

Integrated Simulation Based on TASK Code

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Contents

- BPSI Activity
- TASK Code
- Extensions in Progress
- Summary

Prediction of Burning Plasmas

- **Experiment**

- Progress in Diagnostics: High spatial and time resolution, EM fields
- Large Plasma Experiment: Limited number of shots
- ITER Burning Plasma: 12 years from now

- **Theory**

- Progress in Nonlinear Physics

- **Simulation**

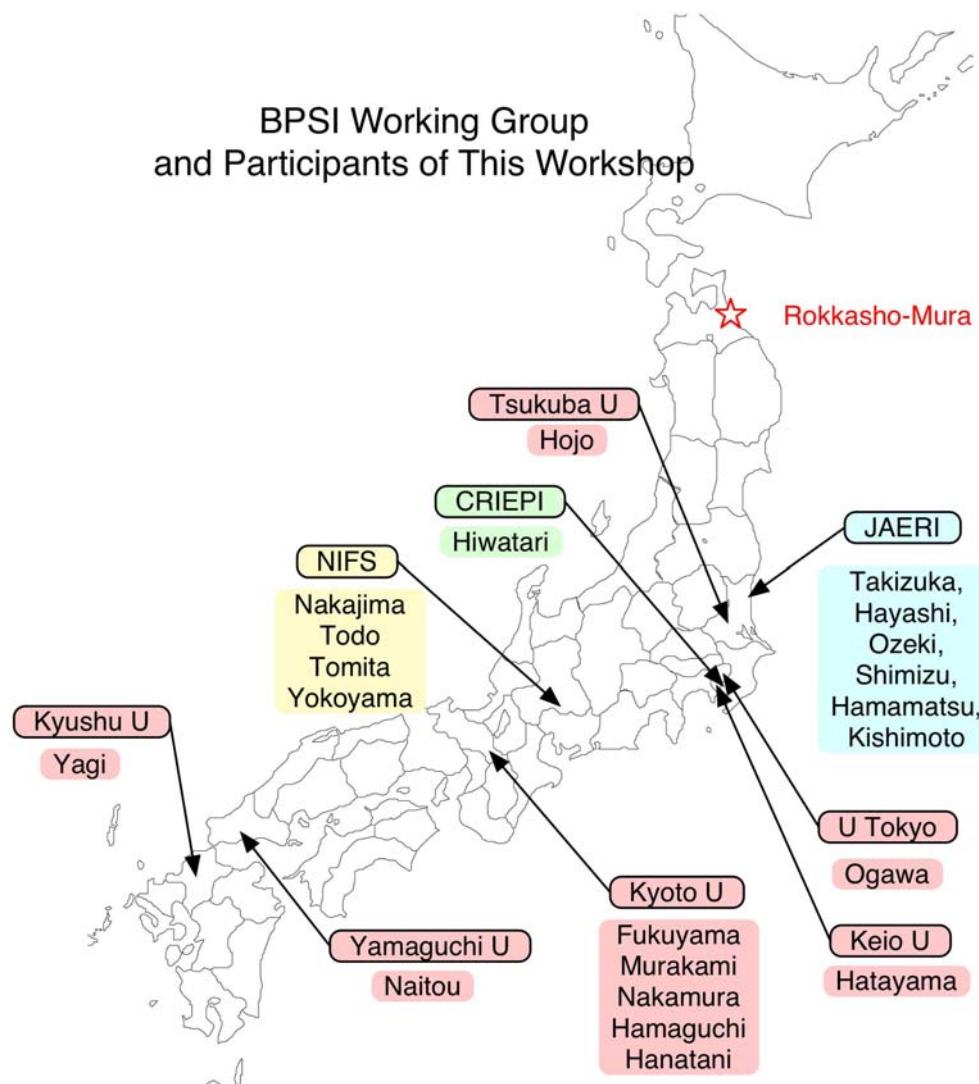
- Exponential Growth of Computation Resources and Network Speed
- Progress in Calculation Technique
- Individual Phenomenon in a Plasma: Detailed simulation by various codes
- Total Description of a Plasma by a Single Code: Impossible

Integrated Simulation of Entire Burning Plasma

- **Indispensable**
- **Feasible in near future**

BPSI: Burning Plasma Simulation Initiative

Research Collaboration among Universities, NIFS and JAERI



Activity of BPSI

- **Framework** for collaboration of various plasma simulation codes
 - Common interface for data transfer
 - Development of reference core code, TASK
- **New Physics** in interaction of phenomena with different time and space scales
 - Transport-MHD Hierarchical Model
 - Core-SOL Interface
 - Interaction with Energetic Particles
- **Advanced technique** of computer science, network computing and visualization
 - **Parallel computing**: PC cluster, Massively Parallel, Vector-Parallel
 - **Distributed computing**: GRID computing, Efficient Use of computer resources
 - **Visualization**: Parallel visualization, Visi GRID

TASK code

- Transport Analyzing System for tokamak

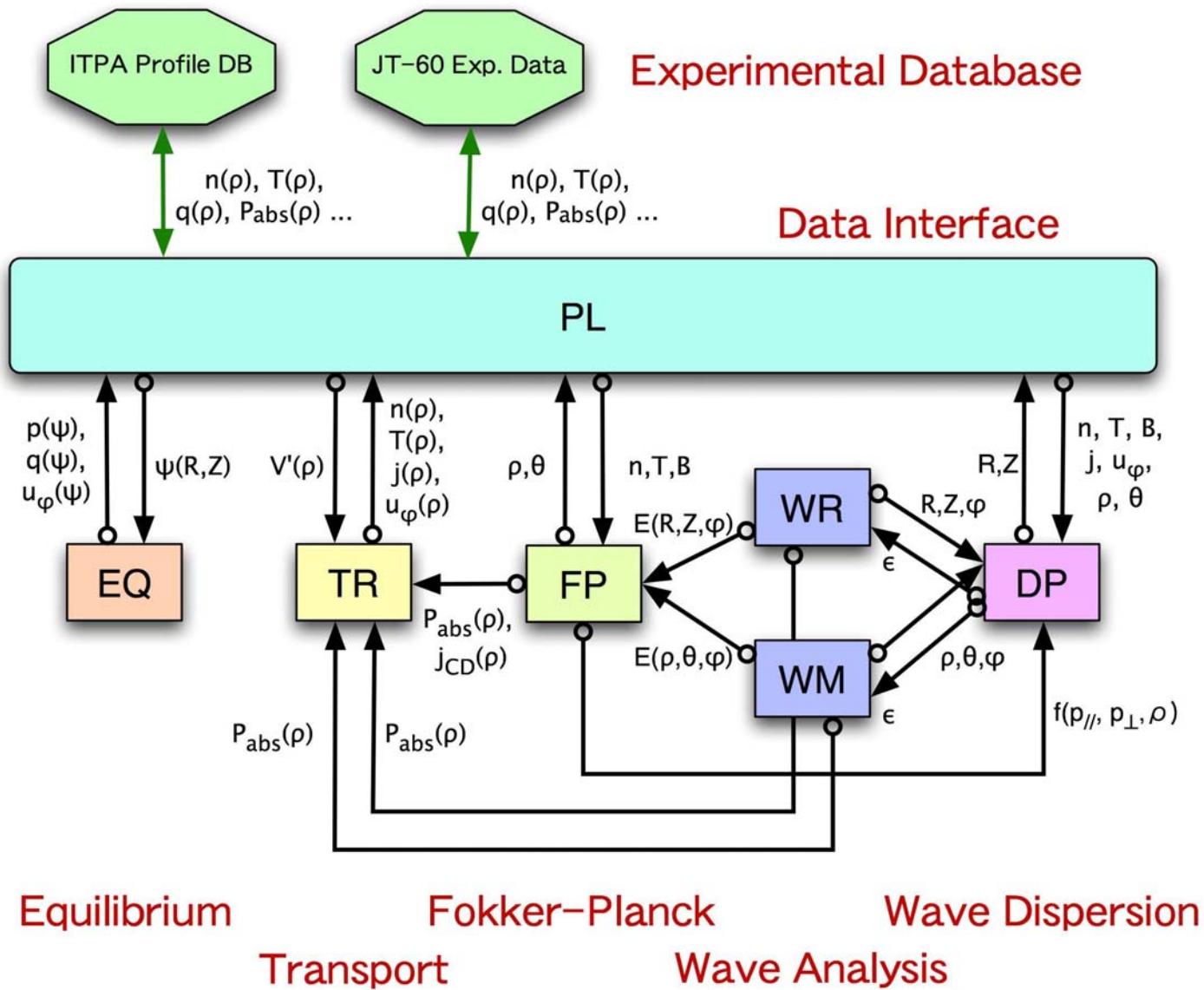
- **Modules**

TASK/EQ	2D Equilibrium	Fixed boundary, Toroidal rotation
PL	Data Conversion	Flux coord. \leftrightarrow Real coord., Profile database
TR	1D Transport	Diffusive Transport, Transport models
WR	Geometrical Optics	EC, LH: Ray tracing, Beam tracing
WM	Full Wave	IC, AW: Antenna excitation, Eigen mode
FP	3D Fokker-Planck	Relativistic, Bounce-averaged
DP	Wave Dispersion	Local dielectric tensor, Arbitrary $f(\nu)$

- **Under Development**

- **TX**: Transport analysis based on flux-averaged fluid equation
- **WA**: Global linear stability analysis
- **WI**: Integro-differential wave analysis (FLR, $k \parallel \nabla B$, $k \parallel \nabla n$)
- **WF**: 3D FEM full wave analysis (mirror configuration, processing plasma)

Structure of TASK code



Features of TASK code

- **Time Evolution of Tokamak Plasma**
 - Module Structure
 - Various Heating and Current Drive Scheme
 - High Portability
 - Development using CVS (concurrent version system)
 - Parallel Processing using MPI library
 - Future Extension for Helical Plasma
- **Status**
 - EQ + TR + PL
 - EQ + WR + FP + DP
 - EQ + WM
 - Coupling of all modules through PL: under development

