2004-06-29
31st EPS PP P-2.131
London, UK

Analysis of Ballooning-Type Transport Model and Its Application to the Transport Barrier Formation

A. Fukuyama, M. Honda, Y. Izumi
Department of Nuclear Engineering,
Kyoto University, Kyoto 606-8501, Japan

Contents

Motivation

- **Development of Robust Transport Model**
  - **L-mode confinement time scaling**
  - **Large transport near the plasma edge in L-mode**
  - **H-mode confinement time scaling for given edge temperature**
  - **Formation of internal transport barrier**
  - **Profile database (ITPA)**
  - **Behavior of fluctuation**

- **Purpose of the present model**
  - **To describe both**
    - **Electrostatic ITG mode**
      - · enhanced transport for large ion temperature gradient
    - **Electromagnetic Ballooning mode (e.g. CDBM)**
      - · transport reduction for negative $s - \alpha$: ITB formation

magnetic shear: $s = \frac{r}{q}\frac{\mathrm{d}q}{\mathrm{d}r}$

pressure gradient (Shafranov shift): $\alpha = -q^2 R \frac{\mathrm{d}\beta}{\mathrm{d}r}$

Turbulent Transport Model

CDBM
Reduced MHD equation
Electromagnetic
Incompressible

Toroidal ITG
Ion Fluid Equation
Electrostatic
Boltzmann Distribution of Electron

Hybrid Ballooning Mode
Reduced Two-Fluid Equation
Electromagnetic
Compressible

- **Small pressure gradient: Electrostatic ITG**
- **Large pressure gradient: Electromagnetic KBM**
- **Without drift motion and compressibility, it reduces to CDBM**
- **Ion parallel viscosity destabilizes the mode**
- $s-\alpha$ **dependence similar to the CDBM mode**

Reduced Two-Fluid Equation (Slab Plasma)

- **Equation of Vorticity**: (Poisson, continuity, perpendicular eq of motion)

$$\left[\frac{n_\mathrm{i}}{\Omega_\mathrm{i}B_0}-\frac{n_\mathrm{e}}{\Omega_\mathrm{e}B}-\frac{\epsilon_0}{e}\right]\frac{\partial\nabla_\perp^2\phi_1}{\partial t}+\frac{n_\mathrm{i}}{\Omega_\mathrm{i}B}\frac{\partial}{\partial t}\left(\frac{\nabla_\perp^2 p_{\mathrm{i}1}}{q_\mathrm{i}n_\mathrm{i}}\right)-\frac{n_\mathrm{e}}{\Omega_\mathrm{e}B}\frac{\partial}{\partial t}\left(\frac{\nabla_\perp^2 p_{\mathrm{e}1}}{q_\mathrm{e}n_\mathrm{e}}\right)$$

$$-\nabla_\parallel(n_\mathrm{i}v_{\parallel\mathrm{i}1}-n_\mathrm{e}v_{\parallel\mathrm{e}1})=\frac{1}{eB}\left(\boldsymbol{b}\times\boldsymbol{\kappa}+\boldsymbol{b}\times\frac{\nabla B}{B}\right)\cdot(\nabla p_{\mathrm{i}1}+\nabla p_{\mathrm{e}1})$$

- **Parallel Equation of Motion** $(j=\mathrm{e},\mathrm{i})$

$$m_j n_{j0}\frac{\partial v_{j\parallel 1}}{\partial t}+\nabla_\parallel p_{j1}+q_j n_{j0}\left(\nabla_\parallel\phi_1+\frac{\partial A_{\parallel 1}}{\partial t}+i\omega_{*pj}A_{\parallel 1}\right)=0$$

- **Equation of State** $(j=\mathrm{e},\mathrm{i})$

$$\frac{\partial p_{j1}}{\partial t}-iq_j n_{j0}\omega_{*pj}\phi_1+\Gamma_j p_{j0}\nabla_\parallel v_{\parallel j1}=0$$

