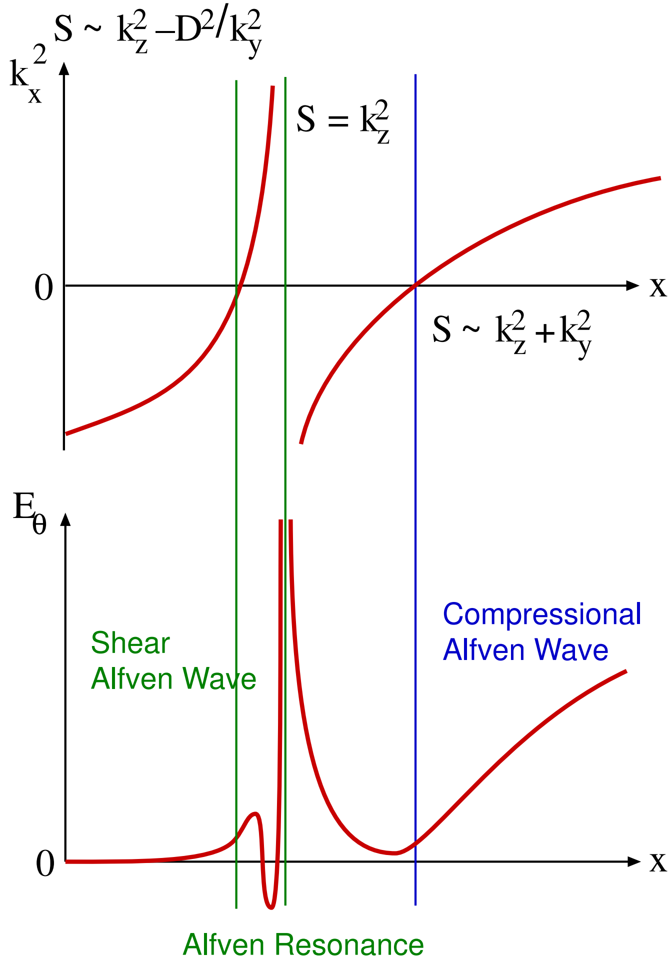
$$\omega/\omega_{\rm ci} \neq 0 \quad \Longrightarrow \quad D \neq 0$$

- **Equation describing wave electric field** :

$$\left(\frac{\partial}{\partial x} - \frac{D}{k_y}\right) \frac{k_y^2}{S - k_z^2 - k_y^2} \left(\frac{\partial}{\partial x} + \frac{D}{k_y}\right) E_y + \left(\frac{\partial^2}{\partial x^2} + S - k_z^2\right) E_y = 0$$

- **Propagation of SAW in the lower density side of Alfvén resonance**

Increase of density



Mode conversion of Shear Alfvén Wave

- **In the vicinity of Alfvén resonance**
  - Short wave length $\Longrightarrow$ Electrostatic
  - Since $E_z \neq 0$, differential eq. of the 4th order
  - Propagation depends of the sign of P

- **Extremely low β** : $\beta < m_\mathrm{e}/m\mathrm{i}$
  - Finite electron mass effect

$$v_\mathrm{Te} < \omega/k_\parallel \sim v_\mathrm{A}, \quad P < 0,$$

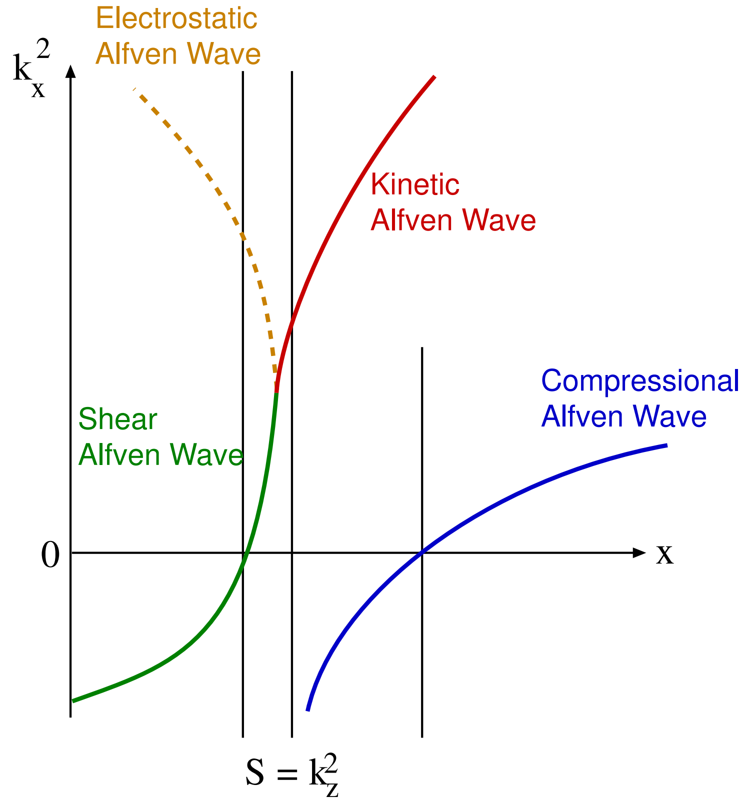
  - Propagation in the **lower density side**

- **Finite β** : $\beta > m_\mathrm{e}/m\mathrm{i}$
  - Finite temperature effect

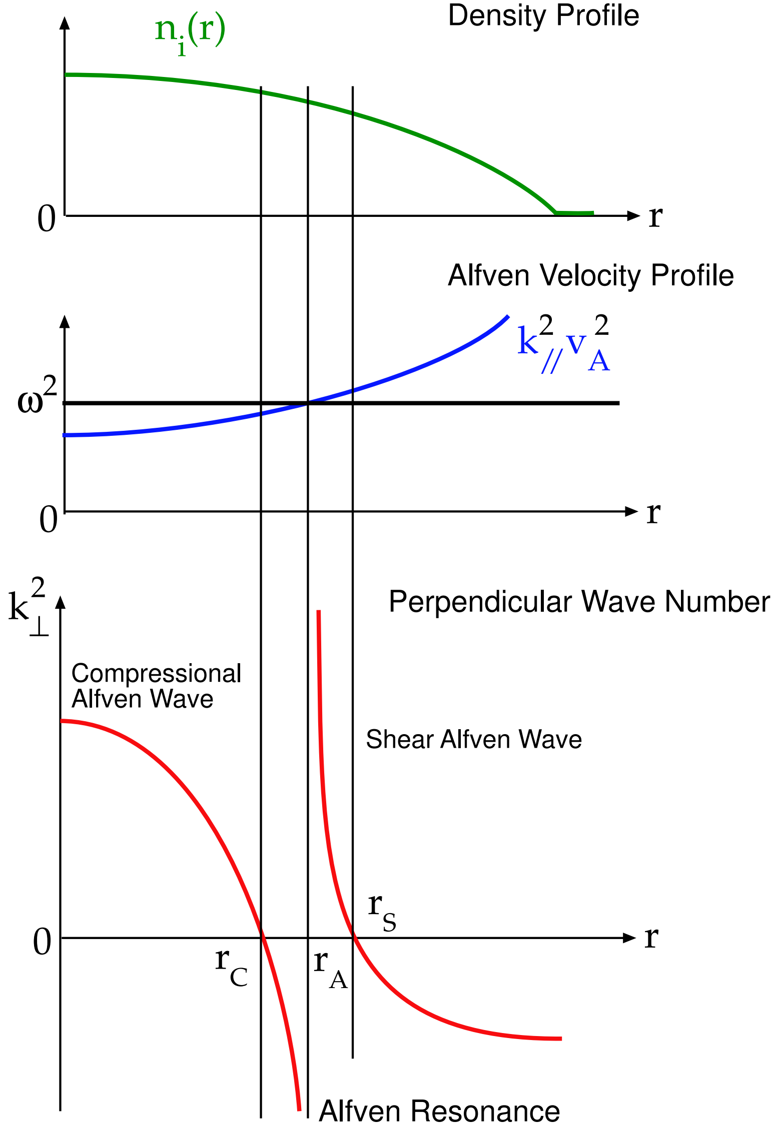
$$v_\mathrm{Te} > \omega/k_\parallel \sim v_\mathrm{A}, \quad P > 0,$$