Wave Dispersion Analysis : TASK/DP

- **Various Models of Dispersion Tensor $\overset{\leftrightarrow}{\epsilon}(\omega, k; \mathbf{r})$:** (Available, Planning)
 - Resistive MHD model
 - Collisional cold plasma model
 - Collisional warm plasma model
 - Kinetic plasma model (Maxwellian, non-relativistic)
 - Kinetic plasma model (Maxwellian, relativistic)
 - Kinetic plasma model (Arbitrary $f(v)$, non-relativistic)
 - Kinetic plasma model (Arbitrary $f(v)$, relativistic)
 - Gyro-kinetic plasma model (Maxwellian, non-relativistic)
 - Gyro-kinetic plasma model (Arbitrary $f(v)$, non-relativistic)
- **Input parameters:**
 - $n, u_{||}, T, B$
 - $n, u_{||}, T, B, \nabla_{\perp} n, \nabla_{\perp} T, \nabla_{\perp} B, E_{\perp}$

Geometrical Optics : TASK/WR

- **Ray Tracing Method:**

- Characteristic length of inhomogeneity $L \gg \lambda$ wave length
- Plane wave: beam size $d \gg \lambda$ wave length

- **Beam Tracing Method**

- Analysis of wave propagation with finite beam size
- Expansion parameter $\delta = \sqrt{\lambda/L} \ll 1$
- **Beam shape** : Gaussian beam (Hermite polynomial H_n)

$$E(\mathbf{r}) = \text{Re} [C(\delta^2 \mathbf{r}) v_e(\delta^2 \mathbf{r}) e^{i s(\mathbf{r}) - \phi(\mathbf{r})}]$$

— amplitude: C , polarization: v_e , phase: $s(\mathbf{r}) + i\phi(\mathbf{r})$

$$s(\mathbf{r}) = s_0(\tau) + k_\alpha^0(\tau)[r^\alpha - r_0^\alpha(\tau)] + \frac{1}{2}s_{\alpha\beta}[r^\alpha - r_0^\alpha(\tau)][r^\beta - r_0^\beta(\tau)]$$

$$\phi(\tau) = \frac{1}{2}\phi_{\alpha\beta}[r^\alpha - r_0^\alpha(\tau)][r^\beta - r_0^\beta(\tau)]$$

— \mathbf{r}_0 : position of beam axis, k^0 : wave number on beam axis

— **Curvature radius**: $R_\alpha = 1/\lambda s_{\alpha\alpha}$, **Beam radius**: $d_\alpha = \sqrt{2/\phi_{\alpha\alpha}}$

Beam Tracing Equation

- From solubility condition:

$$\frac{dr_0^\alpha}{d\tau} = \frac{\partial K}{\partial k_\alpha}$$

$$\frac{dk_\alpha^0}{d\tau} = -\frac{\partial K}{\partial r^\alpha}$$

$$\frac{ds_{\alpha\beta}}{d\tau} = -\frac{\partial^2 K}{\partial r^\alpha \partial r^\beta} - \frac{\partial^2 K}{\partial r^\beta \partial k_\gamma} s_{\alpha\gamma} - \frac{\partial^2 K}{\partial r^\alpha \partial k_\gamma} s_{\beta\gamma} - \frac{\partial^2 K}{\partial k^\gamma \partial k^\delta} s_{\alpha\gamma} s_{\beta\delta} + \frac{\partial^2 K}{\partial k^\gamma \partial k^\delta} \phi_{\alpha\gamma} \phi_{\beta\delta}$$

$$\frac{d\phi_{\alpha\beta}}{d\tau} = -\left(\frac{\partial^2 K}{\partial r^\alpha \partial k^\gamma} + \frac{\partial^2 K}{\partial k^\gamma \partial k_\delta} s_{\alpha\delta}\right) \phi_{\beta\gamma} - \left(\frac{\partial^2 K}{\partial r^\beta \partial k^\gamma} + \frac{\partial^2 K}{\partial k^\gamma \partial k_\delta} s_{\beta\delta}\right) \phi_{\alpha\gamma}$$

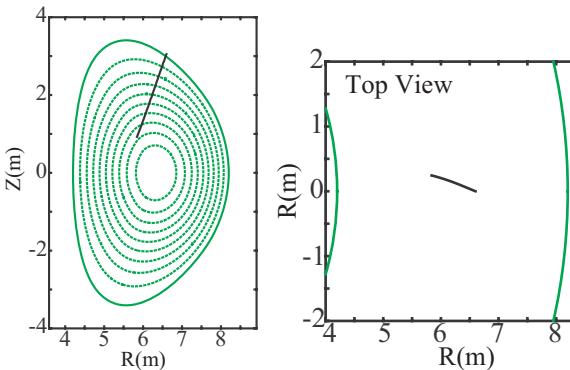
- Integration of 18 ordinary differential equations for beam axis position, typical wave number, curvature radius and beam radius
- Equation for wave amplitude C_{mn}

$$\nabla \cdot (\mathbf{v}_{g0} |C_{mn}|^2) = -2 (\gamma |C_{mn}|^2)$$

\mathbf{v}_{g0} : group velocity, $\gamma \equiv (v_e^* \cdot \overset{\leftrightarrow}{\epsilon}_A \cdot v_e) / (\partial K / \partial \omega)$: Damping rate

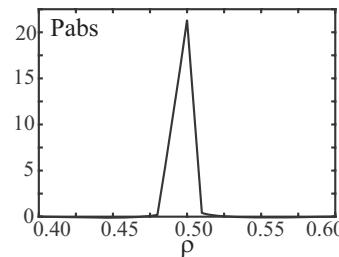
Analysis of ECCD by TASK Code

Poloidal angle	70°
Toroidal angle	20°
Initial beam radius	0.05 m
Initial beam curvature	2 m

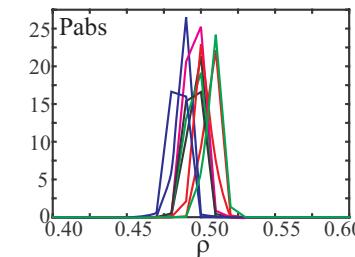


Ray/Beam Profile

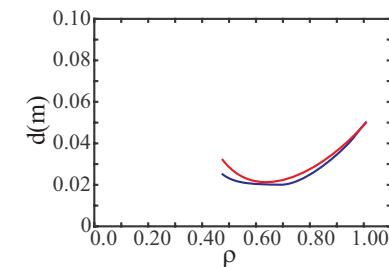
One Ray



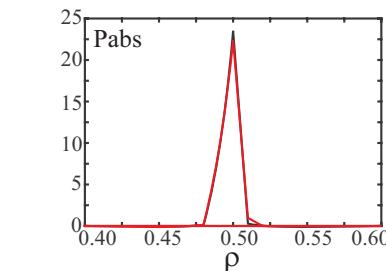
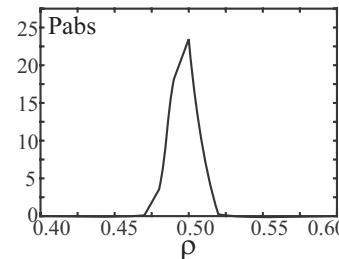
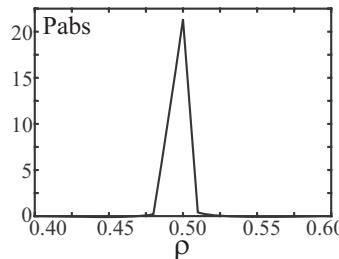
Multi Rays



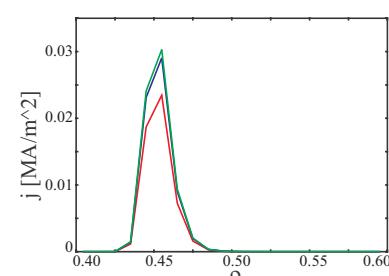
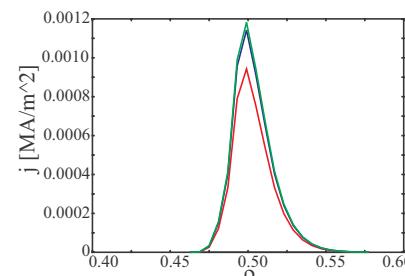
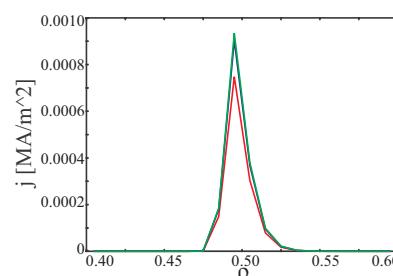
Beam Tracing



P_{abs} Profile



j_{CD} Profile



Fokker-Planck Analysis : TASK/FP

- **Fokker-Planck equation** for **velocity distribution function** $f(p_{\parallel}, p_{\perp}, \psi, t)$

$$\frac{\partial f}{\partial t} = E(f) + C(f) + Q(f) + L(f) \quad (1)$$

- $E(f)$: Acceleration term due to DC electric field
- $C(f)$: Coulomb collision term
- $Q(f)$: Quasi-linear term due to wave-particle resonance
- $L(f)$: Spatial diffusion term

- **Bounce-Averaged**: Trapped particle effect, zero banana width
- **Relativistic**: momentum p , weak relativistic collision term
- **Three-dimensional**: Spatial diffusion (classical, neoclassical, turbulence)

Full wave analysis: TASK/WM

- **magnetic surface coordinate:** (ψ, θ, φ)
- Boundary-value problem of **Maxwell's equation**

