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Analysis of Turbulent Transport Driven by Drift Alfvén Ballooning Mode

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Contents

Motivation

- **Development of Robust Transport Model**
  - **L-mode confinement time scaling**
  - **Large transport near the plasma edge in L-mode**
  - **H-mode confinement time scaling for given edge temperature**
  - **Formation of internal transport barrier**
  - **Profile database (ITPA)**
  - **Behavior of fluctuation**

- **Purpose of the present model**
  - **To describe both**
    - — **Electrostatic ITG mode**
      - · enhanced transport for large ion temperature gradient
    - — **Electromagnetic Ballooning mode (CDBM)**
      - · transport reduction for negative $s - \alpha$: ITB formation

$$\text{magnetic shear: } s = \frac{r}{q}\frac{\mathrm{d}q}{\mathrm{d}r}$$

$$\text{pressure gradient (Shafranov shift): } \alpha = -q^2 R \frac{\mathrm{d}\beta}{\mathrm{d}r}$$

Turbulent Transport Model

CDBM
Reduced MHD equation
Electromagnetic
Incompressible

Toroidal ITG[h
Ion Fluid Equation
Electrostatic
Boltzmann Distribution of Electron

Drift Alfvén Ballooning Mode
Reduced Two-Fluid Equation
Electromagnetic
Compressible

- **Small pressure gradient: Electrostatic ITG**
- **Large pressure gradient: Electromagnetic BM**
- **Without drift motion, it reduces to CDBM**
- **Ion parallel viscosity and compressibility destabilizes the mode**
- $s - \alpha$ **dependence similar to the CDBM mode**

Reduced Two-Fluid Equation (Slab Plasma)

- **Equation of Vorticity**

$$\left[\frac{n_{\rm i}\Lambda_0}{\Omega_{\rm i}B_0} - \frac{\epsilon_0}{e} - \frac{n_{\rm e}\Lambda_{0\rm e}}{\Omega_{\rm e}B}\right]\frac{\partial \nabla_\perp^2\phi_1}{\partial t} + \frac{n_{\rm i}}{\Omega_{\rm i}B}\frac{\partial}{\partial t}\left(\frac{\nabla_\perp^2 p_{1\rm i}}{q_{\rm i}n_{\rm i}}\right) - \frac{n_{\rm e}}{\Omega_{\rm e}B}\frac{\partial}{\partial t}\left(\frac{\nabla_\perp^2 p_{1\rm e}}{q_{\rm e}n_{\rm e}}\right)$$

$$-\nabla_\parallel(n_{\rm i}v_{1\parallel\rm i} - n_{\rm e}v_{1\parallel\rm e}) + \left(\frac{{\rm i}en_{\rm i}\Lambda_{0\rm i}\omega_{*\rm i}}{T_{\rm i}} + \frac{{\rm i}en_{\rm e}\Lambda_{0\rm e}\omega_{*\rm e}}{T_{\rm e}}\right)\phi_1$$

$$= \frac{1}{eB}\left(\boldsymbol{b}\times\boldsymbol{\kappa} + \boldsymbol{b}\times\frac{\nabla B}{B}\right)\cdot(\nabla p_{1\rm i} + \nabla p_{1\rm e})$$

- **Parallel Equation of Motion** $(j = {\rm e}, {\rm i})$

$$m_j n_{0j}\frac{\partial v_{1j\parallel}}{\partial t} + \nabla_\parallel p_{1j} - q_j n_{0j}E_{1\parallel} = 0$$

- **Equation of State** $(j = {\rm e}, {\rm i})$

$$\frac{\partial p_{j1}}{\partial t} + \boldsymbol{v}_{E1}\cdot\nabla p_{j0} + \Gamma_j p_{j0}\nabla_\parallel v_{\parallel j1} = 0$$

- **Ampere's Law**

$$\nabla_\perp^2 A_{1\parallel} = -\mu_0\sum_j(n_0 q_j v_{1j\parallel})$$

Reduced Two-Fluid Equation (Slab Plasma)