- **Ampere's Law**

$$\nabla_\perp^2 A_{\parallel 1}=-\mu_0\sum_j(n_0 q_j v_{j\parallel 1})$$

Reduced Two-Fluid Equation (Toroidal Plasma)

- **Ballooning transformation**: ξ, $\quad \nabla_{\|} \to (B_\theta/rB_0)(\partial/\partial\xi)$, $\nabla_\perp^2 \to (m^2 f^2)/r^2$
- **Equation of Vorticity**

$$\frac{-\mathrm{i}\omega}{\Omega_\mathrm{i} B_0}\frac{m^2}{r^2} f^2 \left(n_{\mathrm{i}0}\phi_1 + \frac{p_{\mathrm{i}1}}{q_\mathrm{i}} \right) + \frac{B_\theta}{rB_0}\frac{\partial}{\partial\xi}(n_{\mathrm{i}0}v_{\mathrm{i}\|1} - n_{\mathrm{e}0}v_{\mathrm{e}\|1}) - \frac{\mathrm{i}mB_\varphi}{erR_0B_0^2} H(\xi)\,(p_{\mathrm{i}1} + p_{\mathrm{e}1}) = 0$$

- **Parallel Equation of Motion** ($j = \mathrm{e}, \mathrm{i}$)

$$-\mathrm{i}\omega m_j n_{j0} v_{j\|1} + \frac{B_\theta}{rB_0}\frac{\partial p_{j1}}{\partial\xi} + q_j n_{j0}\left(\frac{B_\theta}{rB_0}\frac{\partial\phi_1}{\partial\xi} - \mathrm{i}(\omega - \omega_{*pj})A_{\|1} \right) = 0$$

- **Equation of State** ($j = \mathrm{e}, \mathrm{i}$)

$$-\mathrm{i}\omega p_{j1} - \mathrm{i}q_j n_{j0}\omega_{*pj}\phi_1 + \Gamma_j p_{j0}\frac{B_\theta}{rB_0}\frac{\partial v_{j\|1}}{\partial\xi} = 0$$