  - Propagation in the **higher density side**

Increase of density



Alfvén Eigenmode in a Cylindrical Plasma



- **CA Eigenmode** ($|m| \neq 1$)

$$\omega \gtrsim (\pi/a)\, v_A$$

- **CA Surface Eigenmode** ($m = 1$)
  - Nearly constant in plasma, discontinuous on surface
- **CA Surface Eigenmode** ($m = -1$)
  - Localize near surface with the increase of $k_{\|}$
- **Shear Alfvén Wave** : **Strong damping**

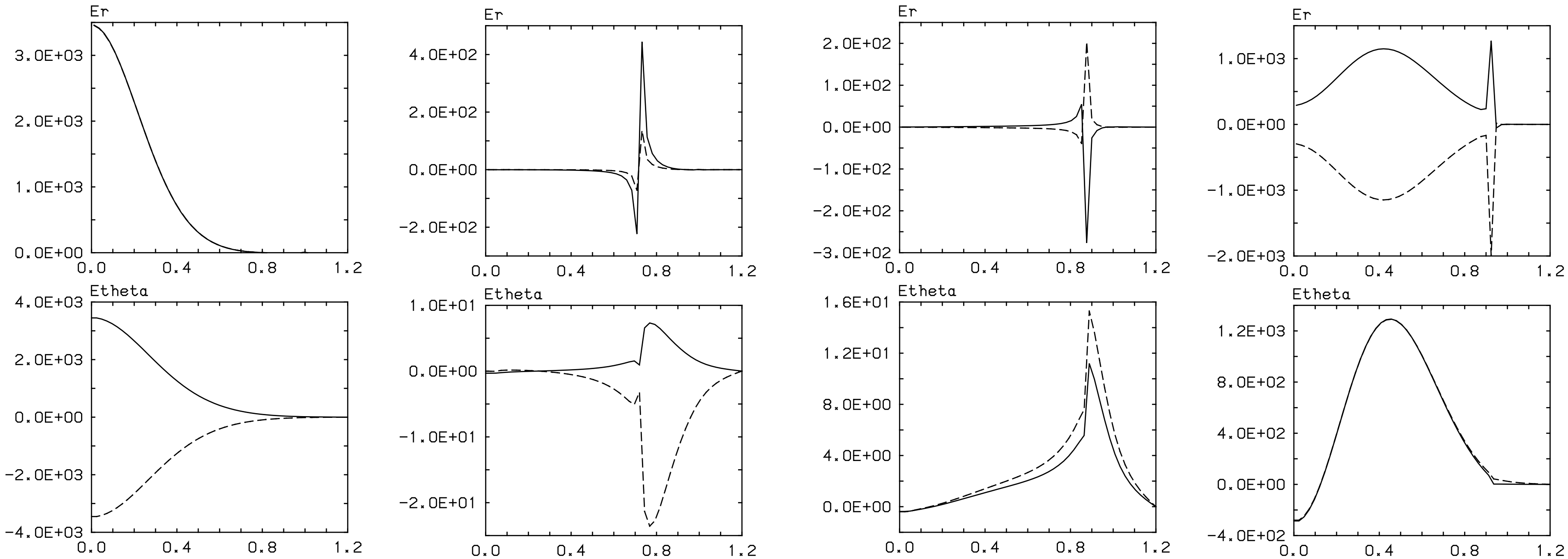
$$k_{\|} v_{\mathrm{Amax}} \gtrsim \omega \gtrsim k_{\|} v_{\mathrm{Amin}}$$

- **Shear Alfvén Eigenmode**
  (GAE : Global Alfvén Eigenmode)

$$\omega \sim k_{\|} v_{\mathrm{Amin}}$$

Examples of Alfvén Eigenmode in a Cylindrical Plasma

- $B = 3\,\mathrm{T}$, $a = 1\,\mathrm{m}$, $n_\mathrm{e}(0) = 10^{20}\,\mathrm{m}^{-3}$, $m = -1$, $k_\parallel = 10\,\mathrm{m}^{-1}$



$f_r = 6.55\,\mathrm{MHz}$
GAE

$f_r = 8.21\,\mathrm{MHz}$
SAW

$f_r = 9.40\,\mathrm{MHz}$
SAW

$f_r = 10.31\,\mathrm{MHz}$
CAW

Alfvén Eigenmode in a Toroidal Plasma (I)

- **Toroidal Plasma**
  - Major radius dependence of B $\Longrightarrow$ Poloidal angle dependence

$$\frac{1}{v_{\mathrm{A}}^2} \propto \frac{1}{B^2} \sim \frac{1+2\varepsilon\cos\theta}{B_0^2}$$

- SAW dispersion **without toroidal effect**

$$k_{\|m}^2 - \frac{\omega^2}{v_{\mathrm{A}}^2} = 0$$

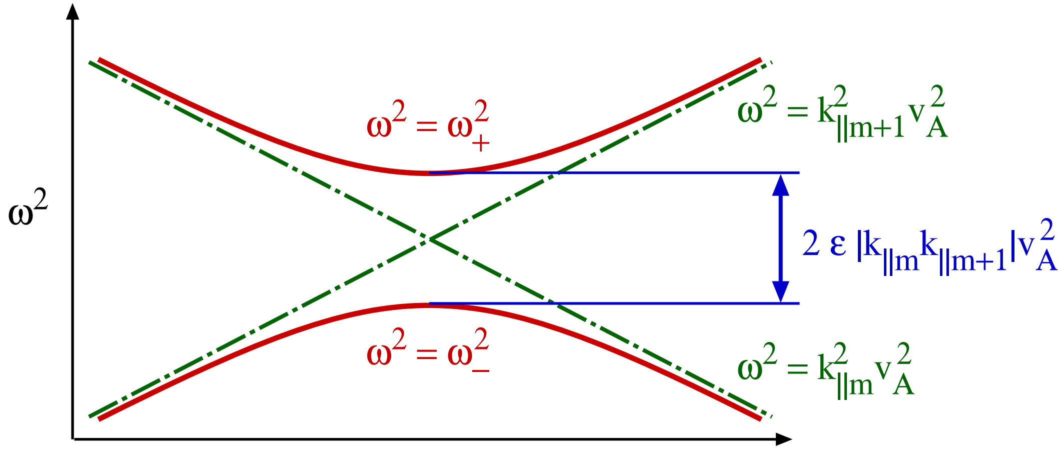
- SAW dispersion **with toroidal effect**

$$\begin{vmatrix} k_{\|m-1}^2 - \frac{\omega^2}{v_{\mathrm{A}}^2} & -\varepsilon\frac{\omega^2}{v_{\mathrm{A}}^2} & 0 \\ -\varepsilon\frac{\omega^2}{v_{\mathrm{A}}^2} & k_{\|m}^2 - \frac{\omega^2}{v_{\mathrm{A}}^2} & -\varepsilon\frac{\omega^2}{v_{\mathrm{A}}^2} \\ 0 & -\varepsilon\frac{\omega^2}{v_{\mathrm{A}}^2} & k_{\|m+1}^2 - \frac{\omega^2}{v_{\mathrm{A}}^2} \end{vmatrix} = 0$$