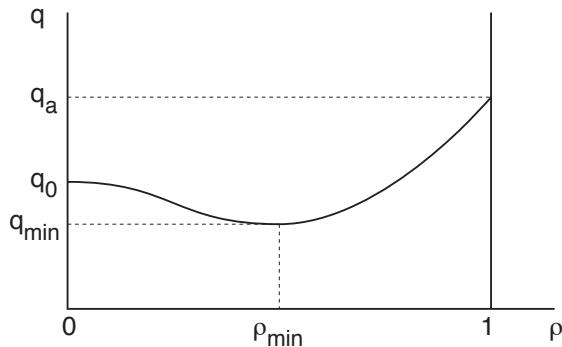
$$\nabla \times \nabla \times \mathbf{E} = \frac{\omega^2}{c^2} \overset{\leftrightarrow}{\epsilon} \cdot \mathbf{E} + i \omega \mu_0 \mathbf{j}_{\text{ext}}$$

- Kinetic **dielectric tensor:** $\overset{\leftrightarrow}{\epsilon}$
 - **Wave-particle resonance:** $Z[(\omega - n\omega_c)/k_{\parallel}v_{\text{th}}]$
 - **Fast ion: Drift-kinetic**
$$\left[\frac{\partial}{\partial t} + v_{\parallel} \nabla_{\parallel} + (\mathbf{v}_d + \mathbf{v}_E) \cdot \nabla + \frac{e_{\alpha}}{m_{\alpha}} (v_{\parallel} E_{\parallel} + \mathbf{v}_d \cdot \mathbf{E}) \frac{\partial}{\partial \varepsilon} \right] f_{\alpha} = 0$$
- Poloidal and toroidal **mode expansion**
 - **Accurate estimation of k_{\parallel}**
- Eigenmode analysis: **Complex eigen frequency** which maximize wave amplitude for fixed excitation proportional to electron density

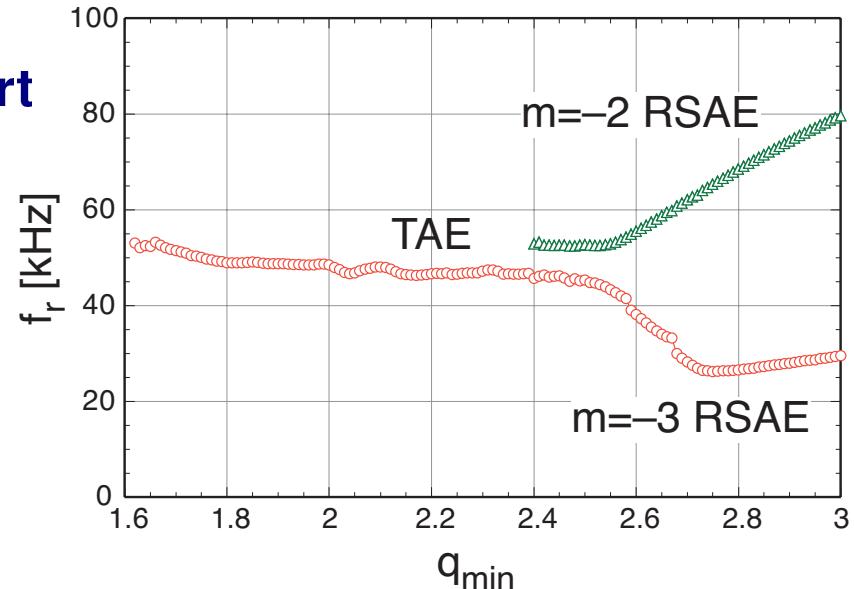
Analysis of TAE in Reversed Shear Configuration

q_{\min} Dependence of Eigenmode Frequency

Assumed q profile



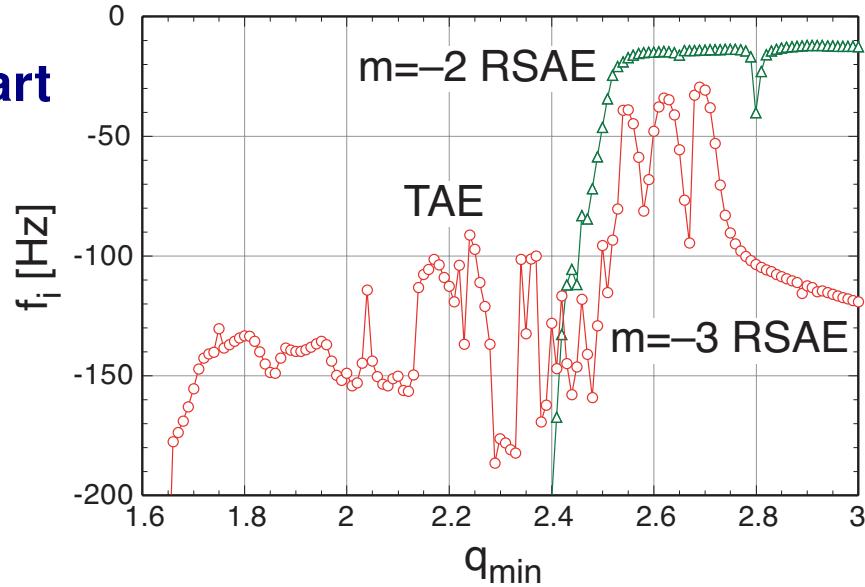
Real part



Plasma Parameters

R_0	3 m
a	1 m
B_0	3 T
$n_e(0)$	10^{20} m^{-3}
$T(0)$	3 keV
$q(0)$	3
$q(a)$	5
ρ_{\min}	0.5
n	1
Flat density profile	

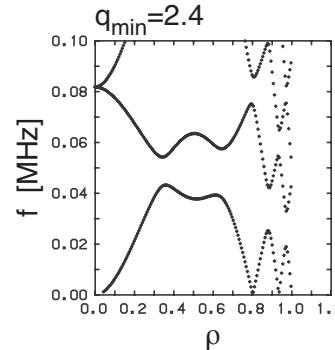
Imag part



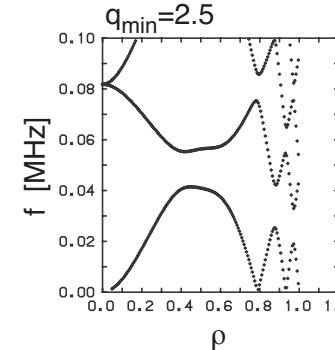
Eigenmode Structure

Alfvén resonance

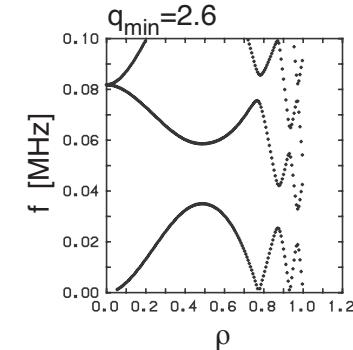
$$q_{\min} = 2.4$$



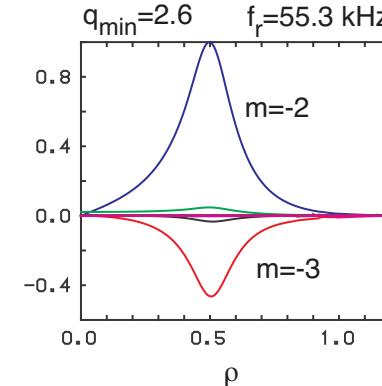
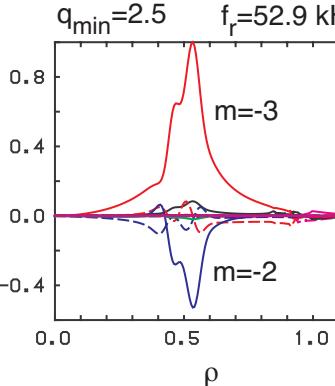
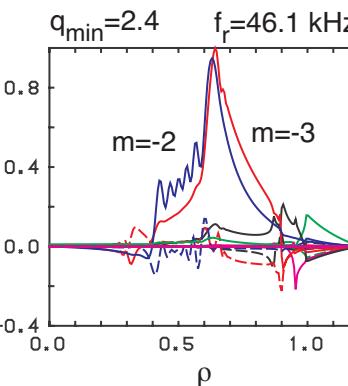
$$q_{\min} = 2.5$$



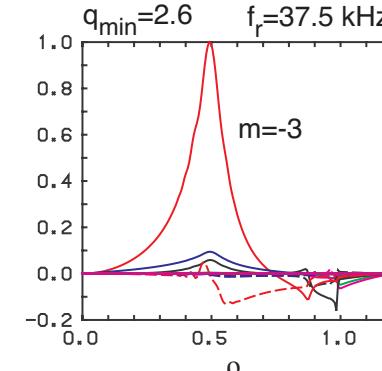
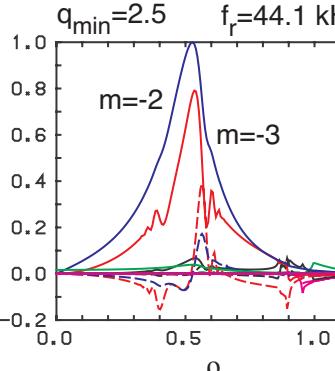
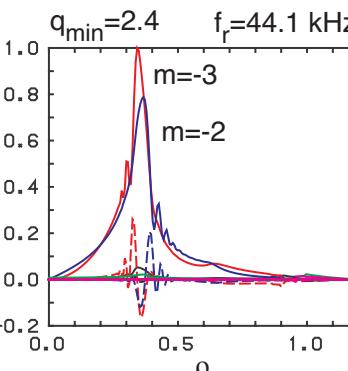
$$q_{\min} = 2.6$$



Higher freq.



Lower freq.