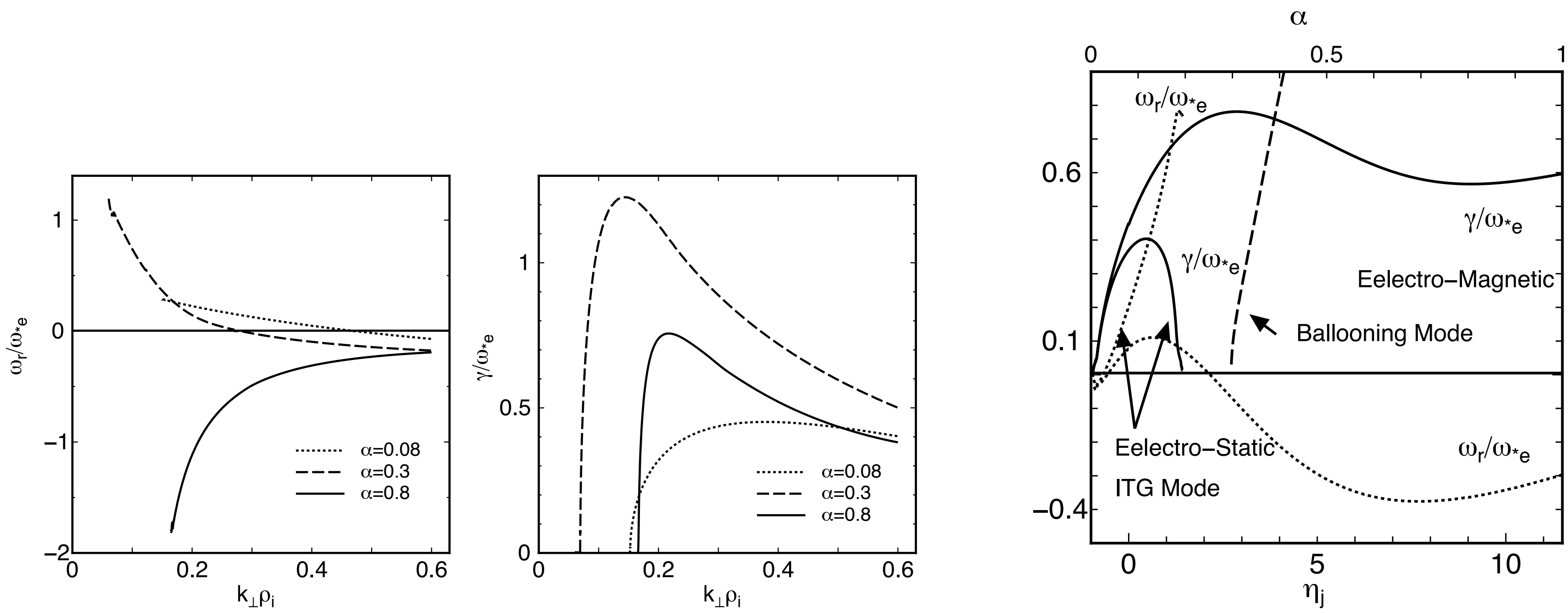
- **Ampere's law**

$$-\frac{m^2}{r^2} f^2 A_{\|1} = -\mu_0 e(n_{\mathrm{i}0}v_{\mathrm{i}\|1} - n_{\mathrm{e}0}v_{\mathrm{e}\|1})$$

- where $H(\xi) \equiv \kappa_0 + \cos\xi + (s\xi - \alpha\sin\xi)\sin\xi$, $f^2(\xi) = 1 + (s\xi - \alpha\sin\xi)^2$

Linear Analysis: Toroidal Plasma

- **Ballooning mode** (Transition from electrostatic to electromagnetic)
  - **Electrostatic Toroidal ITG Mode**
  - **Electromagnetic Ballooning Mode**



- **Electromagnetic effect becomes dominant for large η_i or α**

Nonlinear Reduced Two-Fluid Equation (Toroidal Plasma)

- **Turbulent transport coefficients are included.**: $E \times B$ nonlinearity

$$\frac{\mathrm{d}}{\mathrm{d}t} X_j \to -\mathrm{i}\omega X_j - \chi_j \nabla_\perp^2 X_j, \qquad \chi_j = \frac{\left\langle \phi_1^2 \right\rangle}{\gamma_{nj}}$$

- **Equation of Vorticity**:

$$\left(-\mathrm{i}\omega + \mu_{\mathrm{i}\perp}\frac{m^2 f^2}{r^2}\right)\frac{n_{\mathrm{i}0}}{\Omega_\mathrm{i} B_0}\frac{m^2 f^2}{r^2}\phi_1 + \left(-\mathrm{i}\omega + \chi_\mathrm{i}\frac{m^2 f^2}{r^2}\right)\frac{1}{\Omega_\mathrm{i} B_0}\frac{m^2 f^2}{r^2}\frac{p_{\mathrm{i}1}}{q_\mathrm{i}}$$
$$+\frac{B_\theta}{rB_0}\frac{\partial}{\partial \xi}(n_{\mathrm{i}0}v_{\mathrm{i}\|1} - n_{\mathrm{e}0}v_{\mathrm{e}\|0}) - \frac{\mathrm{i}mB_\varphi}{erR_0 B_0^2} H(\xi)(p_{\mathrm{i}1} + p_{\mathrm{e}1}) = 0$$

- **Parallel Equation of Motion**

$$m_j n_{j0}\left(-\mathrm{i}\omega + \mu_{j\|}\frac{m^2 f^2}{r^2}\right)v_{j\|1} + \frac{B_\theta}{rB_0}\frac{\partial p_{j1}}{\partial \xi} + q_j n_{j0}\left(\frac{B_\theta \Lambda_{j0}}{rB_0}\frac{\partial \phi_1}{\partial \xi} - \mathrm{i}\omega_{*aj}A_{\|1}\right) = 0$$