- **Equation of Vorticity**

$$\left(\frac{\mathrm{i}\omega}{\Omega_\mathrm{i}B_0}+\frac{\mathrm{i}\omega\epsilon_0}{en_{0\mathrm{i}}}-\frac{\mathrm{i}\omega n_{0\mathrm{e}}}{\Omega_\mathrm{e}B_0n_{0\mathrm{i}}}\right)\left(\frac{\partial^2}{\partial x^2}-k_y^2-k_\parallel^2\right)\phi_1$$

$$+\frac{\mathrm{i}\omega}{\Omega_\mathrm{i}B_0}\left(\frac{\partial^2}{\partial x^2}-k_y^2\right)p_{1\mathrm{i}}-\frac{\mathrm{i}\omega}{\Omega_\mathrm{e}B_0}\left(\frac{\partial^2}{\partial x^2}-k_y^2\right)p_{1\mathrm{e}}$$

$$+\mathrm{i}k_\parallel\sum_j(q_jn_{0j}v_{1j\parallel})/e=0$$

- **Parallel Equation of Motion** $(j=\mathrm{e},\mathrm{i})$

$$-\mathrm{i}\omega m_jn_{0j}v_{1j\parallel}+q_jn_{0j}\mathrm{i}k_\parallel\phi_1+q_j\mathrm{i}k_\parallel p_{1j}+q_jn_{0j}(-\mathrm{i}\omega+\omega_{*pj})A_{1\parallel}=0$$

- **Equation of State**: $(j=\mathrm{e},\mathrm{i})$

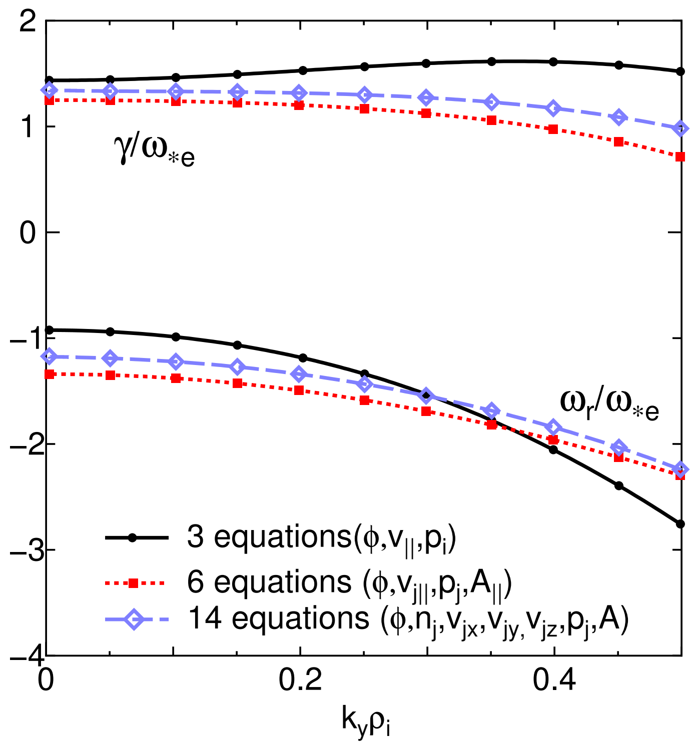
$$-\mathrm{i}\omega p_{1j}-\mathrm{i}\omega_{*pj}\phi_1+\Gamma n_{0j}T_{0j}\mathrm{i}k_\parallel v_{1j\parallel}=0$$