Alfvén Eigenmode in a toroidal Plasma (II)

- Resonance frequency including only m and $m+1$ modes

$$\omega_{\pm}^2 = \frac{k_{\|m}^2 + k_{\|m+1}^2 \pm \sqrt{(k_{\|m}^2 - k_{\|m+1}^2) + 4\varepsilon^2 k_{\|m}^2 k_{\|m+1}^2}}{2(1-\varepsilon^2)}$$



- Condition for Alfvén frequency gap

$$k_{\|m}^2 = k_{\|m+1}^2 \quad \Longrightarrow \quad k_{\|m} = -k_{\|m+1} \quad \Longrightarrow \quad q = -\frac{m+1/2}{n}$$

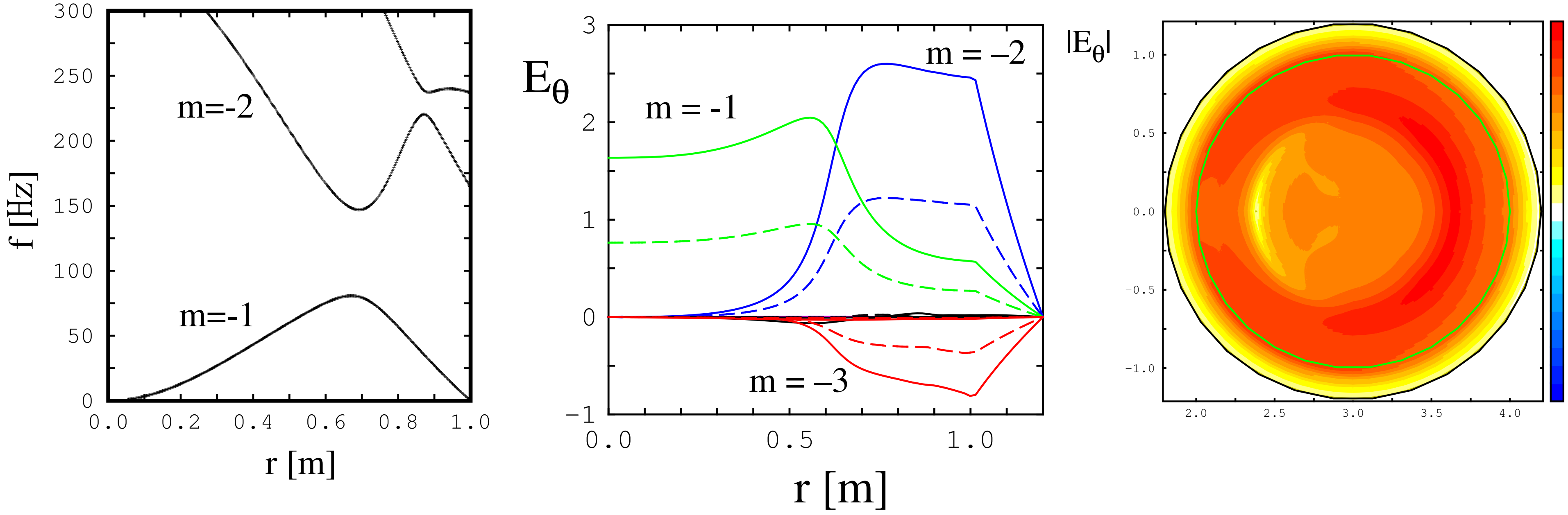
- **Toroidicity-induced Alfvén Eigenmode : (TAE)**

Alfvén Eigenmode in a toroidal Plasma (III)

- Example of low n TAE

  $R = 3\,\mathrm{m},\ a = 1\,\mathrm{m},\ B_0 = 3\,\mathrm{T},\ n_e = 0.5\times10^{20}\,\mathrm{m}^{-3}$

  $q(0) = 1,\ q(a) = 2,\ n = 1,\ f = 126.9\,\mathrm{kHz}$



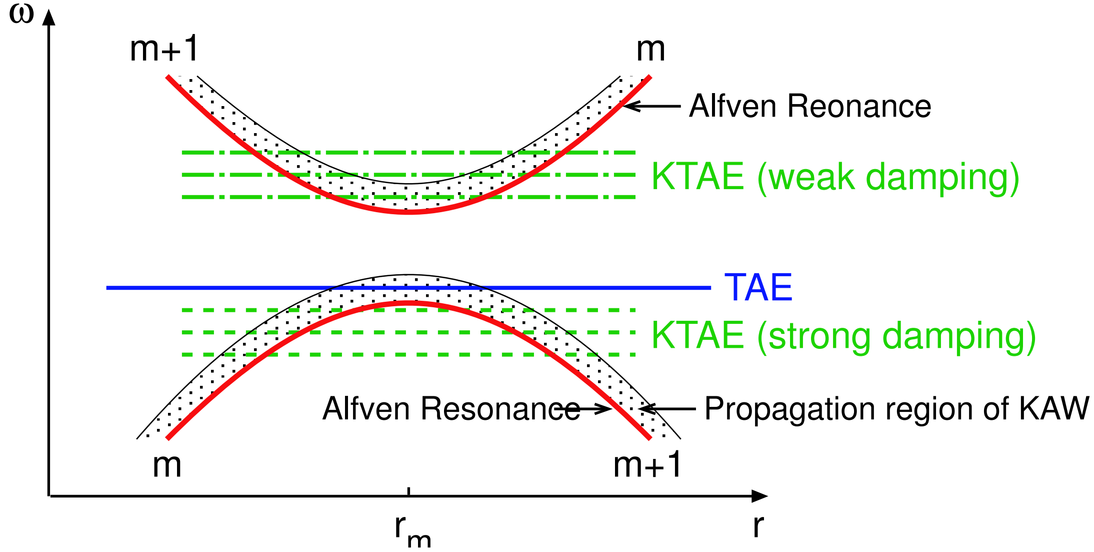
Kinetic Alfvén Eigenmode

- Low-β MHD equation including kinetic effects

$$\left(\mathcal{L}_m + \bar{\rho}^2 \frac{d^4}{dr^4}\right)\phi_m + \bar{\epsilon}(r)\frac{\omega^2}{v_A^2}\frac{d^2}{dr^2}(\phi_{m+1} + \phi_{m-1}) = 0$$

$$\mathcal{L}_m = \frac{d}{dr}\left(\frac{\omega^2}{v_A^2} - k_{\parallel m}^2\right)\frac{d}{dr} - \frac{m^2}{r^2}\left(\frac{\omega^2}{v_A^2} - k_{\parallel m}^2\right), \quad \bar{\rho}^2 \equiv \rho_i^2\left(\frac{3}{4}\frac{\omega^2}{v_A^2} + \frac{T_e}{T_i}k_{\parallel m}^2\right)$$