- **Equation of State**

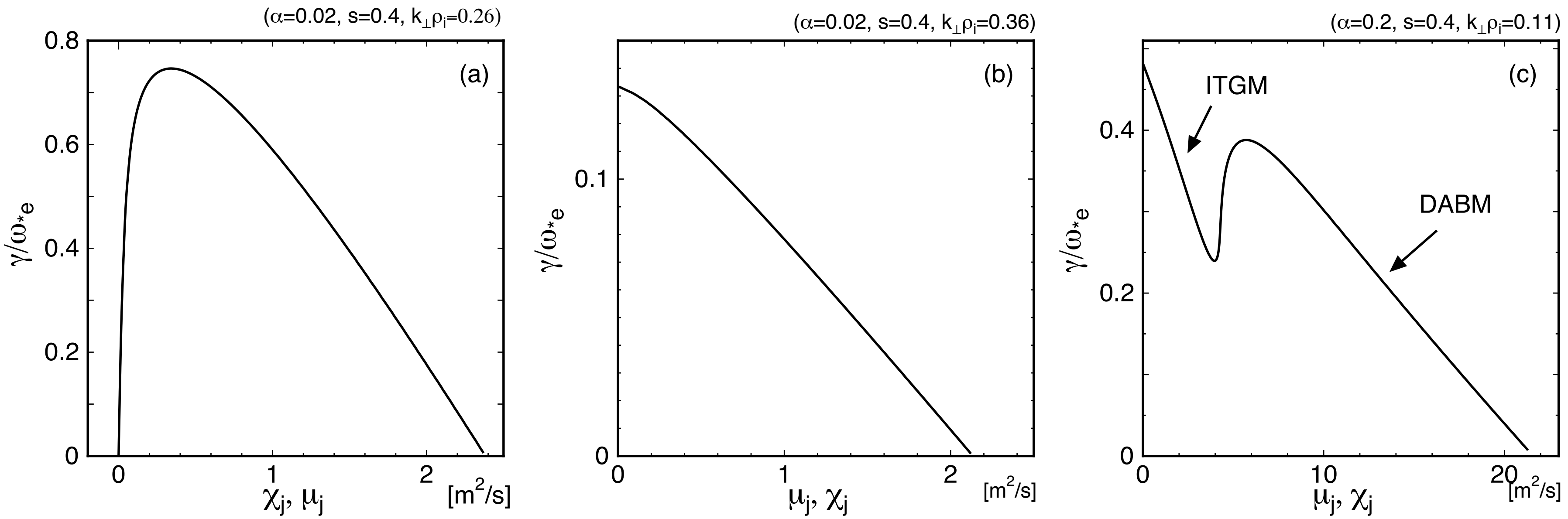
$$\left(-\mathrm{i}\omega + \chi_j\frac{m^2 f^2}{r^2}\right)p_{j1} - \mathrm{i}q_j n_{j0}\omega_{*j}(1+\eta_j)\phi_1 + \frac{\Gamma_j p_{j0} B_\theta}{rB_0}\frac{\partial v_{j\|1}}{\partial \xi} = 0$$