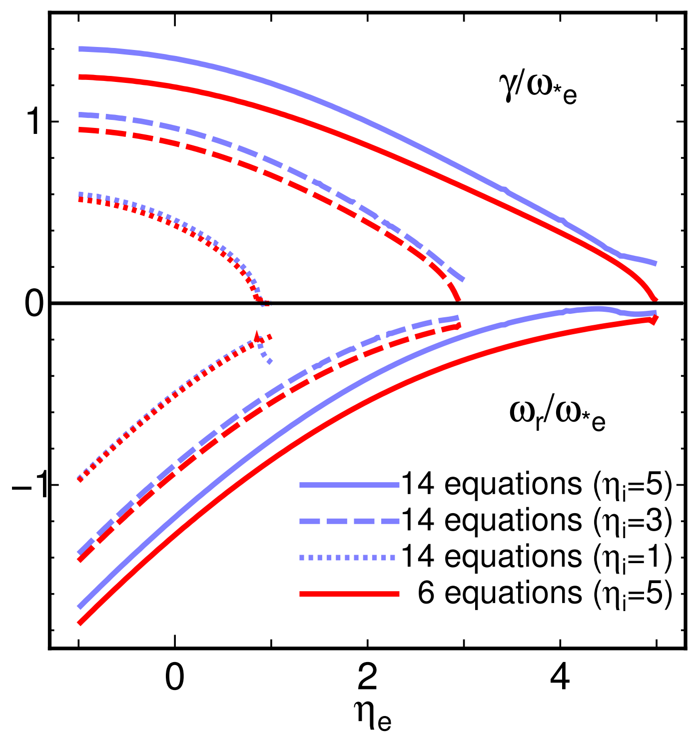
- **Ampere's Law**

$$\mu_0\sum_j(q_jn_{0j}v_{1j\parallel})+\left(\frac{\partial^2}{\partial x^2}-k_y^2-k_\parallel^2\right)A_{1\parallel}=0$$

Linear Analysis: Slab Plasma

- **Slab ITG mode** (Comparison of three models)
  - **Ion Fluid Model** (3 eqs.)
  - **Reduced Two-Fluid Model** (6 eqs.)
  - **Full Two-Fluid Model** (14 eqs.)





- **Reduced two-fluid model is very close to the full two-fluid model.**

Reduced Two-Fluid Equation (Toroidal Plasma)

- **Ballooning transformation**: ξ
- **Equation of Vorticity**

$$\frac{-\mathrm{i}\omega}{\Omega_\mathrm{i} B_0}\frac{m^2}{r^2} f^2 \left(n_{0\mathrm{i}}\Lambda_{0\mathrm{i}}\phi_1 + \frac{p_{1\mathrm{i}}}{q_\mathrm{i}} \right) - \frac{-\mathrm{i}\omega}{\Omega_\mathrm{e} B_0}\frac{m^2}{r^2} f^2 \left(n_{0\mathrm{e}}\Lambda_{0\mathrm{e}}\phi_1 + \frac{p_{1\mathrm{e}}}{q_\mathrm{e}} \right)$$

$$-\frac{-\mathrm{i}\omega e m^2 f^2 \phi_1}{\epsilon_0 r^2} + \frac{B_\theta}{r B_0}\frac{\partial}{\partial \xi}(n_{0\mathrm{i}} v_{1j\parallel} - n_{0\mathrm{e}} v_{1\mathrm{e}\parallel}) - \frac{\mathrm{i} m B_\varphi}{e r R_0 B_0^2} H(\xi)(p_{1\mathrm{i}} + p_{1\mathrm{e}}) = 0$$

- **Parallel Equation of Motion** $(j = \mathrm{e}, \mathrm{i})$

$$-\mathrm{i}\omega m_j n_{0j} v_{1j\parallel} + \frac{B_\theta}{r B_0}\frac{\partial p_1}{\partial \xi} + q_j n_{0j} \left(\frac{B_\theta \Lambda_{0j}}{r B_0}\frac{\partial \phi}{\partial \xi} - \mathrm{i}\omega_{*aj} A_\parallel \right) = 0$$

- **Equation of State** $(j = \mathrm{e}, \mathrm{i})$

$$-\mathrm{i}\omega p_{1j} - \mathrm{i} q_j n_{0j} \Lambda_{0j} \omega_{*j}(1 + \eta_j)\phi + \frac{\Gamma_j p_{0j} B_\theta}{r B_0}\frac{\partial v_{1j\parallel}}{\partial \xi} = 0$$