- Frequency gap and kinetic Alfvén waves



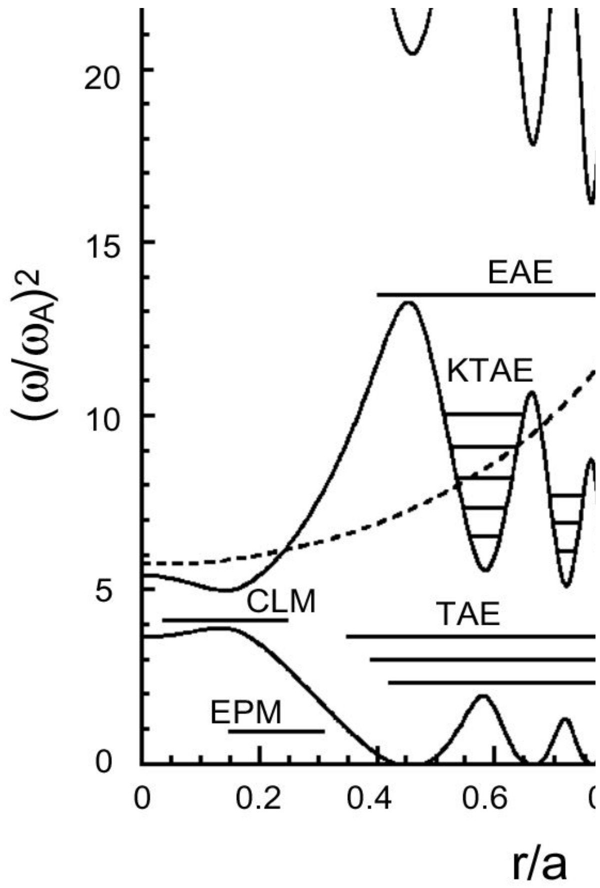
Various Alfvén Eigenmodes

- **Non-circular tokamak**
  - Coupling between m and $m + \ell$ modes ($\ell = 2$: Elongation, $\ell = 3$: Triangularity)
  - Coupling condition:

$$q = -\frac{m + \ell/2}{n}$$

- **Helical Plasma**
  - Coupling between n and $n + \ell' N_{\mathrm{h}}$ modes ((N_{h} : Helical coil turn)
  - Coupling condition:

$$q = -\frac{m + \ell/2}{n + \ell' N_{\mathrm{h}}/2}$$



Excitation of Alfvén Eigenmode by Energetic Ions

- **Destabilization requires**
  - **Wave-particle resonance** condition

$$v_{\parallel} > \frac{\omega}{k_{\parallel}} \sim v_{\mathrm{A}}$$

— Existence of energetic ions is required.

  - **Diamagnetic drift velocity** faster than poloidal phase velocity

$$v_{\mathrm{df}} = \frac{T_{\mathrm{f}}}{e_{\mathrm{f}} B} \frac{\mathrm{d} \ln n_{\mathrm{f}}}{\mathrm{d} r} > \frac{\omega r}{m}$$

or

$$\omega_{*\mathrm{fm}} = \frac{m}{r} \frac{T_{\mathrm{f}}}{e_{\mathrm{f}} B} \frac{\mathrm{d} \ln n_{\mathrm{f}}}{\mathrm{d} r} > \omega$$

— Low frequency made can be easily excited.

- **Growth rate of low *n* TAE** :

$$\frac{\gamma}{\omega} \sim \frac{9}{4} \left[\beta_{\mathrm{f}} \left(\frac{\omega_{*f}}{\omega_0} - \frac{1}{2} \right) F \left(\frac{v_{\mathrm{A}}}{v_{\mathrm{f}}} \right) - \beta_{\mathrm{e}} \frac{v_{\mathrm{A}}}{v_{\mathrm{e}}} \right]$$

Damping Mechanism of Alfvń Eigenmodes

- **MHD model**
  - Absorption near Alfvén resonance
    (**Continuous spectrum damping**)
- **Perturbative treatment of kinetic Alfv’en waves**
  (**Eigen function: MHD, Damping: Kinetic**)
  - Radiative damping
    (**power propagating outward**)
  - Landau damping
    (**Estimation of parallel wave electric field**)
- **Kinetic absorption mechanism**
  - Electron Landau damping
  - Landau damping of energetic ions

TASK/WM

- **Magnetic flux coordinates**: (ψ, θ, φ)
  - **Non-orthogonal system** (including 3D helical configuration)
- **Maxwell's equation** for stationary wave electric field $\boldsymbol{E}$

$$\nabla \times \nabla \times \boldsymbol{E} = \frac{\omega^2}{c^2} \overleftrightarrow{\epsilon} \cdot \boldsymbol{E} + \mathrm{i}\,\omega\mu_0\, \boldsymbol{j}_{\rm ext}$$

  - $\overleftrightarrow{\epsilon}$ **: Dielectric tensor with kinetic effects**: $Z[(\omega - n\omega_{\rm c})/k_{\parallel}]$
- **Fourier expansion** in poloidal and toroidal directions
  - **Exact parallel wave number**: $k_{\parallel}^{m,n} = (mB^{\theta} + nB^{\varphi})/B$
- **Destabilization by energetic ions** included in $\overleftrightarrow{\epsilon}$
  - **Drift kinetic equation**

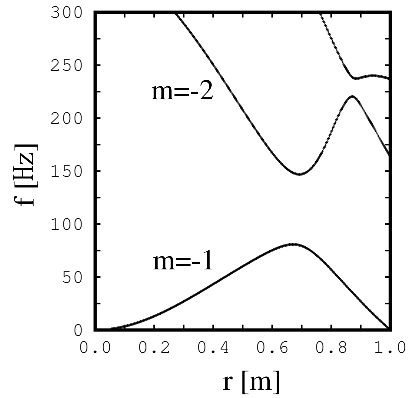
$$\left[\frac{\partial}{\partial t} + v_{\parallel}\nabla_{\parallel} + (\boldsymbol{v}_{\rm d} + \boldsymbol{v}_{\rm E}) \cdot \nabla + \frac{e_{\alpha}}{m_{\alpha}}(v_{\parallel}E_{\parallel} + \boldsymbol{v}_{\rm d} \cdot \boldsymbol{E})\frac{\partial}{\partial \varepsilon}\right] f_{\alpha} = 0$$

- **Eigenvalue problem** for complex wave frequency
  - **Maximize wave amplitude** for finite excitation proportional to $n_{\rm e}$