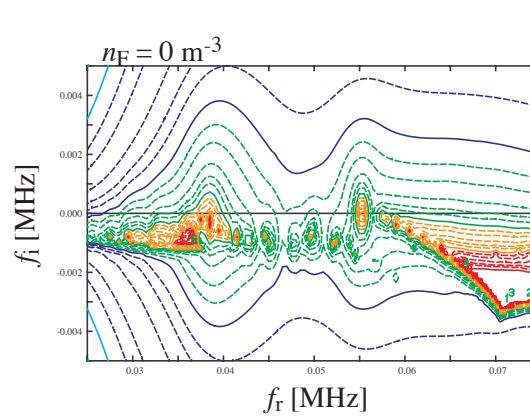
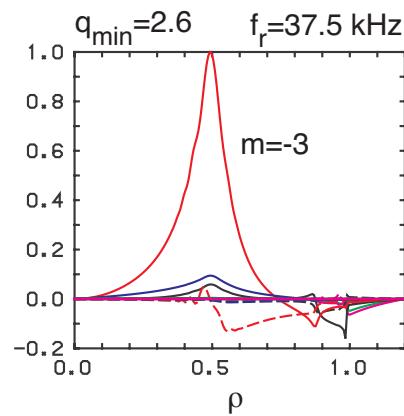
TAEs

Double TAE

RSAE

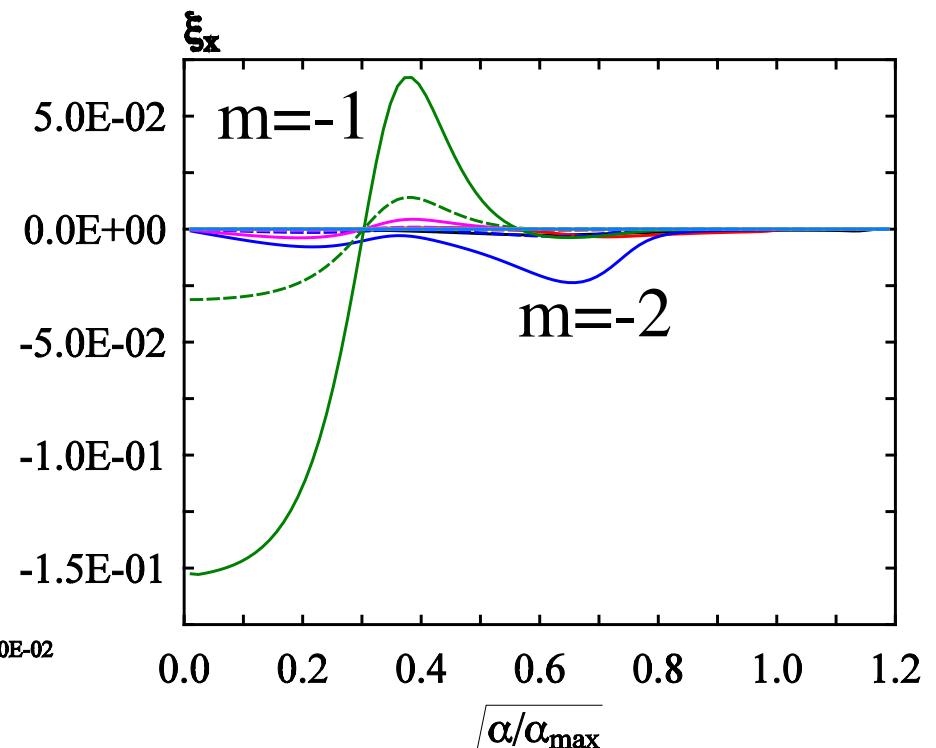
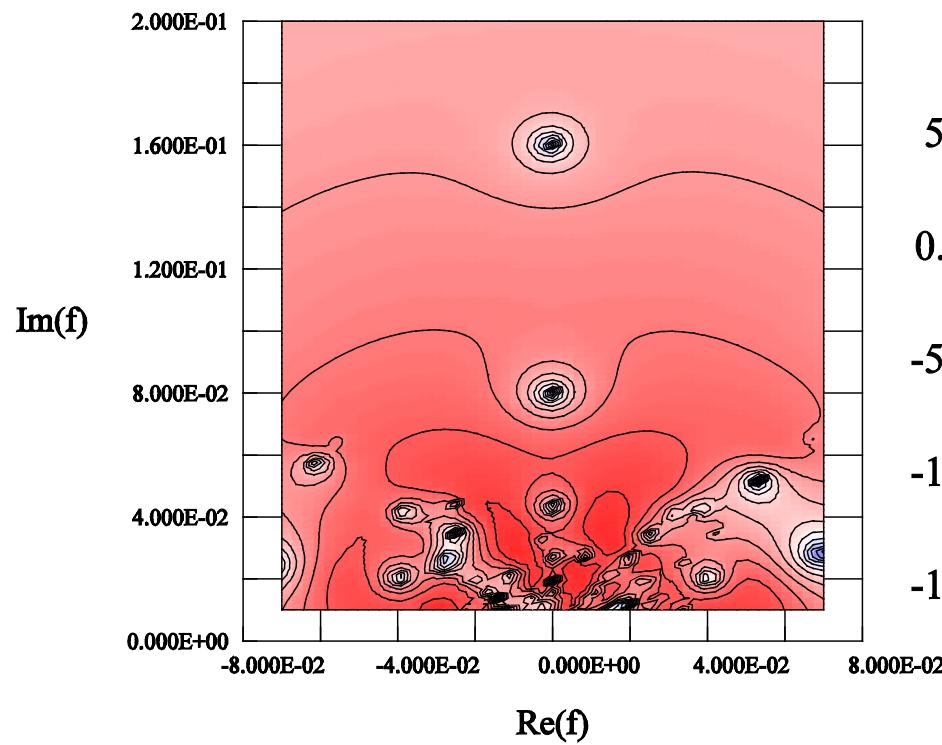
Excitation by Energetic Particles ($q_{\min} = 2.6$)

- Without EP



Internal Kink mode I

- Contour maps of $[\log(\int |\vec{E}|^2 dV)]^{-1}$ in the complex frequency space ($n = 1$, $q_0 = 0.7$, $q_a = 3$).



Displacement in the radial direction

Extension of Transport Analysis

- **Level of Analysis:**
 - Diffusive transport equation: Flux-Gradient relation
 - Fluid-like transport equation: Flux-averaged fluid equation (plasma rotation, transient phenomena)
 - Kinetic transport equation: Bounce-averaged Fokker-Plank equation
- **Diffusive transport equation:** V : Volume, ρ : Normalized radius, $V' = dV/d\rho$

- **Particle transport**

$$\frac{1}{V'} \frac{\partial}{\partial t} (n_s V') = - \frac{\partial}{\partial \rho} \left(\langle |\nabla \rho| \rangle n_s V_s - \langle |\nabla \rho|^2 \rangle D_s \frac{\partial n_s}{\partial \rho} \right) + S_s$$

- **Heat transport**

$$\frac{1}{V'^{5/3}} \frac{\partial}{\partial t} \left(\frac{3}{2} n_s T_s V'^{5/3} \right) = - \frac{1}{V'} \frac{\partial}{\partial \rho} \left(V' \langle |\nabla \rho| \rangle \frac{3}{2} n_s T_s V_{Es} - V' \langle |\nabla \rho|^2 \rangle n_s \chi_s \frac{\partial T_s}{\partial \rho} \right) + P_s$$

- **Current diffusion**

$$\frac{\partial B_\theta}{\partial t} = \frac{\partial}{\partial \rho} \left[\frac{\eta}{FR_0 \langle R^{-2} \rangle} \frac{R_0 F^2}{\mu_0 V'} \frac{\partial}{\partial \rho} \left(\frac{V' B_\theta}{F} \left\langle \frac{|\nabla \rho|^2}{R^2} \right\rangle \right) - \frac{\eta}{FR_0 \langle R^{-2} \rangle} \langle \mathbf{J} \cdot \mathbf{B} \rangle_{\text{ext}} \right]$$

CDBM Turbulence Model

- Marginal Stability Condition ($\gamma = 0$)

$$\chi_{\text{TB}} = F(s, \alpha, \kappa, \omega_{E1}) \alpha^{3/2} \frac{c^2}{\omega_{pe}^2} \frac{v_A}{qR}$$

Magnetic shear

$$s \equiv \frac{r}{q} \frac{dq}{dr}$$

Pressure gradient

$$\alpha \equiv -q^2 R \frac{d\beta}{dr}$$

Magnetic curvature

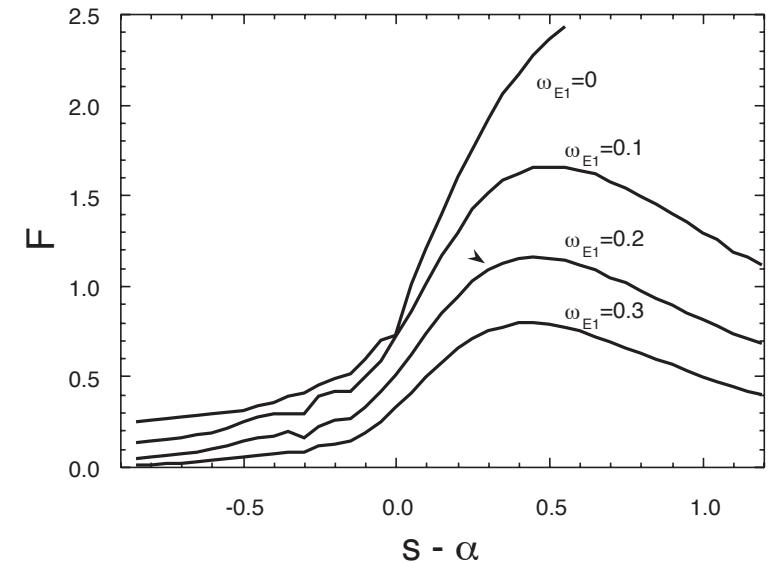
$$\kappa \equiv -\frac{r}{R} \left(1 - \frac{1}{q^2} \right)$$

$E \times B$ rotation shear

$$\omega_{E1} \equiv \frac{r^2}{sv_A} \frac{d}{dr} \frac{E}{rB}$$

- Weak and negative magnetic shear, Shafranov shift and $E \times B$ rotation shear reduce thermal diffusivity.