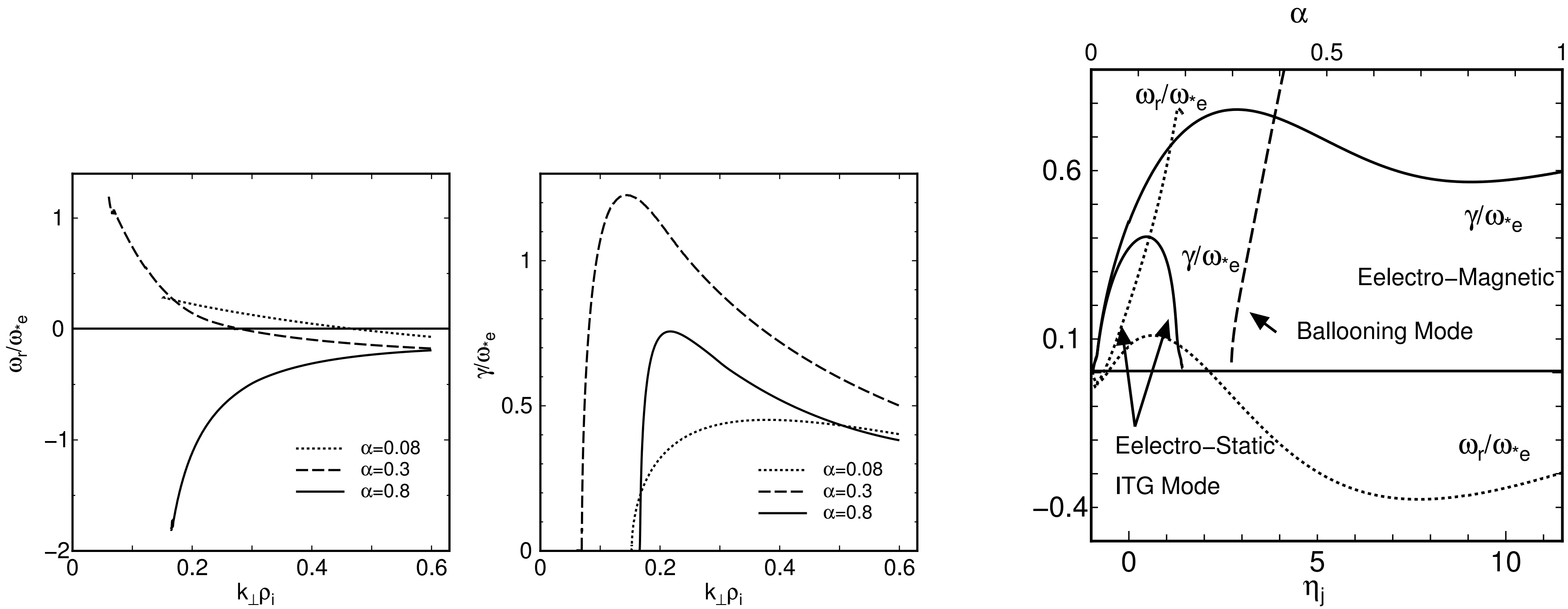
- **Ampere's law**

$$-\frac{m^2}{r^2} f^2 A_\parallel = -\mu_0 e (n_{0\mathrm{i}} v_{1\mathrm{i}\parallel} - n_{0\mathrm{e}} v_{1\mathrm{e}\parallel})$$

- $H(\xi) \equiv \kappa_0 + \cos\xi + (s\xi - \alpha \sin\xi)\sin\xi,\ f^2(\xi) = 1 + (s\xi - \alpha\sin\xi)^2$

Linear Analysis: Toroidal Plasma

- **Ballooning mode** (Transition from electrostatic to electromagnetic)
  - **Electrostatic Toroidal ITG Mode**
  - **Electromagnetic Ballooning Mode**



- **Electromagnetic effect becomes dominant for large η_i or α**

Nonlinear Reduced Two-Fluid Equation (Toroidal Plasma)

- **Turbulent transport coefficients are included.**

$$\frac{\mathrm{d}}{\mathrm{d}t}X_j \longrightarrow -\mathrm{i}\omega X_j - \chi_j \nabla_\perp^2 X_j, \qquad \chi_j = \frac{\langle \phi_1^2 \rangle}{\gamma_{nj}} \sim \sqrt{\langle \phi_1^2 \rangle}$$

- **Equation of Vorticity**:

$$\left[(-\mathrm{i}\omega + \mu_\mathrm{i}\frac{m^2 f^2}{r^2})\frac{n_{0\mathrm{i}}}{\Omega_\mathrm{i} B_0} - \frac{-\mathrm{i}\omega e}{\epsilon_0} - (-\mathrm{i}\omega + \mu_\mathrm{e}\frac{m^2 f^2}{r^2})\frac{n_{0\mathrm{e}}}{\Omega_\mathrm{e} B_0}\right]\frac{m^2 f^2}{r^2}\phi_1$$