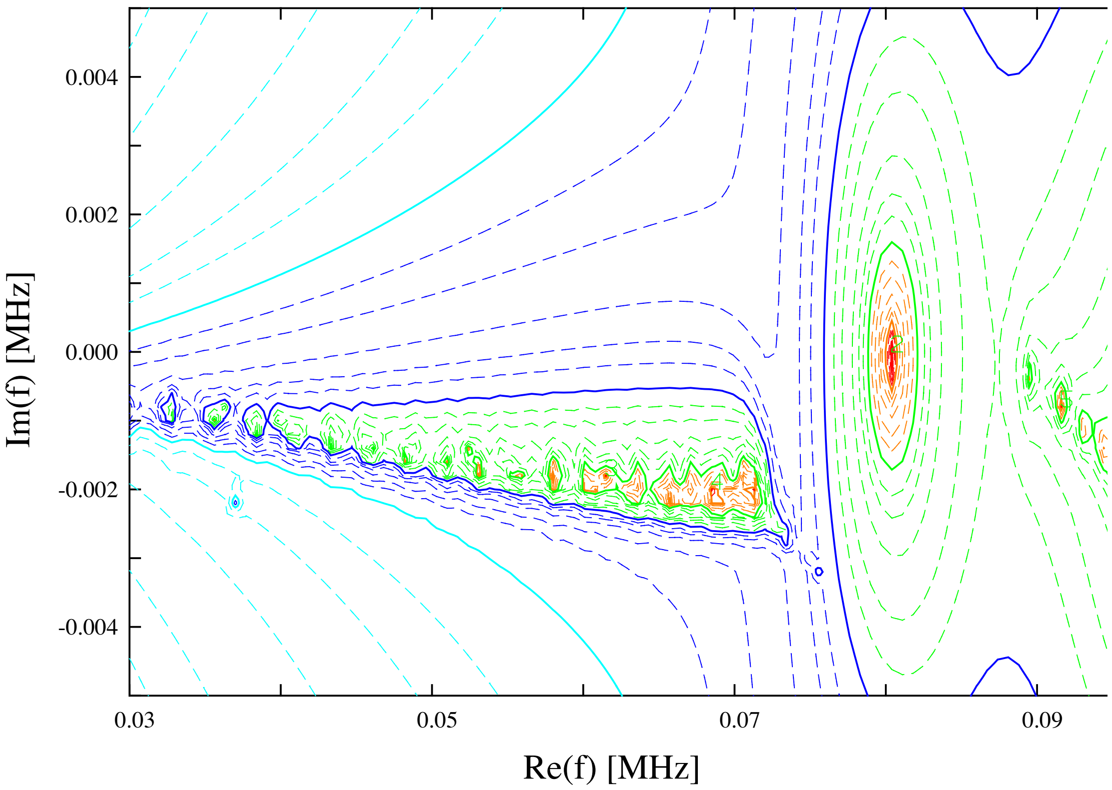
Typical TAE with Positive Magnetic Shear

- **Configuration**
  - $q(\rho) = q_0 + (q_a - q_0)\rho^2,\ q_0 = 1,\ q_a = 2$
  - Flat Density Profile

Alfvén Frequency

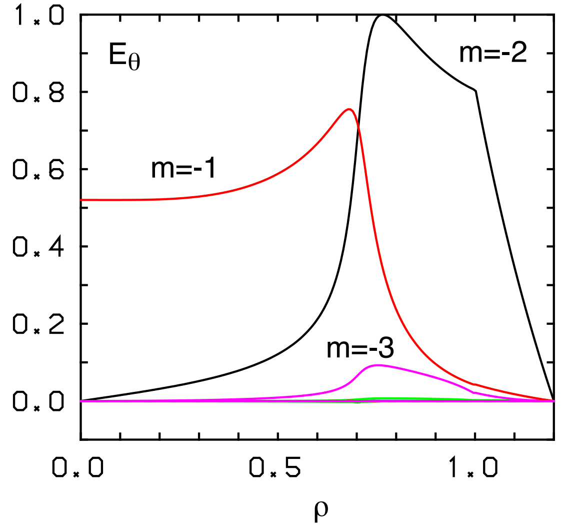


Contour of $|E|^2$ in Complex Frequency Space



Eigen function

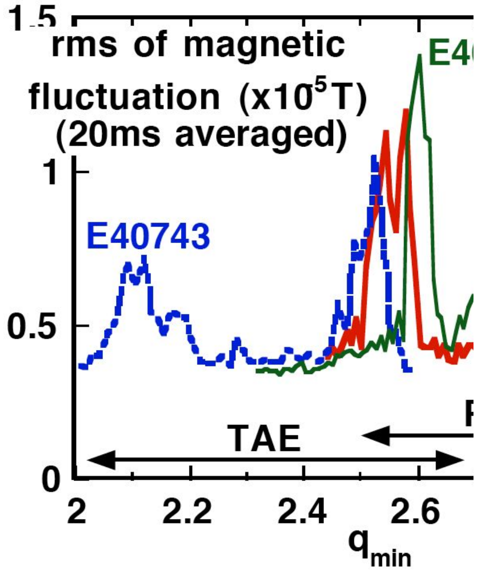
$$f_r = 81.95\,\text{kHz}$$
$$f_i = -20.32\,\text{Hz}$$



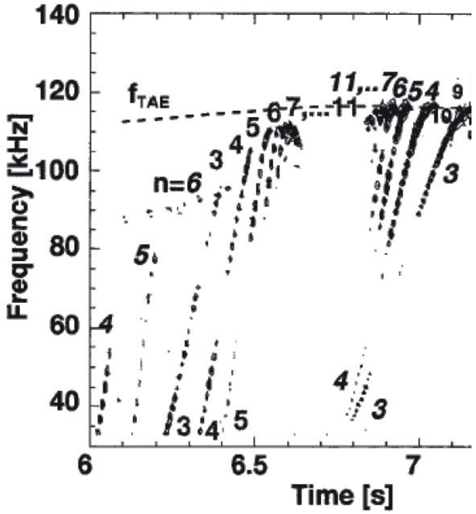
AE in the Reversed Magnetic Shear Configuration (JT-60U)