$s - \alpha$ dependence of $F(s, \alpha, \kappa, \omega_{E1})$

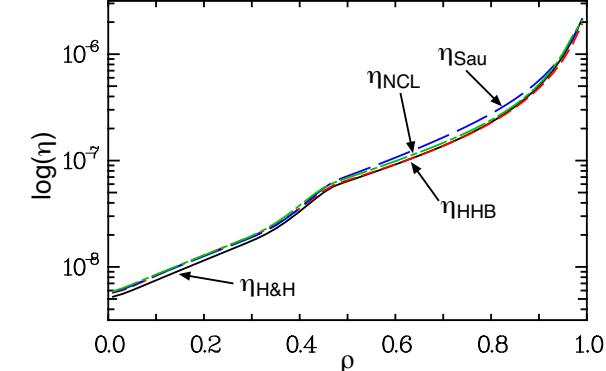
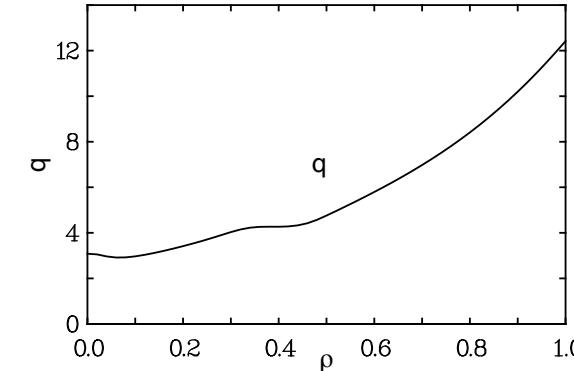
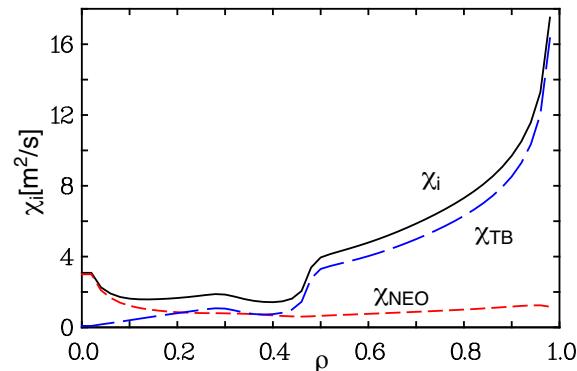
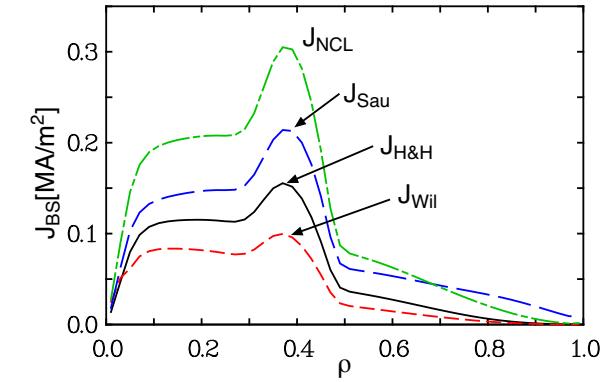
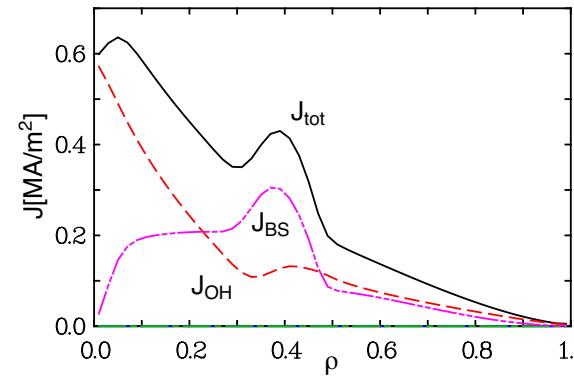
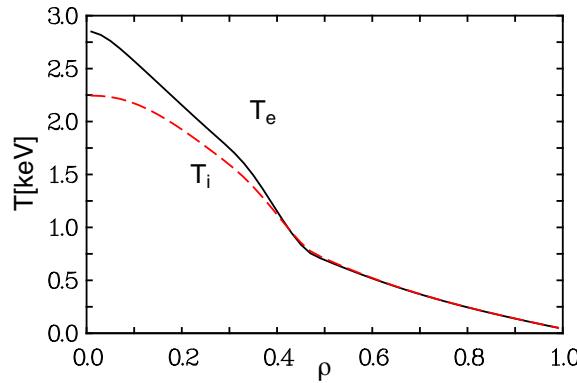


Fitting Formula

$$F = \begin{cases} \frac{1}{1 + G_1 \omega_{E1}^2} \frac{1}{\sqrt{2(1 - 2s')(1 - 2s' + 3s'^2)}} \\ \text{for } s' = s - \alpha < 0 \\ \frac{1}{1 + G_1 \omega_{E1}^2} \frac{1 + 9\sqrt{2}s'^{5/2}}{\sqrt{2}(1 - 2s' + 3s'^2 + 2s'^3)} \\ \text{for } s' = s - \alpha > 0 \end{cases}$$

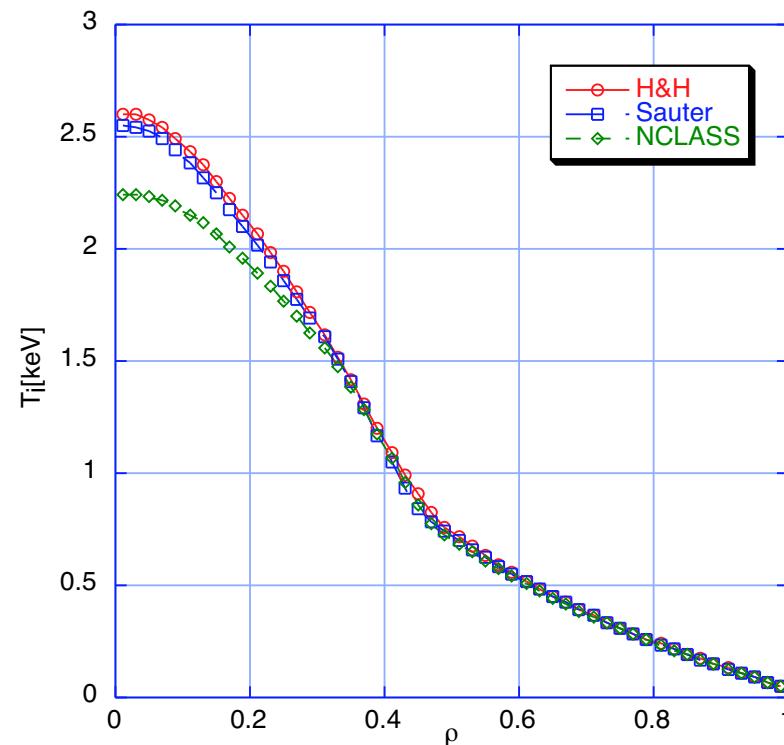
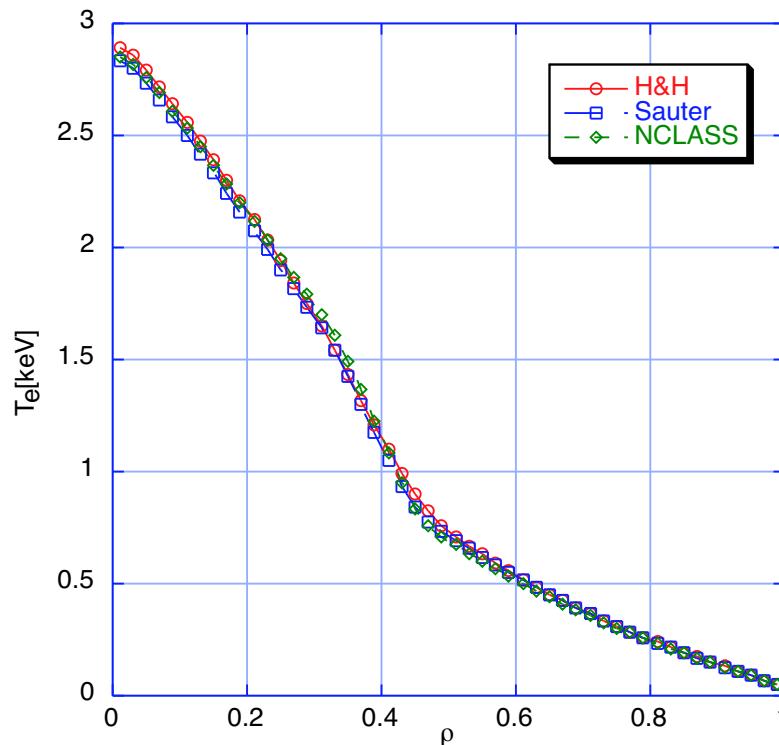
High β_p mode

- All conditions are the same as those of L mode except $\kappa = 1.5$ and $I_p = 1$ MA, therefore we compare neoclassical models on the **same profile**.
- **On-axis heating of 10 MW is switched on at $t = 1$ s during one second.**
- **Resistivities are similar to each other.**
- **Magnitudes of bootstrap currents are very different although graphic forms resemble respectively.**