$$+\left(-\mathrm{i}\omega + \chi_\mathrm{i}\frac{m^2 f^2}{r^2}\right)\frac{1}{\Omega_\mathrm{i} B_0}\frac{m^2}{r^2}f^2\frac{p_{1\mathrm{i}}}{q_\mathrm{i}} - \left(-\mathrm{i}\omega + \chi_\mathrm{e}\frac{m^2 f^2}{r^2}\right)\frac{1}{\Omega_\mathrm{e} B_0}\frac{m^2}{r^2}f^2\frac{p_{1\mathrm{e}}}{q_\mathrm{e}}$$

$$+\frac{B_\theta}{rB_0}\frac{\partial}{\partial \xi}(n_{0\mathrm{i}} v_{1j\parallel} - n_{0\mathrm{e}} v_{1\mathrm{e}\parallel}) - \frac{\mathrm{i} m B_\varphi}{e r R_0 B_0^2} H(\xi)(p_{1\mathrm{i}} + p_{1\mathrm{e}}) = 0$$

- **Parallel Equation of Motion**

$$m_j n_{0j}\left(-\mathrm{i}\omega + \mu_j \frac{m^2 f^2}{r^2}\right) v_{1j\parallel} + \frac{B_\theta}{rB_0}\frac{\partial p_1}{\partial \xi} + q_j n_{0j}\left(\frac{B_\theta \Lambda_{0j}}{rB_0}\frac{\partial \phi}{\partial \xi} - \mathrm{i}\omega_{*aj} A_\parallel\right) = 0$$

- **Equation of State**

$$\left(-\mathrm{i}\omega + \chi_j \frac{m^2 f^2}{r^2}\right) p_{1j} - \mathrm{i} q_j n_{0j} \Lambda_{0j} \omega_{*j}(1+\eta_j)\phi + \frac{\Gamma_j p_{0j} B_\theta}{rB_0}\frac{\partial v_{1j\parallel}}{\partial \xi} = 0$$

- **Ampere's Law**

$$-\frac{m^2}{r^2}f^2 A_{\parallel} = -\mu_0 e(n_{0\mathrm{i}}v_{1\mathrm{i}\parallel} - n_{0\mathrm{e}}v_{1\mathrm{e}\parallel})$$

- **CDBM Eigenmode Equation**

$$\frac{\partial}{\partial\xi}\frac{\gamma f}{\gamma + \eta_{\mathrm{r}} m^2 f + \lambda m^4 f^2}\frac{\partial\phi_1}{\partial\xi} - (\gamma^2 f + \gamma\mu m^2 f)\phi_1 + \frac{\alpha\gamma}{\gamma + \chi m^2 f}H(\xi)\phi = 0$$

Marginal Stability Condition ($\gamma = 0$)

$$\frac{1}{\lambda}\frac{\partial^2\phi}{\partial\xi^2} - \mu m^6 f^3\phi + \frac{\alpha m^2 f}{\chi}H(\xi)\phi = 0$$

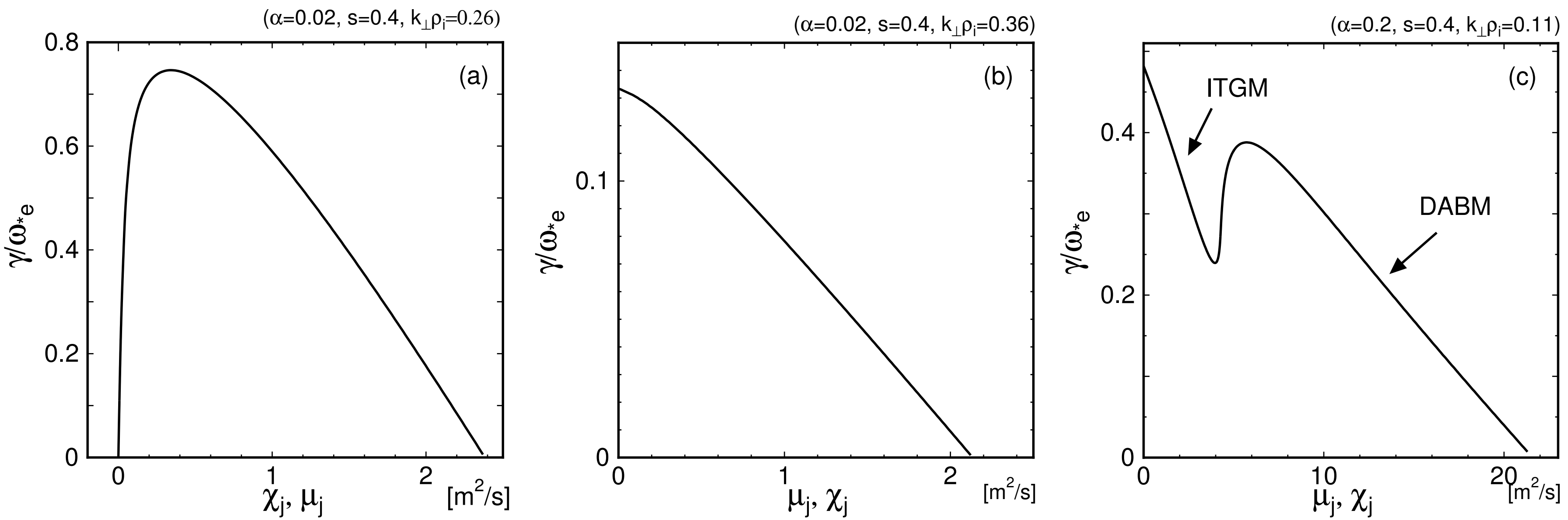
- **Low β DABM Eigenmode Equation**: Marginal Stability Condition