- Takechi et al. IAEA 2002 (Lyon) EX/W-6

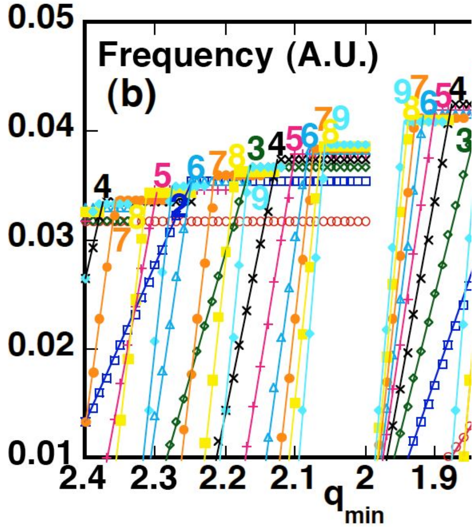
Fluctuation Amplitude



Observed frequency



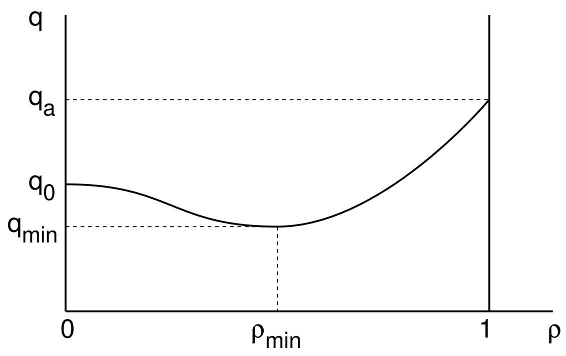
calculated frequency



Analysis of AE in Reversed Shear Configuration

$q_{\rm min}$ Dependence of Eigenmode Frequency

Assumed q profile

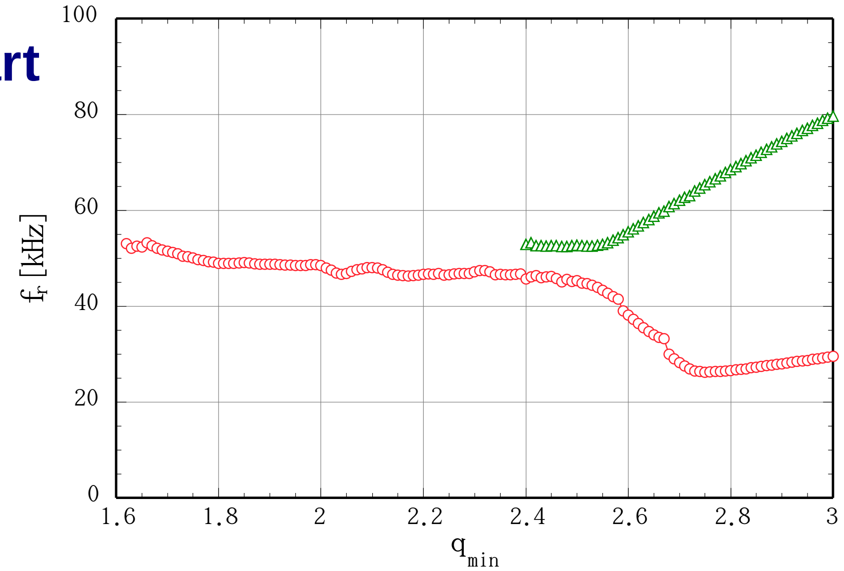


Plasma Parameters

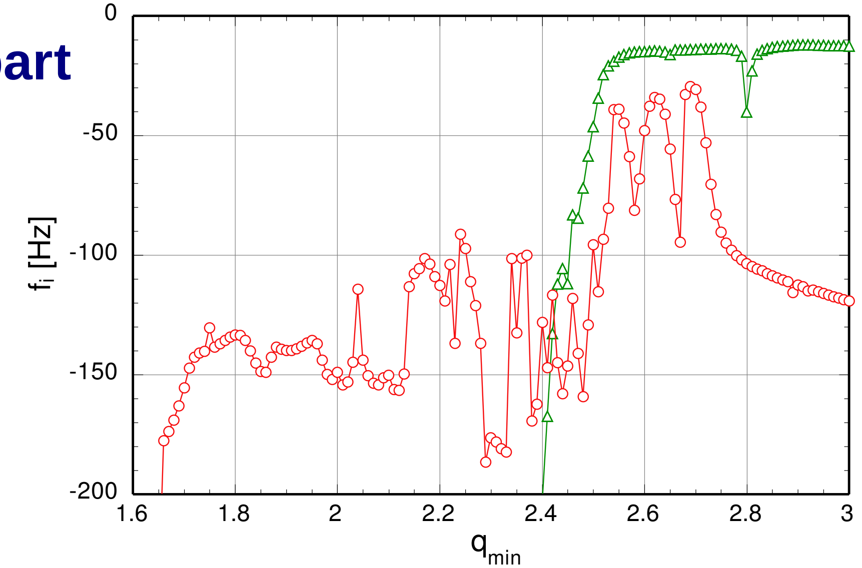
R_0	$3\,\mathrm{m}$
a	$1\,\mathrm{m}$
B_0	$3\,\mathrm{T}$
$n_e(0)$	$10^{20}\,\mathrm{m}^{-3}$
$T(0)$	$3\,\mathrm{keV}$
$q(0)$	3
$q(a)$	5
$\rho_{\rm min}$	0.5
n	1

Flat density profile

Real part

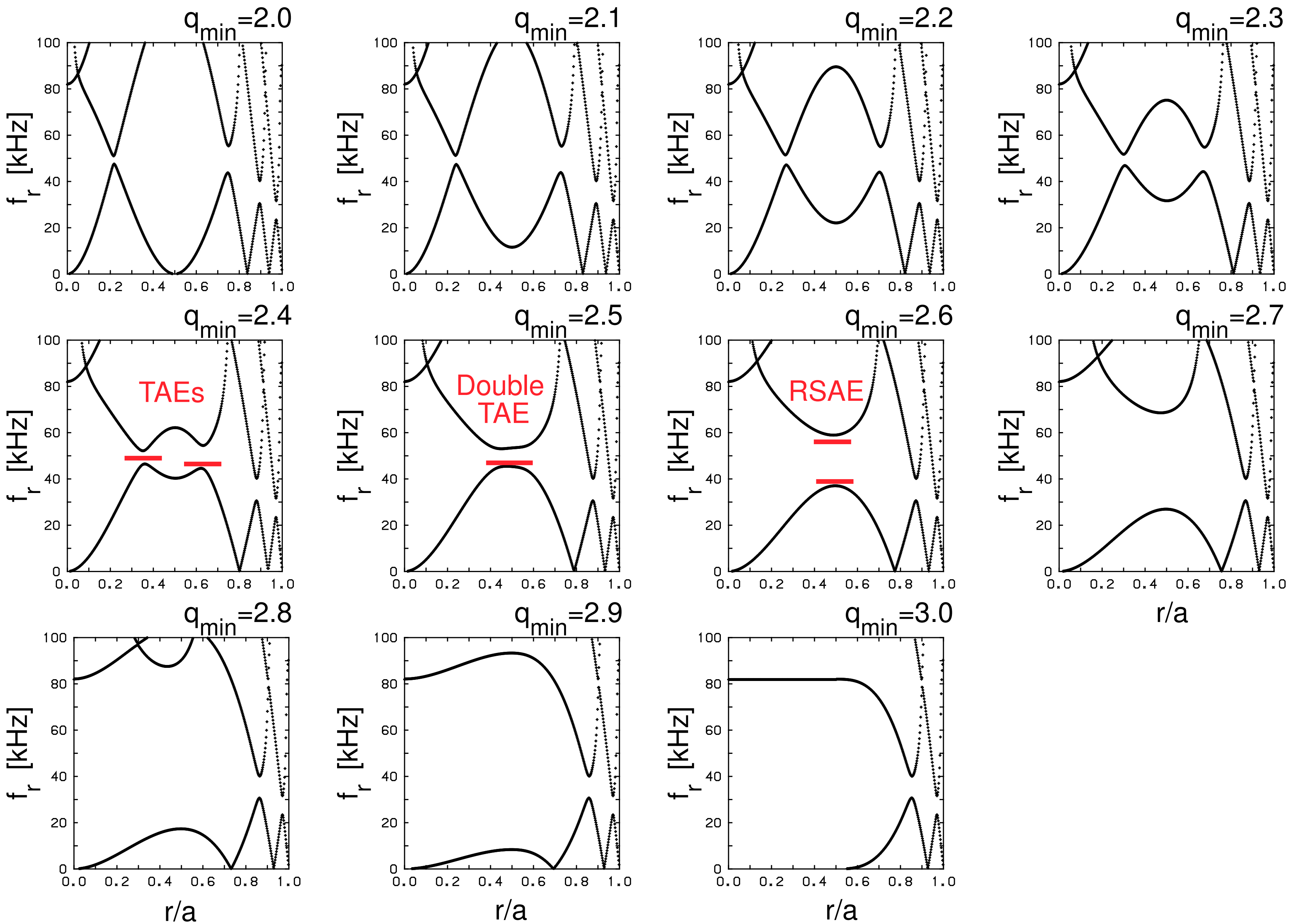


Imag part



- **RSAE (reversed-shear-induced Alfvén eigenmode)** for $\ell + \frac{1}{2} < q_{\rm min} < \ell + 1$

