Analysis of Ballooning-Type Transport Model and Its Application to the Transport Barrier Formation

A. Fukuyama, M. Honda, Y. Izumi

Department of Nuclear Engineering, Kyoto University, Kyoto 606-8501, Japan

In order to realize high performance operation of fusion reactor, development of a reliable and robust transport model for burning plasmas is one of the key issues in fusion research. Transport models based on the ion temperature gradient (ITG) mode turbulence have recently attracted most attentions. We have proposed the current diffusive ballooning mode (CDBM) model [1], and successfully reproduced the L-mode confinement time scaling and the formation of internal transport barrier (ITB) [2,3]. Though these microscopic modes, such as ITG and CDBM, have been studied independently, it is desirable to give a unified view and clarifies the applicable range of these models.

Basic Equations: Starting from the full set of two-fluid equations for electrons and ions, we have derived a reduced set of two-fluid equations, which is composed of six equations, the equation of vorticity, parallel component of the equations of motion for electrons and ions, equations of state for electrons and ions, and Ampere's law [4]:

$$\left[\frac{n_\mathrm{i}}{\Omega_\mathrm{i}B_0}-\frac{n_\mathrm{e}}{\Omega_\mathrm{e}B}-\frac{\epsilon_0}{e}\right]\frac{\partial\nabla_\perp^2\phi_1}{\partial t}+\frac{n_\mathrm{i}}{\Omega_\mathrm{i}B}\frac{\partial}{\partial t}\left(\frac{\nabla_\perp^2 p_\mathrm{i1}}{q_\mathrm{i}n_\mathrm{i}}\right)-\frac{n_\mathrm{e}}{\Omega_\mathrm{e}B}\frac{\partial}{\partial t}\left(\frac{\nabla_\perp^2 p_\mathrm{e1}}{q_\mathrm{e}n_\mathrm{e}}\right)$$
$$-\nabla_\parallel(n_\mathrm{i}v_{\parallel\mathrm{i}1}-n_\mathrm{e}v_{\parallel\mathrm{e}1})=\frac{1}{eB}\left(\boldsymbol{b}\times\boldsymbol{\kappa}+\boldsymbol{b}\times\frac{\nabla B}{B}\right)\cdot(\nabla p_\mathrm{i1}+\nabla p_\mathrm{e1}) \tag{1}$$

$$m_j n_{j0}\frac{\partial v_{j\parallel 1}}{\partial t}+\nabla_\parallel p_{j1}+q_j n_{j0}\left(\nabla_\parallel\phi_1+\frac{\partial A_{\parallel 1}}{\partial t}+i\omega_{*pj}A_{\parallel 1}\right)=0 \tag{2}$$

$$\frac{\partial p_{j1}}{\partial t}-iq_j n_{j0}\omega_{*pj}\phi_1+\Gamma_j p_{j0}\nabla_\parallel v_{\parallel j1}=0 \tag{3}$$

$$\nabla_\perp^2 A_{\parallel 1}=-\mu_0\sum_j(n_0 q_j v_{j\parallel 1}) \tag{4}$$

where Ω_j is the cyclotron frequency, Γ_j is the specific heat, $\boldsymbol{\kappa}$ is the magnetic curvature, $j=\mathrm{e}$ or i denotes electron and ion, and other notations are standard. The equation of vorticity was obtained by combining the continuity equation, perpendicular component of the equations of motion, and the Poisson equation with the assumption of $\omega \ll \Omega_\mathrm{i}$.

In a toroidal configuration, we obtain the following set of equations after the transformation to the ballooning variable ξ and the renormalization of the $E\times B$ nonlinearities [5], e.g. $\mathrm{d}X_j/\mathrm{d}t \to -\mathrm{i}\omega X_j-\chi_j\nabla_\perp^2 X_j$, where χ_j is a turbulent transport coefficient proportional to $|\phi_1|^2$, which may destabilize or stabilize the modes

$$\left(-\mathrm{i}\omega+\mu_{\mathrm{i}\perp}\frac{m^2f^2}{r^2}\right)\frac{n_{0\mathrm{i}}}{\Omega_\mathrm{i}B_0}\frac{m^2f^2}{r^2}\phi_1+\left(-\mathrm{i}\omega+\chi_\mathrm{i}\frac{m^2f^2}{r^2}\right)\frac{1}{\Omega_\mathrm{i}B_0}\frac{m^2f^2}{r^2}\frac{p_\mathrm{i1}}{q_\mathrm{i}}$$
$$+\frac{B_\theta}{rB_0}\frac{\partial}{\partial\xi}(n_\mathrm{i0}v_{\mathrm{i}\parallel 1}-n_\mathrm{e0}v_{\mathrm{e}\parallel 0})-\frac{\mathrm{i}mB_\varphi}{erR_0B_0^2}H(\xi)(p_\mathrm{i1}+p_\mathrm{e1})=0 \tag{5}$$