$$\frac{\partial^2\phi}{\partial\xi^2} + \mu_{\mathrm{i}}\chi_{\mathrm{e}}\frac{\tau_{PA}^2}{r^4}\frac{v_A^2 m^4 f^2}{v_s^2}\phi - \mu_{\mathrm{i}}\chi_{\mathrm{i}}\frac{\tau_{AP}^2 c^2 m^6 f^3}{r^6\omega_{p\mathrm{i}}^2}\phi + \frac{m^2 c^2\alpha_{\mathrm{i}}}{2r^2\omega_{p\mathrm{i}}^2}H(\xi)\phi = 0$$

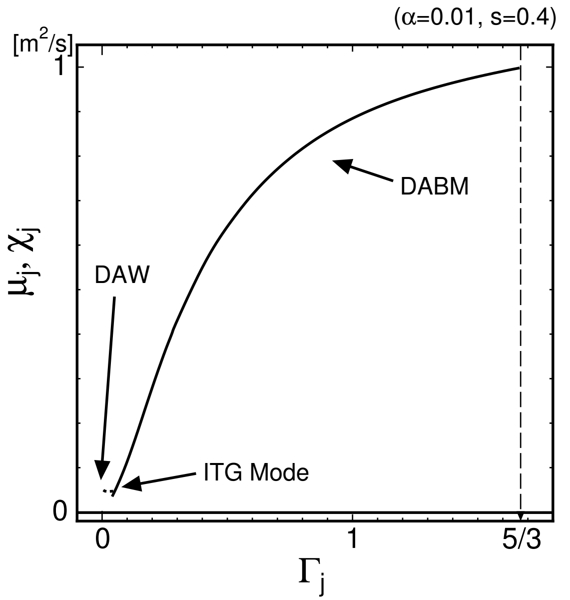
- **Eigenmode equations for CDBM and DABM are similar.**

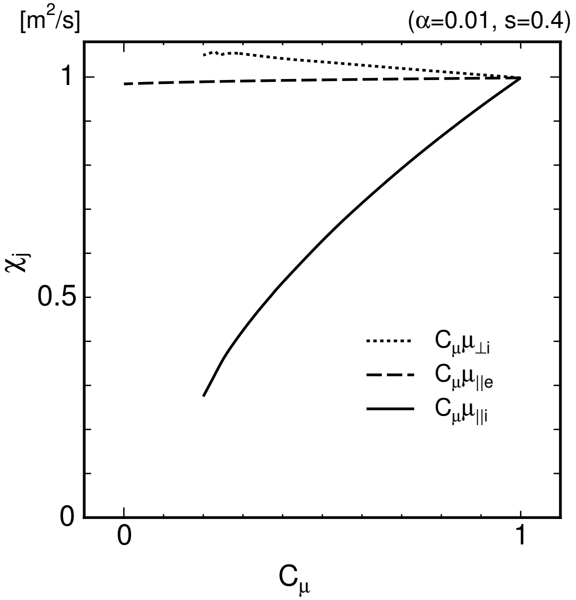
Nonlinear Analysis (Toroidal Plasma)