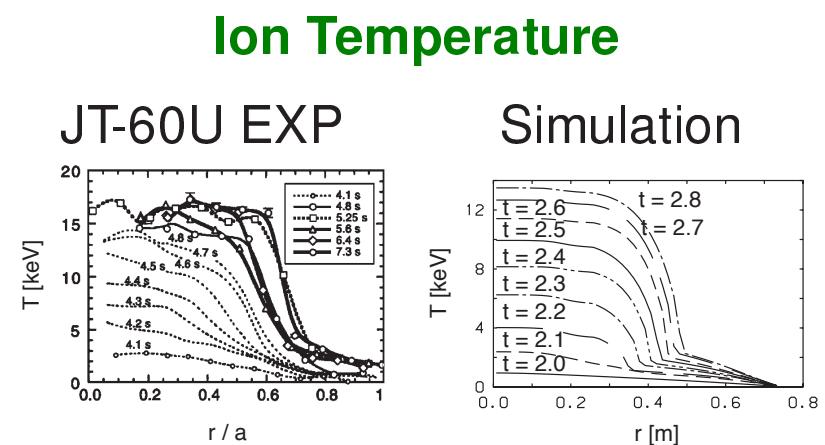
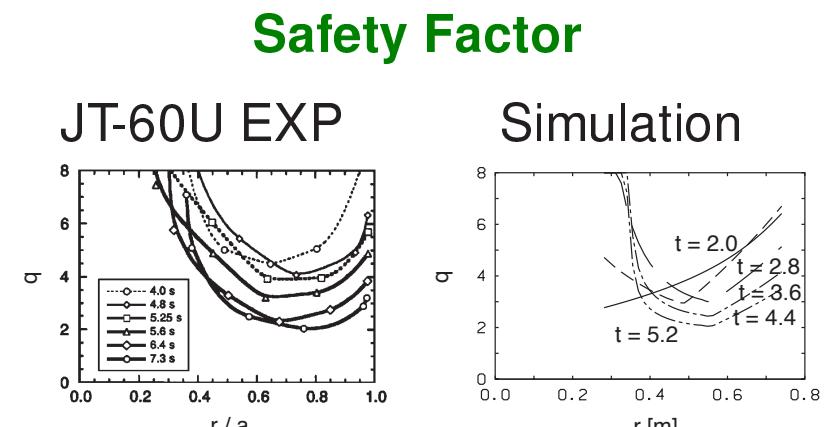
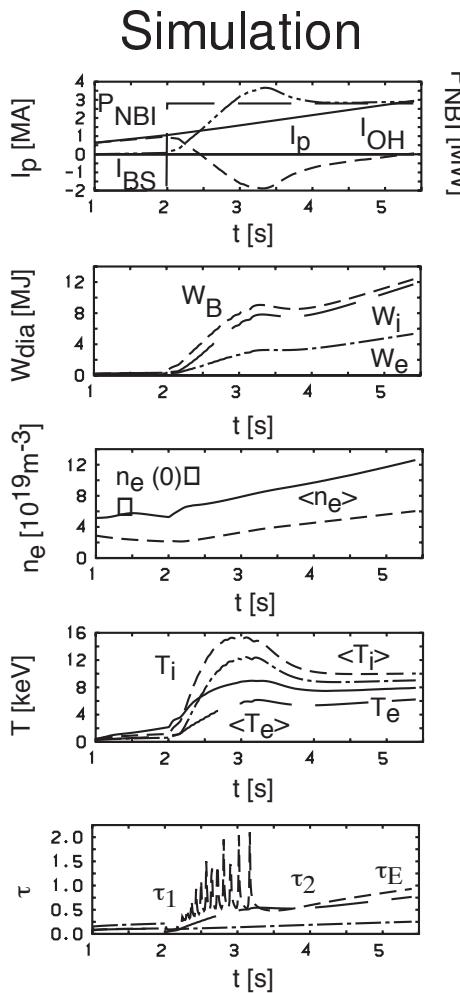
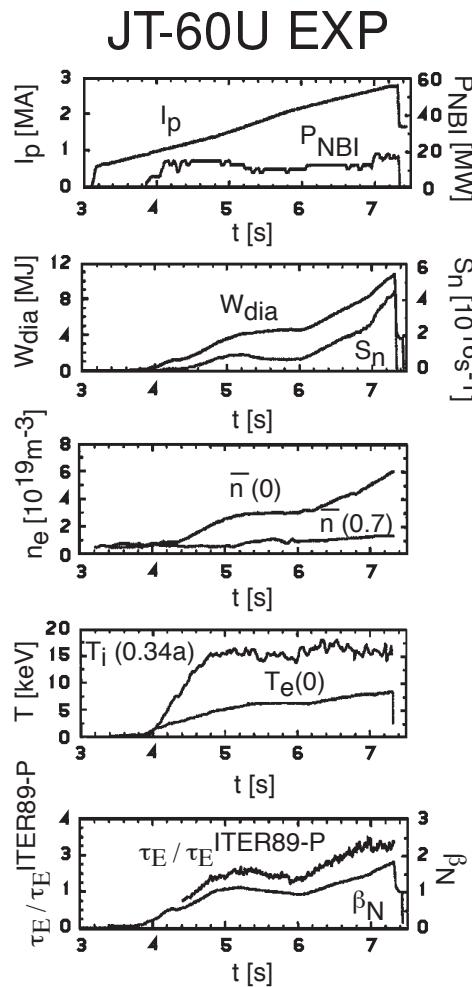
Difference due to Neoclassical Transport Models

- We carry out transport simulations on each neoclassical model on the **same initial condition** in the high β_p mode.
- The profiles below are electron and ion temperature ones at $t = 2$ s.
- **Quite a good agreement in the electron temperature profile.**
- **The reason the ion temperature near the axis in NCLASS is relatively low may be the strong influence of the neoclassical thermal diffusivity whose value is almost double in comparison with others.**



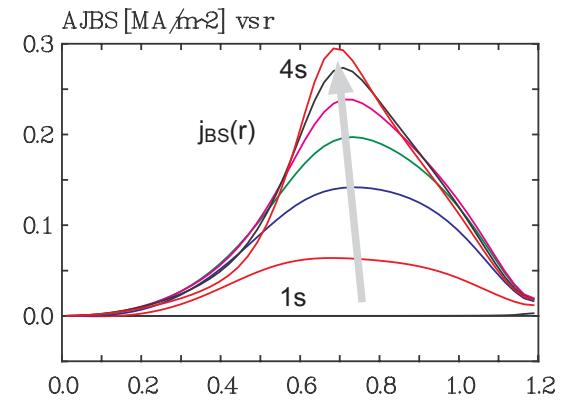
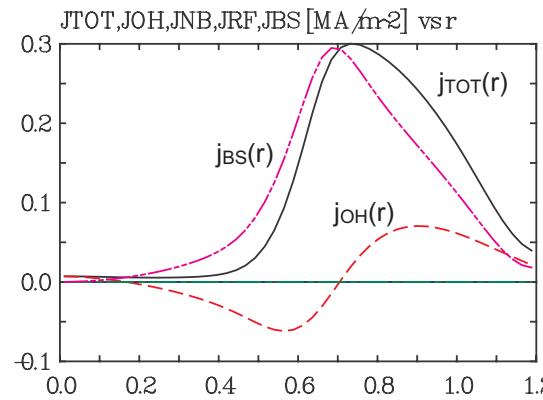
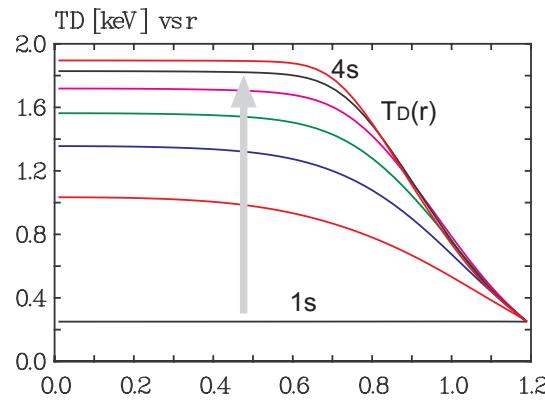
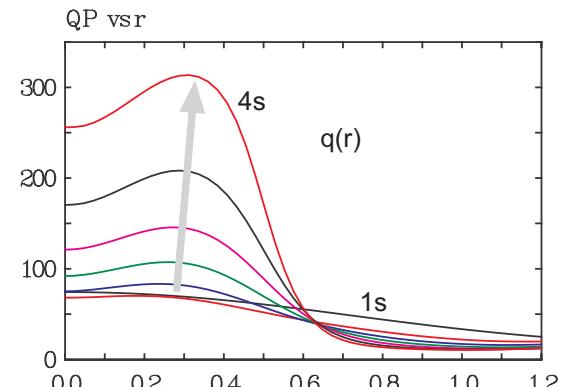
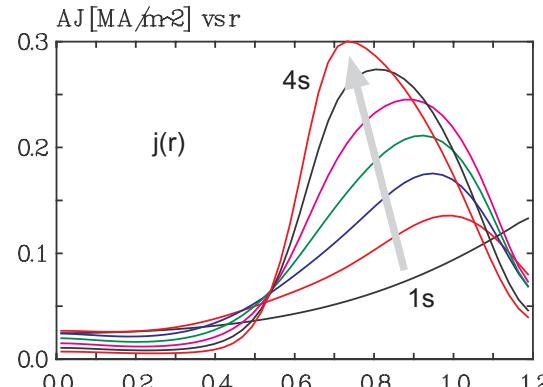
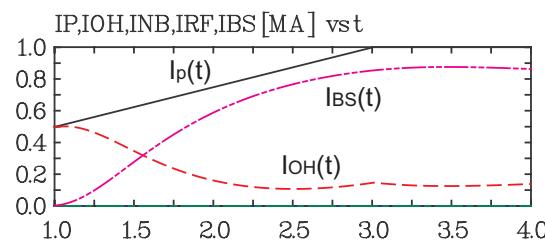
Simulation of ITB Formation

Comparison with JT-60U experimental data: on-going
Time Evolution



Simulation of Current Hole Formation

- Current ramp up: $I_p = 0.5 \rightarrow 1.0 \text{ MA}$
- Moderate heating: $P_H = 5 \text{ MW}$
- **Current hole** is formed.
- The formation is sensitive to the edge temperature.