$$m_j n_{j0}\left(-\mathrm{i}\omega+\mu_{j\|}\frac{m^2f^2}{r^2}\right)v_{j\|1}+\frac{B_\theta}{rB_0}\frac{\partial p_{j1}}{\partial\xi}+q_j n_{j0}\left(\frac{B_\theta\Lambda_{j0}}{rB_0}\frac{\partial\phi_1}{\partial\xi}-\mathrm{i}\omega_{*aj}A_{\|1}\right)=0 \tag{6}$$

$$\left(-\mathrm{i}\omega+\chi_j\frac{m^2f^2}{r^2}\right)p_{j1}-\mathrm{i}q_j n_{j0}\omega_{*j}(1+\eta_j)\phi_1+\frac{\Gamma_j p_{j0}B_\theta}{rB_0}\frac{\partial v_{j\|1}}{\partial\xi}=0 \tag{7}$$

$$-\frac{m^2}{r^2}f^2A_{\|1}=-\mu_0 e(n_{\mathrm{i}0}v_{\mathrm{i}\|1}-n_{\mathrm{e}}v_{\mathrm{e}\|1}) \tag{8}$$

where $H(\xi)\equiv\kappa_0+\cos\xi+(s\xi-\alpha\sin\xi)\sin\xi$ and $f^2(\xi)=1+(s\xi-\alpha\sin\xi)^2$

Two Field Model: By eliminating v_{j1} and p_{j1} from eqs. (5) to (8) and assuming $k_\|^2v_{\mathrm{Te}}^2/\omega^2\gg 1$ and $k_\| v_{\mathrm{Ti}}^2/\omega^2\ll 1$, we obtain a set of equations for two fields, ϕ_1 and ψ_1;

$$\left(1+iM_{\mathrm{i}\perp}-\frac{\omega_{*\mathrm{pi}}}{\omega}\right)\frac{m^2f^2}{r^2}\rho_{\mathrm{i}}^2\,\phi_1+\frac{m_{\mathrm{i}}}{m_{\mathrm{e}}}\frac{v_{\mathrm{Ti}}^2}{\omega^2}\frac{m^2f^2}{r^2}\frac{c^2}{\omega_{\mathrm{pe}}^2}\nabla_\|^2\,\psi_1$$
$$+\frac{m}{r}\frac{v_{\mathrm{Ti}}\rho}{\omega R}\left(1-\frac{1}{1+iX_{\mathrm{i}}}\frac{\omega_{*\mathrm{pi}}}{\omega}\right)H(\xi)\,\phi_1-\frac{m}{r}\frac{v_{\mathrm{Ti}}\rho}{\omega R}\left(1-\frac{\omega_{*\mathrm{pe}}}{\omega}\right)H(\xi)\,\psi_1=0 \tag{9}$$

$$\left[\frac{m^2f^2}{r^2}\frac{c^2}{\omega_{\mathrm{pe}}^2}\Pi_{\mathrm{e}}\nabla_\|^2+\frac{m_{\mathrm{e}}/m_{\mathrm{i}}}{1+iM_{\mathrm{i}\|}}\left(1-\frac{\omega_{*\mathrm{pi}}}{\omega}\right)\Pi_{\mathrm{e}}\nabla_\|^2+\frac{1+iX_{\mathrm{e}}}{1+iM_{\mathrm{e}\|}}\left(1-\frac{\omega_{*\mathrm{pi}}}{\omega}\right)\right]\psi_1$$
$$=\left[\frac{m_{\mathrm{e}}/m_{\mathrm{i}}}{1+iM_{\mathrm{i}\|}}\frac{1}{1+iX_{\mathrm{i}}}\left(1+iX_{\mathrm{i}}-\frac{\omega_{*\mathrm{pi}}}{\omega}\right)\Pi_{\mathrm{e}}\nabla_\|^2+\frac{1}{1+iM_{\mathrm{e}\|}}\left(1+iX_{\mathrm{e}}-\frac{\omega_{*\mathrm{pe}}}{\omega}\right)\right]\phi_1 \tag{10}$$