$q_{\min}$ Dependence of Radial Structure of Alfvén resonance

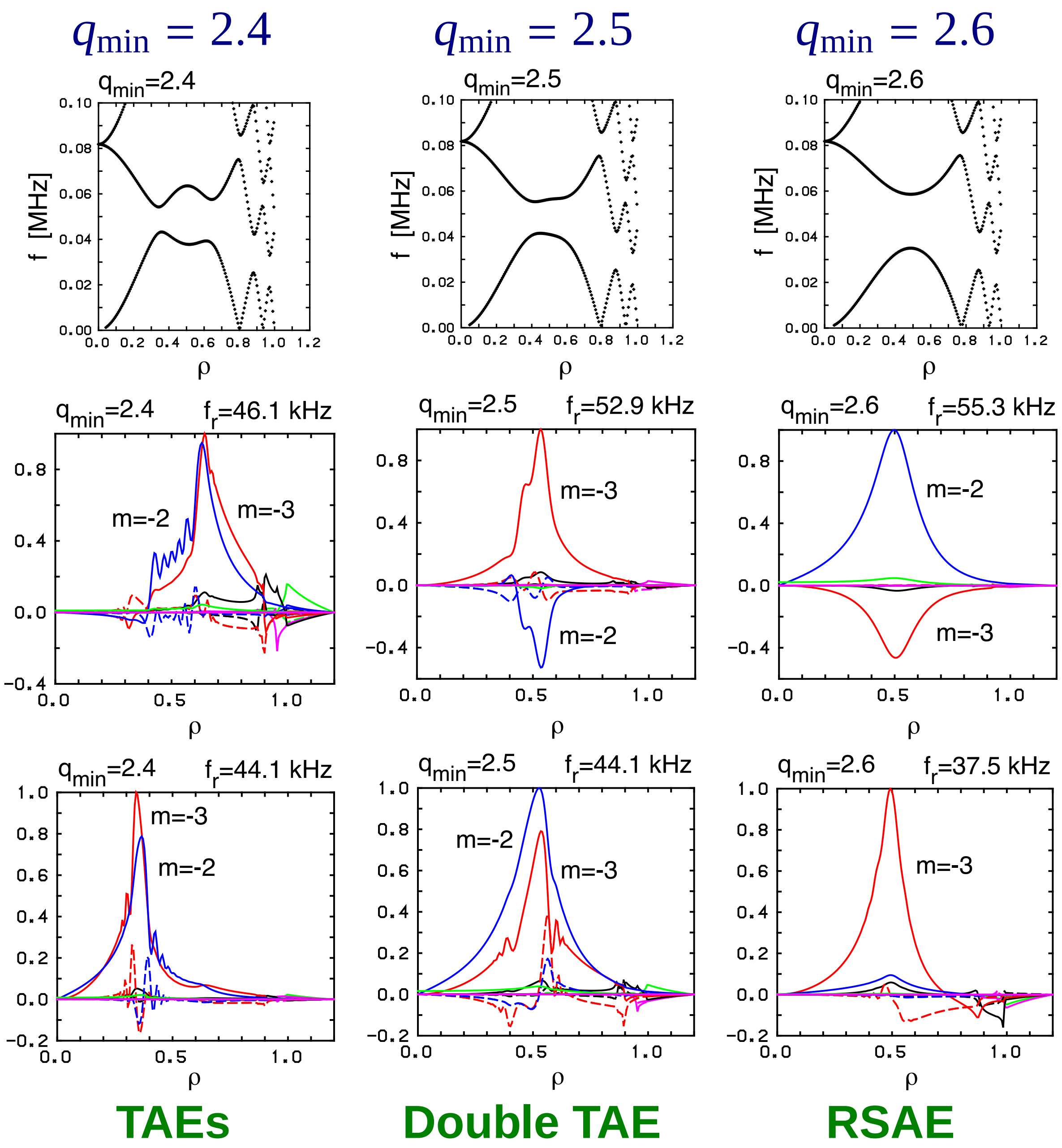


Eigenmode Structure (n = 1)

Alfvén resonance

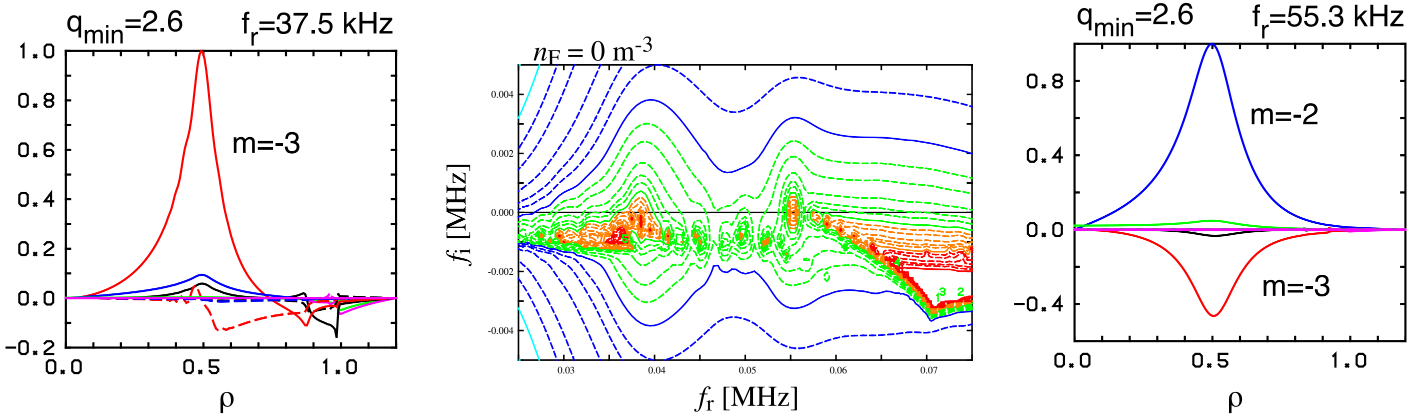
Higher freq.

Lower freq.