Modeling of ETB Formation

- **Transport Simulation including Core and SOL Plasmas**
- **Role of Separatrix**
 - Closed magnetic surface \iff Open magnetic field line
 - Difference of dominant transport process
- **Radial Electric Field**
 - Poloidal rotation, Toroidal rotation
 - Polarization current
 - Poisson equation
- **Atomic Processes**
 - Ionization, Charge exchange, Recycling

Transport Model

- **1D Transport code (TASK/TX)** *Ref. Fukuyama et al.*
- **Two fluid equation for electrons and ions**
 - Flux surface average
 - Coupled with Maxwell equation
 - Neutral diffusion equation
- **Neoclassical transport**
 - Included as a poloidal viscosity term
 - Diffusion, resistivity, bootstrap current, Ware pinch
- **Anomalous transport**
 - Current diffusive ballooning mode
 - Ambipolar diffusion through poloidal momentum transfer
 - Perpendicular viscosity

Model Equation (1)

- **Fluid equations (electrons and ions)**

$$\frac{\partial n_s}{\partial t} = -\frac{1}{r} \frac{\partial}{\partial r} (rn_s u_{sr}) + S_s$$

$$\frac{\partial}{\partial t} (m_s n_s u_{sr}) = -\frac{1}{r} \frac{\partial}{\partial r} (rm_s n_s u_{sr}^2) + \frac{1}{r} m_s n_s u_{s\theta}^2 + e_s n_s (E_r + u_{s\theta} B_\phi - u_{s\phi} B_\theta) - \frac{\partial}{\partial r} n_s T_s$$

$$\frac{\partial}{\partial t} (m_s n_s u_{s\theta}) = -\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 m_s n_s u_{sr} u_{s\theta}) + e_s n_s (E_\theta - u_{sr} B_\phi) + \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^3 n_s m_s \mu_s \frac{\partial}{\partial r} \frac{u_{s\theta}}{r} \right)$$

$$+ F_{s\theta}^{\text{NC}} + F_{s\theta}^{\text{C}} + F_{s\theta}^{\text{W}} + F_{s\theta}^{\text{X}} + F_{s\theta}^{\text{L}}$$

$$\frac{\partial}{\partial t} (m_s n_s u_{s\phi}) = -\frac{1}{r} \frac{\partial}{\partial r} (rm_s n_s u_{sr} u_{s\phi}) + e_s n_s (E_\phi + u_{sr} B_\theta) + \frac{1}{r} \frac{\partial}{\partial r} \left(rn_s m_s \mu_s \frac{\partial}{\partial r} u_{s\phi} \right)$$

$$+ F_{s\phi}^{\text{C}} + F_{s\phi}^{\text{W}} + F_{s\phi}^{\text{X}} + F_{s\phi}^{\text{L}}$$

$$\frac{\partial}{\partial t} \frac{3}{2} n_s T_s = -\frac{1}{r} \frac{\partial}{\partial r} r \left(\frac{5}{2} u_{sr} n_s T_s - n_s \chi_s \frac{\partial}{\partial r} T_e \right) + e_s n_s (E_\theta u_{s\theta} + E_\phi u_{s\phi})$$

$$+ P_s^{\text{C}} + P_s^{\text{L}} + P_s^{\text{H}}$$

Model Equation (2)

- **Neutral Transport**

$$\frac{\partial n_0}{\partial t} = -\frac{1}{r} \frac{\partial}{\partial r} \left(-r D_0 \frac{\partial n_0}{\partial r} \right) + S_0$$

- **Maxwell equations**

$$\frac{1}{r} \frac{\partial}{\partial r} (r E_r) = \frac{1}{\epsilon_0} \sum_s e_s n_s$$

$$\frac{\partial B_\theta}{\partial t} = \frac{\partial E_\phi}{\partial r}, \quad \frac{\partial B_\phi}{\partial t} = -\frac{1}{r} \frac{\partial}{\partial r} (r E_\phi)$$

$$\frac{1}{c^2} \frac{\partial E_\theta}{\partial t} = -\frac{\partial}{\partial r} B_\phi - \mu_0 \sum_s n_s e_s u_{s\theta}, \quad \frac{1}{c^2} \frac{\partial E_\phi}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} (r B_\theta) - \mu_0 \sum_s n_s e_s u_{s\phi}$$

Transport Model (1)

- **Neoclassical transport**

- Viscosity force arises when plasma rotates in the poloidal direction.
- Banana-Plateau regime

$$F_{s\theta}^{\text{NC}} = - \sqrt{\pi} q^2 n_s m_s \frac{v_{Ts}}{qR} \frac{v_s^*}{1 + v_s^*} u_{s\theta}$$
$$v_s^* \equiv \frac{v_s q R}{\epsilon^{3/2} v_{Ts}}$$

- **This poloidal viscosity force induces**

- Neoclassical radial diffusion
- Neoclassical resistivity
- Bootstrap current
- Ware pinch

Transport Model (2)

- **Turbulent Diffusion**

- **Poloidal momentum exchange between electron and ion through the turbulent electric field**
- **Ambipolar flux (electron flux = ion flux)**

$$F_{i\theta}^W = - F_{e\theta}^W$$

$$= - ZeB_\phi n_i D_i \left[-\frac{1}{n_i} \frac{dn_i}{dr} + \frac{Ze}{T_i} E_r - \langle \frac{\omega}{m} \rangle \frac{ZeB_\phi}{T_i} - \left(\frac{\mu_i}{D_i} - \frac{1}{2} \right) \frac{1}{T_i} \frac{dT_i}{dr} \right]$$

- **Perpendicular viscosity**

- **Non-ambipolar flux (electron flux \neq ion flux):** $\mu_s = \text{constant} \times D$
- **Diffusion coefficient (proportional to $|E|^2$)**
 - **Current-diffusive ballooning mode turbulence model**

Modeling of Scrape-Off Layer Plasma

- **Particle, momentum and heat losses along the field line**

- **Decay time**

$$\nu_L = \begin{cases} 0 & (0 < r < a) \\ \frac{C_s}{2\pi r R \{1 + \log[1 + 0.05/(r - a)]\}} & (a < r < b) \end{cases}$$

- **Electron source term**

$$S_e = n_0 \langle \sigma_{\text{ion}} v \rangle n_e - \nu_L (n_e - n_{e,\text{div}})$$

- **Recycling from divertor**

- **Recycling rate:** $\gamma_0 = 0.8$

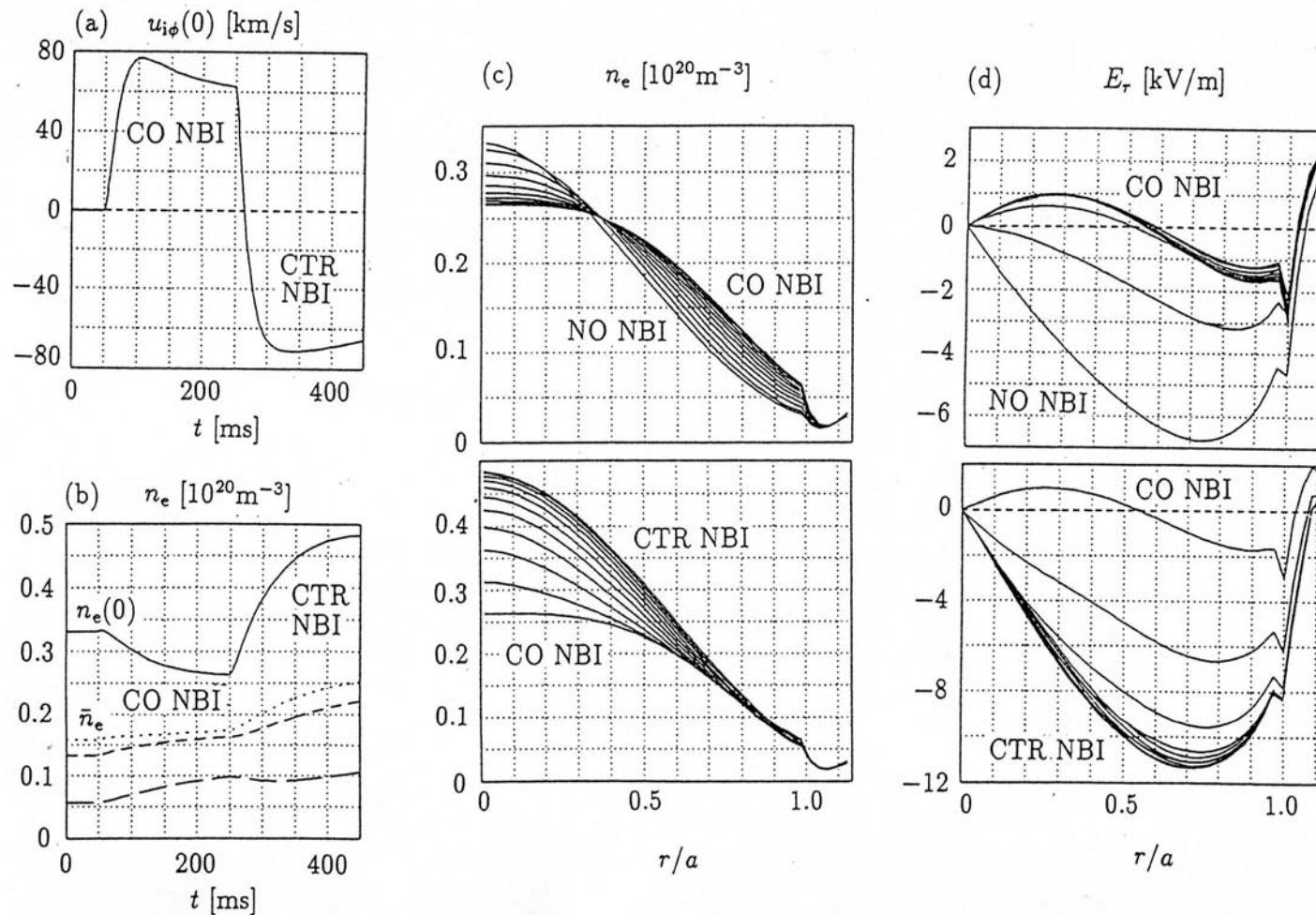
- **Neutral source**

$$S_0 = \frac{\gamma_0}{Z_i} \nu_L (n_e - n_{e,\text{div}}) - \frac{1}{Z_i} n_0 \langle \sigma_{\text{ion}} v \rangle n_e + \frac{P_b}{E_b}$$