where the normalized transport coefficients are defined by

$$M_{j\|}=\frac{\mu_{j\|}}{\omega}\frac{m^2f^2}{r^2},\quad M_{j\perp}=\frac{\mu_{j\perp}}{\omega}\frac{m^2f^2}{r^2},\quad X_j=\frac{\chi_j}{\omega}\frac{m^2f^2}{r^2},\quad \Pi_j=\frac{\Gamma_j v_{\mathrm{T}j}^2}{\omega^2}\frac{1}{1+iM_{j\|}},$$

and the parallel derivative $\nabla_\|=(B_\theta/rB_0)(\mathrm{d}/\mathrm{d}\xi)$. This set of equations describes both the toroidal ITG mode and the ballooning mode.

In the electrostatic limit where $k_\perp^2c^2/\omega^2\to\infty$ and $\psi_1\to 0$ while $c^2\psi_1$ is finite, eqs. (9) and (10) are combined to yield

$$\frac{v_{\mathrm{Ti}}^2}{\omega^2}\frac{1}{1+iM_{\mathrm{i}\|}}\left(1-\frac{1}{1+iX_{\mathrm{i}}}\frac{\omega_{*\mathrm{pi}}}{\omega}\right)\nabla_\|^2\phi_1+\left(1+iM_{\mathrm{i}\perp}-\frac{\omega_{*\mathrm{pi}}}{\omega}\right)\frac{m^2f^2}{r^2}\rho_{\mathrm{i}}^2\,\phi_1$$
$$+\frac{T_{\mathrm{i}0}}{\Gamma_{\mathrm{e}}T_{\mathrm{e}0}}\left(1+iX_{\mathrm{e}}-\frac{\omega_{*\mathrm{pe}}}{\omega}\right)\phi_1+\frac{m}{r}\frac{v_{\mathrm{Ti}}\rho}{\omega R}\left(1-\frac{1}{1+iX_{\mathrm{i}}}\frac{\omega_{*\mathrm{pi}}}{\omega}\right)H(\xi)\,\phi_1=0. \tag{11}$$

From this ordinary differential equation, we found that the ion parallel viscosity has a destabilizing effect by reducing the field bending term [4]. Without nonlinear transport terms (M_j and X_j), eq.(11) is reduced to the equation describing the toroidal ITG mode:

$$\left[\frac{v_{\mathrm{Ti}}^2}{\omega^2}\left(1-\frac{\omega_{*\mathrm{pi}}}{\omega}\right)\nabla_\|^2+\left(1-\frac{\omega_{*\mathrm{pi}}}{\omega}\right)\frac{m^2f^2}{r^2}\rho_{\mathrm{i}}^2\right.$$
$$\left.+\frac{T_{\mathrm{i}0}}{\Gamma_{\mathrm{e}}T_{\mathrm{e}0}}\left(1-\frac{\omega_{*\mathrm{pe}}}{\omega}\right)+\frac{m}{r}\frac{v_{\mathrm{Ti}}\rho}{\omega R}\left(1-\frac{\omega_{*\mathrm{pi}}}{\omega}\right)H(\xi)\right]\phi_1=0 \tag{12}$$

In the other limit $k_\perp^2 c^2/\omega^2 \ll 1$, we obtain $\phi_1 \sim \psi_1$ and

$$\frac{m_\mathrm{i}}{m_\mathrm{e}}\frac{v_\mathrm{Ti}^2}{\omega^2}\frac{m^2 f^2}{r^2}\frac{c^2}{\omega_\mathrm{pe}^2}\nabla_\parallel^2\phi_1 + \left(1 + iM_{\mathrm{i}\perp} - \frac{\omega_{*\mathrm{pi}}}{\omega}\right)\frac{m^2 f^2}{r^2}\rho_\mathrm{i}^2\,\phi_1 + \frac{m}{r}\frac{v_\mathrm{Ti}\rho}{\omega R}\left(\frac{\omega_{*\mathrm{pe}}}{\omega} - \frac{1}{1+iX_\mathrm{i}}\frac{\omega_{*\mathrm{pi}}}{\omega}\right) H(\xi)\,\phi_1 = 0 \quad (13)$$