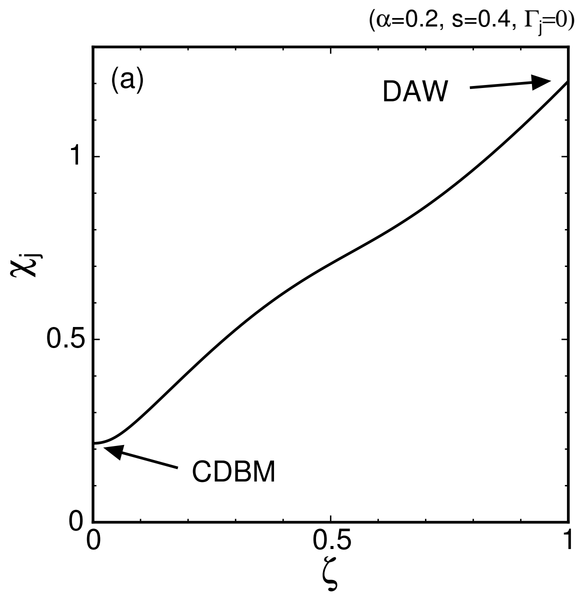
- **Amplitude Dependence of the Growth Rate**
  - **Linear growth rate ($\chi_j = 0$) is sensitive to $k_\perp \rho_\mathrm{i}$.**
  - **For large α, electromagnetic effect becomes important.**
  - **Saturation level can be estimated from the marginal condition.**



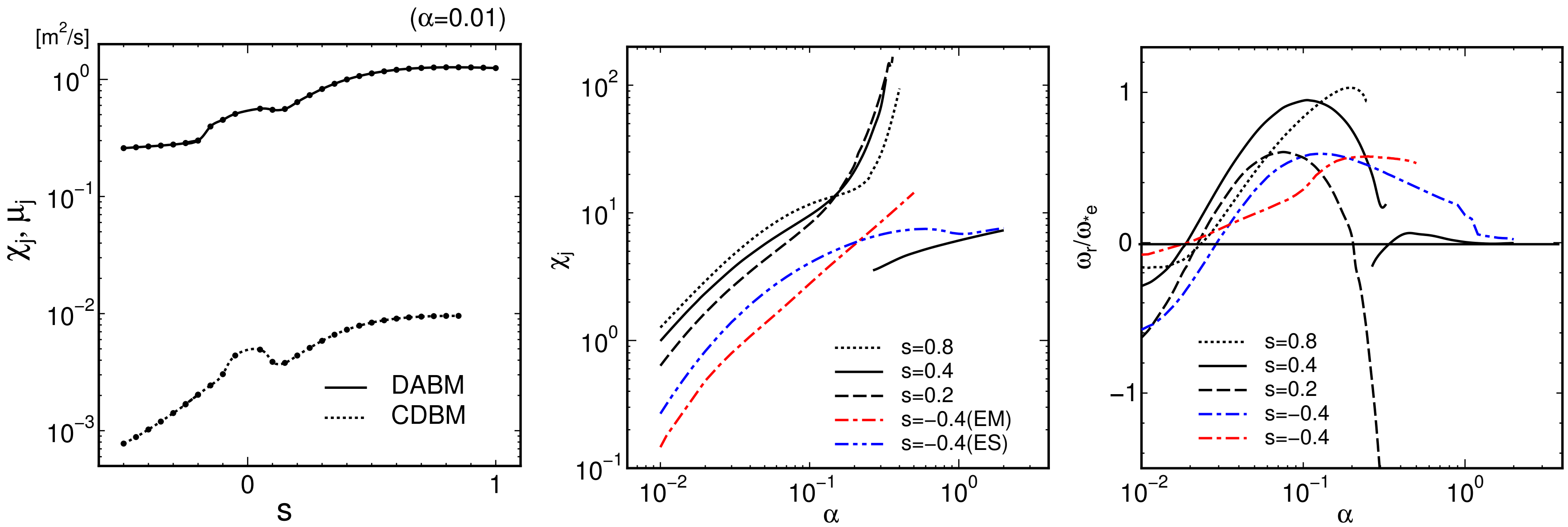
- Sensitivity on Various Parameters
  - Compressibility enhances the transport. (Γ_j: Adiabatic index)
  - Parallel viscosity of ion enhances the transport.
  - Finite drift frequency enhances the transport.