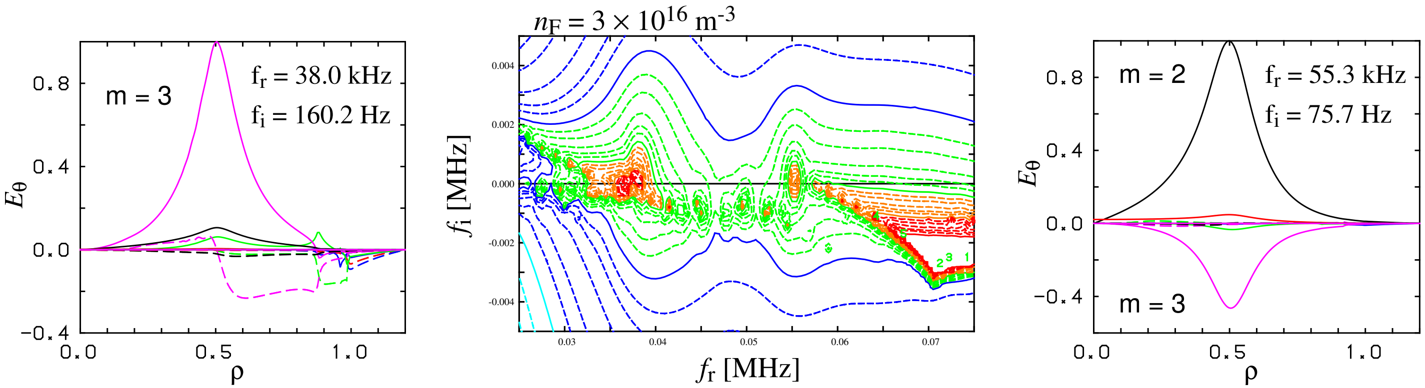
Excitation by Energetic Particles ($q_{min} = 2.6$)

- Without EP



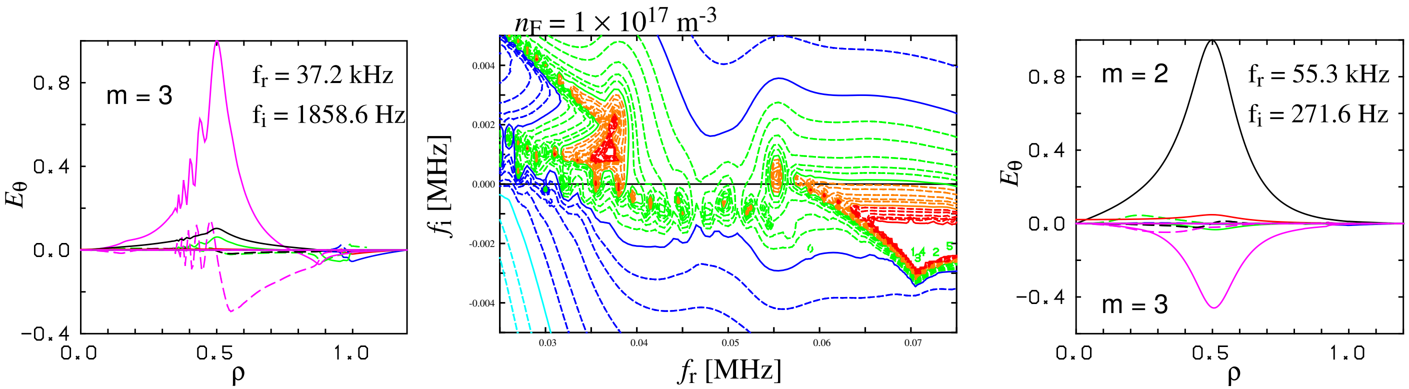
- With EP

  $3 \times 10^{16}\,\mathrm{m}^{-3}$
  360 keV
  0.5 m



- With EP

  $1 \times 10^{17}\,\mathrm{m}^{-3}$
  360 keV
  0.5 m



Summary

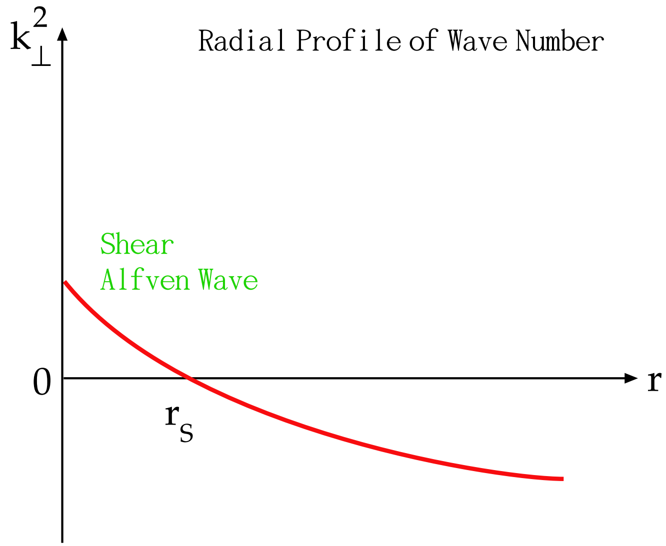
- **Various kinds of Alfvén eigenmodes** have possibilities to be excited by energetic ions in toroidal plasmas.
- The study of linear stability requires **global kinetic analysis**, because the mode structure is sensitive to the q profile and the damping is sensitive to the parallel wave electric field.
- The existence of **RSAE** in the reversed magnetic shear configuration explains the large-scale frequency increase in the reverse magnetic configuration observed in JT-60U and JET.
- The calculated **threshold of fast ion pressure** is consistent with experimental conditions.
- **Remaining problems**
  - Coupling with drift waves in the low frequency range
  - Nonlinear analysis to estimate the loss of energetic ions

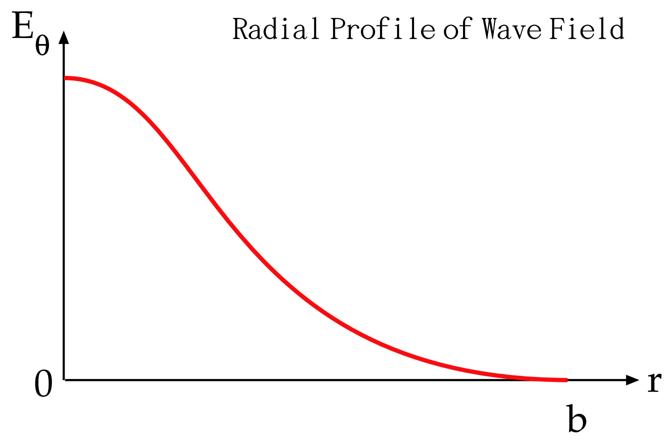
Alfvén Eigenmode in a Cylindrical Plasma (II)

- **GAE : Global Alfvén Eigenmode**

$$\omega \sim k_{\|} v_{\text{Amin}}$$

  - Shear Alfvén wave can propagate.
  - No Alfvén resonance
  - Weak damping, easily excited





- **Effect of poloidal Magnetic Field**
  - Toroidal mode number : n
  - Poloidal mode number : m
  - Safety factor : $q = rB_\phi / RB_\theta$
  - Wave number parallel to the long field line : $k_{\|} = \dfrac{m + nq}{qR}$

- If $k_{\|} v_A$ has local minimum, **GAE** may exist. $\omega \sim (k_{\|} v_A)_{\min}$

Structure of TASK code system

- **Integrated code for the analysis of toroidal plasmas**