In the presence of electron compressibility, the current diffusivity (electron parallel viscosity) does not contribute to destabilization. In the linear case without nonlinear transport terms, eq.(13) is reduced to the kinetic ballooning mode equation:

$$\left[\frac{m_\mathrm{i}}{m_\mathrm{e}}\frac{v_\mathrm{Ti}^2}{\omega^2}\frac{m^2 f^2}{r^2}\frac{c^2}{\omega_\mathrm{pe}^2}\nabla_\parallel^2 + \left(1 - \frac{\omega_{*\mathrm{pi}}}{\omega}\right)\frac{m^2 f^2}{r^2}\rho_\mathrm{i}^2 + \frac{m}{r}\frac{v_\mathrm{Ti}\rho}{\omega R}\left(\frac{\omega_{*\mathrm{pe}}}{\omega} - \frac{\omega_{*\mathrm{pi}}}{\omega}\right) H(\xi)\right]\phi_1 = 0. \quad (14)$$

Analytic Formula of Transport Coefficients: In the strong ballooning limit where the eigenmode localizes around $\xi \sim 0$, eqs. (11) and (13) are approximated by the Weber equation. From the eigenvalue condition, we can derive the dispersion relation of the mode. In order to obtain an analytic expression of the transport coefficients, we consider the simplest case, an electrostatic low-frequency limit. With the low frequency marginal stability condition, $\omega = i\gamma = 0$, eq.(11) is reduced to

$$\frac{1}{\mu_{\mathrm{i}\parallel}}\frac{\mathrm{d}^2\phi_1}{\mathrm{d}\xi^2} - \frac{\chi_\mathrm{e} T_\mathrm{i0}}{\Gamma_\mathrm{e} T_\mathrm{e0}}\frac{q^2 R^2}{v_\mathrm{Ti}^2}\frac{m^4 f^4(\xi)}{r^4}\phi_1 + \frac{\alpha_\mathrm{i}}{\chi_\mathrm{i}}\frac{c^2}{\omega_\mathrm{pi}^2}\frac{m^2}{r^2}H(\xi)\,\phi_1 = 0 \quad (15)$$

In the strong ballooning limit ($\xi \ll 1$), the eigenvalue problem of the Weber equation can be easily solved. The lowest eigenvalue condition leads to

$$\frac{\chi_\mathrm{i}^2\chi_\mathrm{e}}{\mu_{\mathrm{i}\parallel}} = \alpha_\mathrm{i}^2\frac{\Gamma_\mathrm{e} T_\mathrm{e0}}{T_\mathrm{i0}}\frac{v_\mathrm{Ti}^2}{q^2 R^2}\frac{c^4}{\omega_\mathrm{pi}^4}\frac{N^2(1+\kappa-N^2)^2}{(\alpha - s + 1/2) + 2N^2(\alpha - s)^2} \quad (16)$$

where the normalized wave number

$$N^2 \equiv \frac{\chi_\mathrm{i}\chi_\mathrm{e}}{\alpha_\mathrm{i}}\frac{T_\mathrm{i0}}{\Gamma_\mathrm{e} T_\mathrm{e0}}\frac{q^2 R^2}{v_\mathrm{Ti}^2 r^2}\frac{\omega_\mathrm{pi}^2}{c^2} m^2. \quad (17)$$

The RHS of eq.(16) has an approximate maximum around $N^2 \sim (1+\kappa)/2$. In order to stabilize all modes with respect to m, the transport coefficients ought to be

$$\chi_{TB} \sim \chi_\mathrm{e} \sim \chi_\mathrm{i} \sim \mu_{\mathrm{i}\parallel} = \alpha_\mathrm{i}\sqrt{\frac{\Gamma_\mathrm{e} T_\mathrm{e}}{T_\mathrm{i}}}\frac{c^2}{\omega_\mathrm{pi}^2}\frac{v_\mathrm{Ti}}{qR}\,g(s,\alpha,\kappa) \quad (18)$$

where

$$g(s,\alpha,\kappa) \sim \sqrt{\frac{(1+\kappa)^3}{8[\alpha - s + 1/2 + (1+\kappa)(\alpha - s)^2]}}$$

This model is slightly different from the CDBM model; (1) dependence on pressure gradient: $\chi_\mathrm{TB} \propto \alpha_\mathrm{i}$, while $\chi_\mathrm{CDBM} \propto (\alpha_\mathrm{e} + \alpha_\mathrm{i})^{3/2}$, (2) dependence on plasma parameters: $\chi_\mathrm{TB}/\chi_\mathrm{CDBM} \propto (v_\mathrm{Ti}/v_\mathrm{A})(m_\mathrm{i}/m_\mathrm{e})$, (3) $s-\alpha$ dependence: $f(s,\alpha,\kappa)$ vs. $g(s,\alpha,\kappa)$.