- **Ampere's Law**

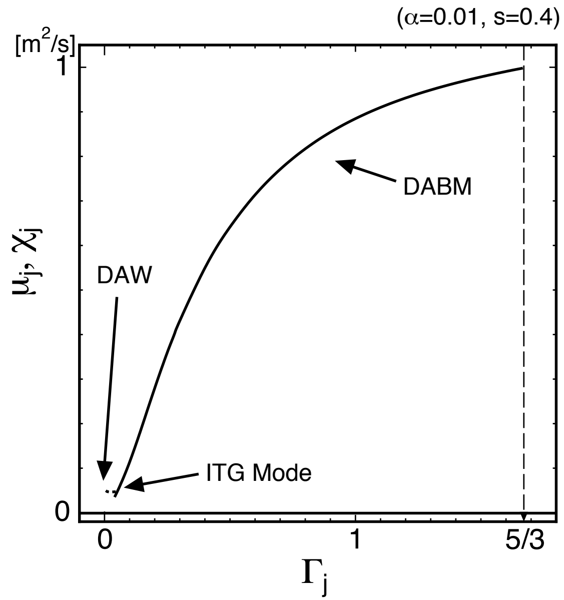
$$-\frac{m^2}{r^2} f^2 A_{||1} = -\mu_0 e(n_{i0} v_{i||1} - n_e v_{e||1})$$

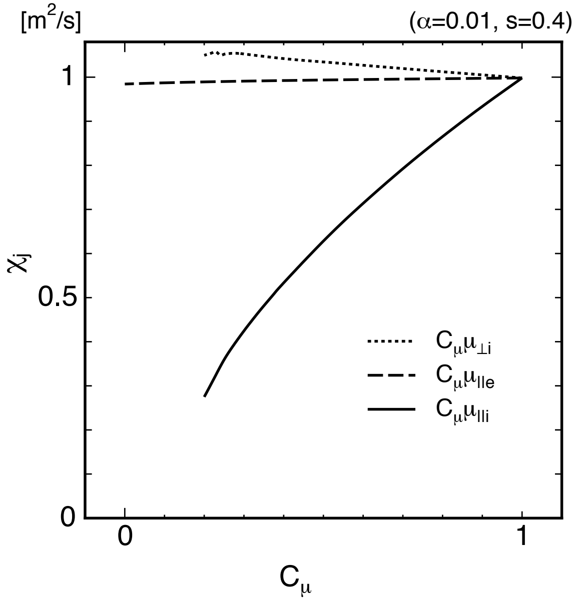
Nonlinear Analysis (Toroidal Plasma)

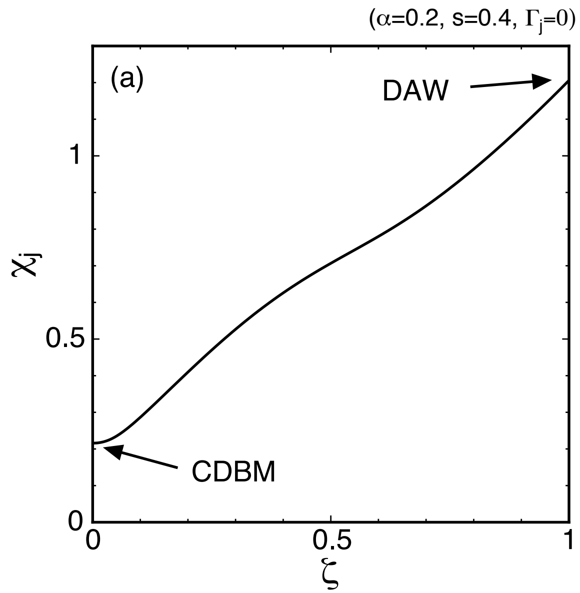
- **Amplitude Dependence of the Growth Rate**
  - **Linear growth rate ($\chi_j = 0$) is sensitive to $k_\perp \rho_\mathrm{i}$.**
  - **For large α, electromagnetic effect becomes important.**
  - **Saturation level can be estimated from the marginal condition.**