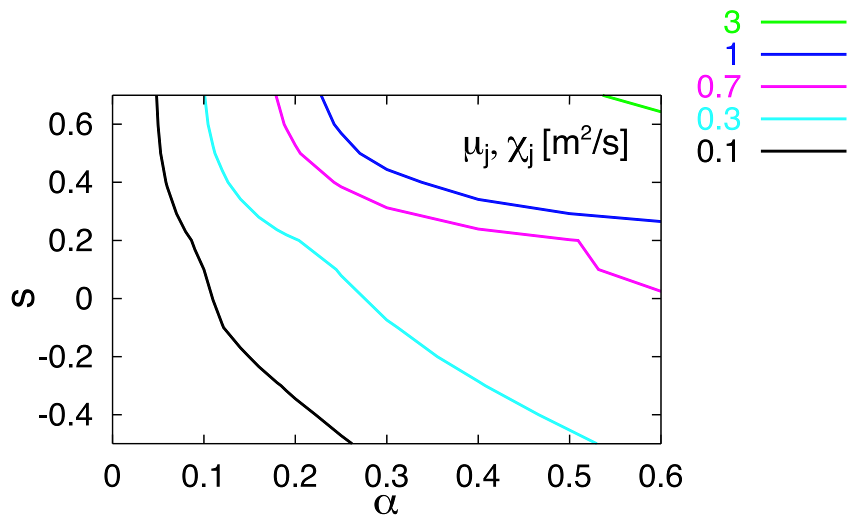




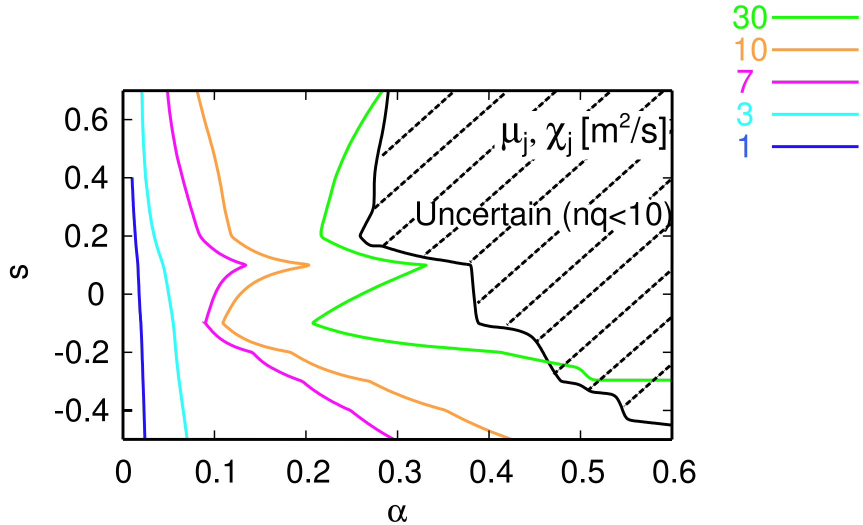
- **Dependence on s and α**
  - **Transport of DABM is much larger than that of CDBM.**
  - **Negative magnetic shear reduces the transport.**
  - **χ is proportional to $\alpha^{3/2}$ for small α.**
  - **There exists critical α above which transport is strongly enhanced.**



- **Contour of χ on s-α plane**



CDBM



DABM

Summary

- In order to describe both the electrostatic ion temperature gradient (ITG) mode and the electromagnetic current diffusive ballooning mode (CDBM), we have derived a set of reduced two-fluid equations in both slab and toroidal configurations and numerically solved them as an eigenvalue problem.

- Linear analysis in a toroidal configuration describes a ballooning mode, which we call DABM (Drift Alfvén Ballooning Mode).
  - Small pressure gradient: Toroidal ITG mode
  - Large pressure gradient: Ballooning mode with stabilizing ω_*

- Based on the theory of self-sustained turbulence, we have estimated the transport coefficients from the marginal stability condition.
  - When α is small, χ is approximately proportional to $\alpha^{3/2}$.
  - χ is a increasing function of $s - \alpha$; similar to CDBM model which successfully reproduces the ITB formation.

- **When α exceeds a critical value, χ starts to increase strongly with α, which may suggest the stiffness of the profile**

- **The DABM model is a promising candidate to explain the turbulent transport in tokamaks.**