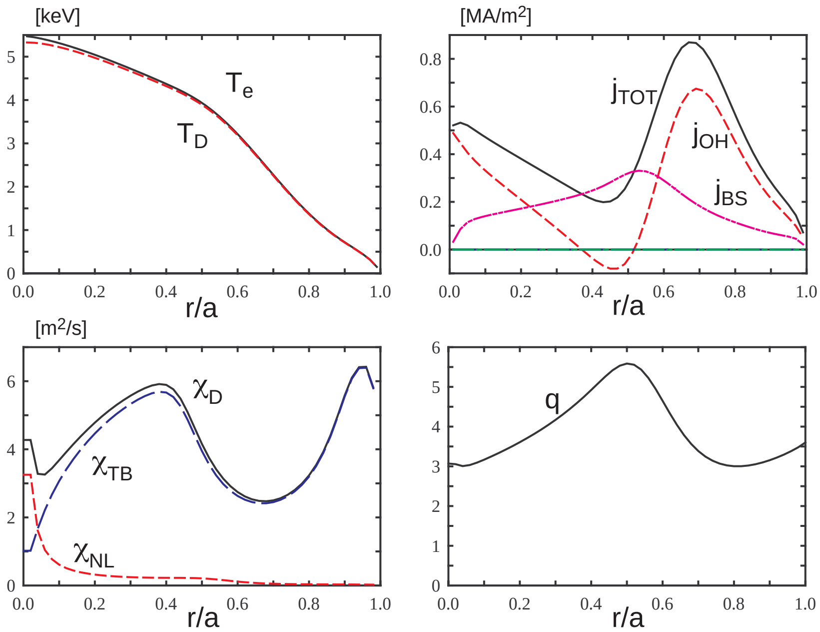

Figure 1 Typical radial profiles obtained by the transport simulation using eq.(18) in a reversed magnetic shear configuration.

Preliminary Transport Simulation of ITB Formation: Preliminary results of transport simulations by TASK/TR using eq.(18) are shown in Fig.1. Device Parameters are $R_0 = 3\,\mathrm{m}$, $a = 1.2\,\mathrm{m}$, $\kappa = 1.5$, $B_0 = 3\,\mathrm{T}$. We assume the thermal diffusivity of the form, $\chi_i = \chi_e = \chi_{NC} + 0.3\chi_{TB}$ and the adjustable parameter 0.3 was chosen so that the L-mode energy confinement time scaling law be reproduced for $I_p = 3\,\mathrm{MA}$ and the heating power $P_H = 30\,\mathrm{MW}$ ($P_{He} = P_{Hi}$). Figure 1 depicts typical radial profiles for the reversed magnetic shear configuration with current ramp up ($1 \rightarrow 3\,\mathrm{MA}$ in $1\,\mathrm{s}$) and $P_H = 30\,\mathrm{MW}$. Relatively weak ITB is formed between $r/a = 0.5$ and 0.8 where $s < 0$, and ITB formation requires more heating power than the case with CDBM model.

Summary: We have introduced a two-field model to describe both the toroidal ITG mode and the kinetic ballooning mode. Based on the theory of self-sustained turbulence [5], we obtained an approximate formula of the transport coefficients from the marginal stability condition of the electrostatic mode. This mode is destabilized by the ion parallel viscosity and stabilized by the electron thermal diffusivity. Preliminary result of transport simulation suggests that the internal transport barrier formation is more difficult for this model than the previous CDBM model. Improvement of the model including the finite mode frequency and the electromagnetic case is underway.

[1] K. Itoh et al.: Plasma Phys. Control. Fusion, **35** (1993) 543.
[2] A. Fukuyama et al.: Plasma Phys. Control. Fusion, **37** (1995) 611.
[3] A. Fukuyama et al.: Nucl. Fusion, **35** (1995) 1669.
[4] M. Uchida and A. Fukuyama: Proc. 30th EPS Conf. on Cotnrol. Fusion and Plasma Phys. (St. Petersburg, 2003) P-2.118
[5] K. Itoh et al.: Plasma Phys. Control. Fusion, **36** (1994) 279.