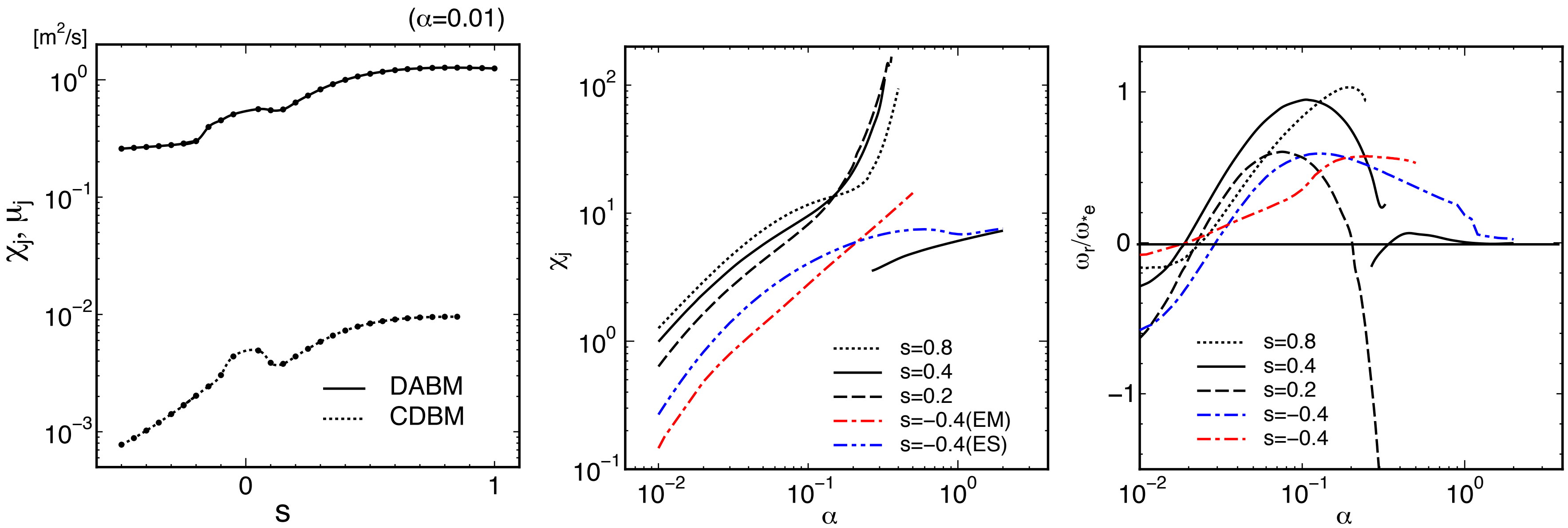
- **Sensitivity on Various Parameters**
  - **Compressibility enhances the transport. (Γ_j: Adiabatic index)**
  - **Parallel viscosity of ion enhances the transport.**
  - **Finite drift frequency enhances the transport.**







- **Dependence on s and α**
  - **Transport due to Hybrid BM is much larger than that of CDBM.**
  - **Negative magnetic shear reduces the transport.**
  - **χ is proportional to $\alpha^{3/2}$ for small α.**
  - **There exists critical α above which transport is strongly enhanced.**



Reduction to Two Field Equation for Hybrid BM

- **Eliminate v_{j1} and p_{j1}, and assume $\Pi_e \nabla_\parallel^2 \gg 1$ and $\Pi_i \nabla_\parallel^2 \ll 1$**

$$\left(1 + iM_{i\perp} - \frac{\omega_{*pi}}{\omega}\right)\frac{m^2 f^2}{r^2}\rho_i^2\,\phi_1 + \frac{m_i}{m_e}\frac{v_{Ti}^2}{\omega^2}\frac{m^2 f^2}{r^2}\frac{c^2}{\omega_{pe}^2}\nabla_\parallel^2\,\psi_1$$

$$+\frac{m}{r}\frac{v_{Ti}\rho}{\omega R}\left(1 - \frac{1}{1+iX_i}\frac{\omega_{*pi}}{\omega}\right)H(\xi)\,\phi_1 - \frac{m}{r}\frac{v_{Ti}\rho}{\omega R}\left(1 - \frac{\omega_{*pe}}{\omega}\right)H(\xi)\,\psi_1 = 0$$