- **Gas puff from wall**

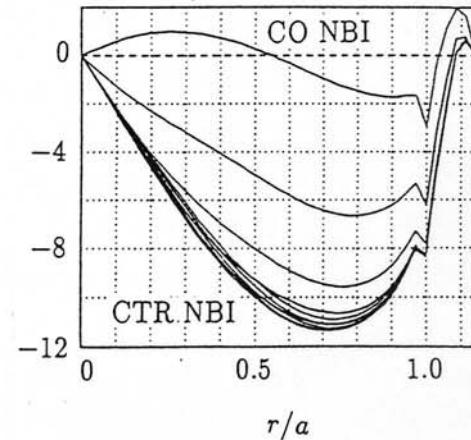
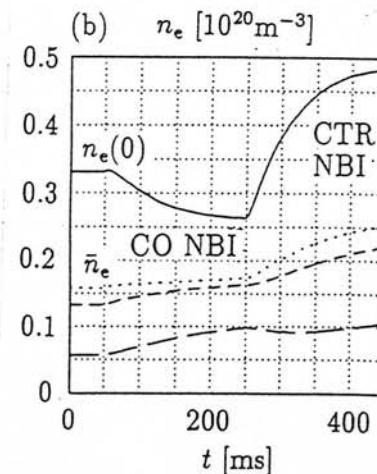
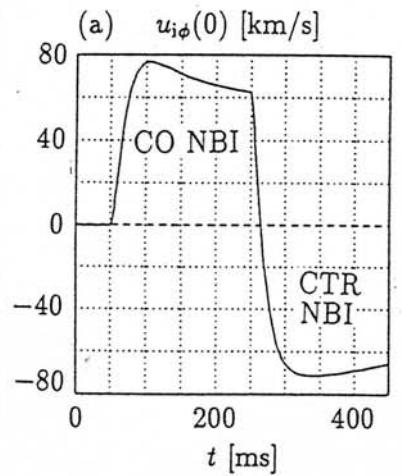
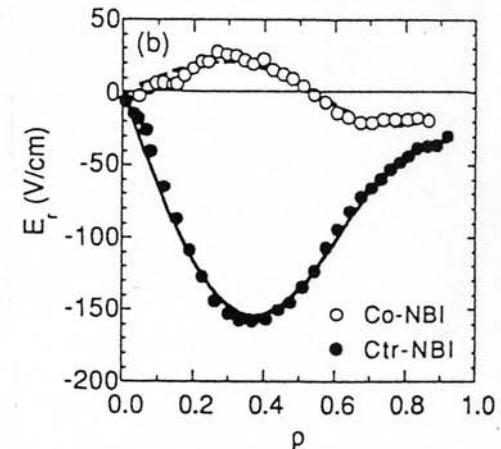
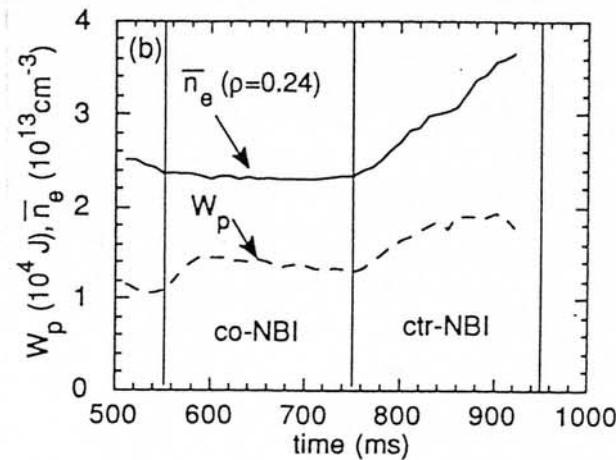
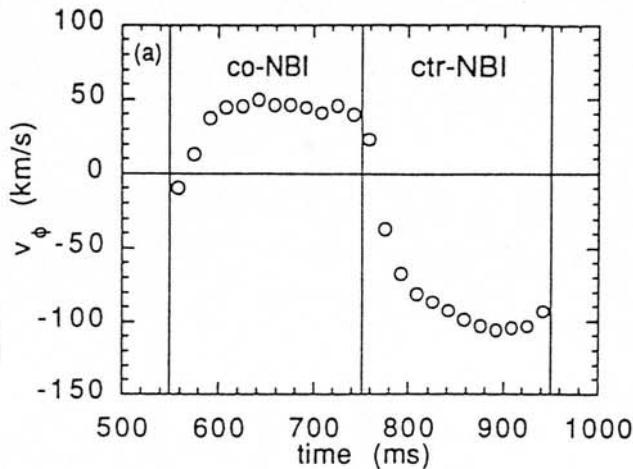
Simulation of plasma rotation and radial electric field

- **JFT-2M parameter:** NBI co-injection \rightarrow counter-injection
- Toroidal rotation \implies Negative E_r \implies Density peaking
- **TASK/TX:** Particle Diffusivity: $0.3 \text{ m}^2/\text{s}$, Ion viscosity: $10 \text{ m}^2/\text{s}$



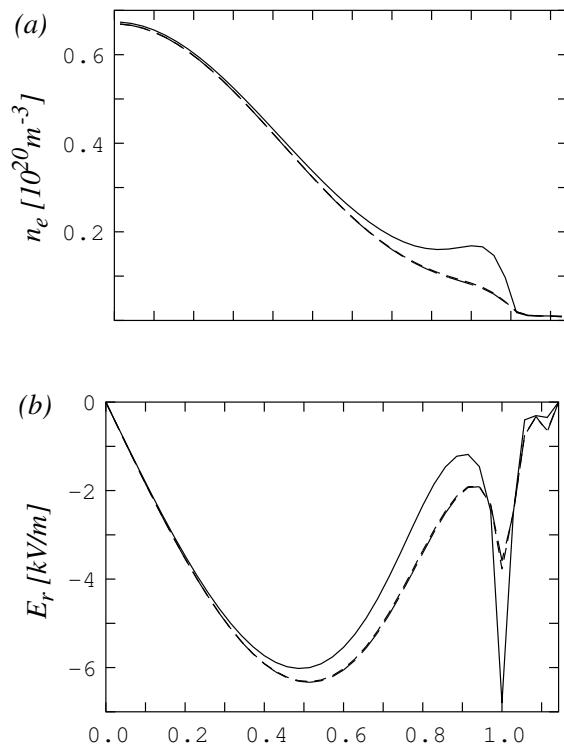
Comparison with JFT-2M Experiment

- **JFT-2M Experiment:** Ida et al.: Phys. Rev. Lett. 68 (1992) 182
- Good agreement with experimental observation

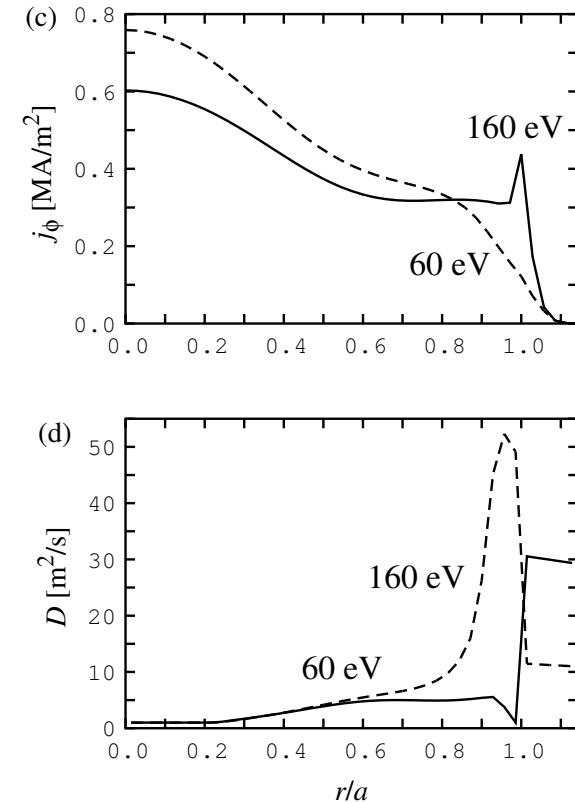
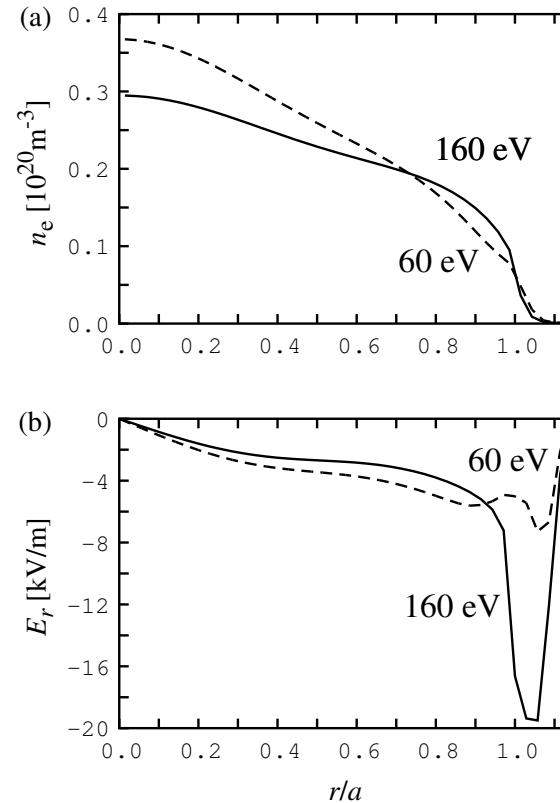


Typical Profiles

$D_{\text{TB}} = 0$



Edge Temperature Dependence



Summary

- We are developing TASK code as a reference core code for burning plasma simulation based on transport analysis.
- The TASK code is composed of modules: equilibrium, transport, wave analysis, velocity space analysis, and data interface.
- Analysis of ECCD by beam tracing, Alfvén eigenmode analysis by full wave code, transport analysis of ITB formation and radial electric field formation are shown as examples of TASK code results.
- To Do
 - Standard data interface with other simulation code
 - Open source
 - Comparison with experimental data
 - Improvement of modules and development of new modules