$$\left[\frac{m^2 f^2}{r^2}\frac{c^2}{\omega_{pe}^2}\Pi_e\nabla_\parallel^2 + \frac{m_e}{m_i}\frac{1}{1+iM_{i\parallel}}\left(1 - \frac{\omega_{*pi}}{\omega}\right)\Pi_e\nabla_\parallel^2 + \frac{1+iX_e}{1+iM_{e\parallel}}\left(1 - \frac{\omega_{*pi}}{\omega}\right)\right]\psi_1$$

$$= \left[\frac{m_e}{m_i}\frac{1}{1+iM_{i\parallel}}\frac{1}{1+iX_i}\left(1 + iX_i - \frac{\omega_{*pi}}{\omega}\right)\Pi_e\nabla_\parallel^2 + \frac{1}{1+iM_{e\parallel}}\left(1 + iX_e - \frac{\omega_{*pe}}{\omega}\right)\right]\phi_1$$

where

$$M_{j\parallel} = \frac{\mu_{j\parallel}}{\omega}\frac{m^2 f^2}{r^2}\Lambda_j, \quad M_{j\perp} = \frac{\mu_{j\perp}}{\omega}\frac{m^2 f^2}{r^2}\Lambda_j, \quad X_j = \frac{\chi_j}{\omega}\frac{m^2 f^2}{r^2}\Lambda_j,$$

$$\Pi_j = \frac{\Gamma_j v_{Tj}^2}{\omega^2}\frac{1}{1+iM_{j\parallel}}, \quad \Lambda_j = I_0(\lambda)e^{-\lambda_j}, \quad \lambda_j = \frac{m^2}{r^2}\rho_j^2, \quad \nabla_\parallel = \frac{B_\theta}{rB_0}\frac{\partial}{\partial\xi}$$

ES Limits

- **Electrostatic Limit**: Since $c^2 \to \infty$, $\psi_1 \to 0$

$$\frac{v_{\mathrm{Ti}}^2}{\omega^2}\frac{1}{1+iM_{\mathrm{i}\|}}\left(1-\frac{1}{1+iX_{\mathrm{i}}}\frac{\omega_{*\mathrm{pi}}}{\omega}\right)\nabla_\|^2\phi_1+\left(1+iM_{\mathrm{i}\perp}-\frac{\omega_{*\mathrm{pi}}}{\omega}\right)\frac{m^2 f^2}{r^2}\rho_{\mathrm{i}}^2\,\phi_1$$

$$+\frac{T_{\mathrm{i0}}}{\Gamma_{\mathrm{e}}T_{\mathrm{e0}}}\left(1+iX_{\mathrm{e}}-\frac{\omega_{*\mathrm{pe}}}{\omega}\right)\phi_1+\frac{m}{r}\frac{v_{\mathrm{Ti}}\rho}{\omega R}\left(1-\frac{1}{1+iX_{\mathrm{i}}}\frac{\omega_{*\mathrm{pi}}}{\omega}\right)H(\xi)\,\phi_1=0$$

- **Linear case: Toroidal ITG mode**

$$\frac{v_{\mathrm{Ti}}^2}{\omega^2}\left(1-\frac{\omega_{*\mathrm{pi}}}{\omega}\right)\nabla_\|^2\phi_1+\left(1-\frac{\omega_{*\mathrm{pi}}}{\omega}\right)\frac{m^2 f^2}{r^2}\rho_{\mathrm{i}}^2\,\phi_1$$

$$+\frac{T_{\mathrm{i0}}}{\Gamma_{\mathrm{e}}T_{\mathrm{e0}}}\left(1-\frac{\omega_{*\mathrm{pe}}}{\omega}\right)\phi_1+\frac{m}{r}\frac{v_{\mathrm{Ti}}\rho}{\omega R}\left(1-\frac{\omega_{*\mathrm{pi}}}{\omega}\right)H(\xi)\,\phi_1=0$$

- Ion parallel viscosity $M_{\mathrm{i}\|}$ destabilizes the toroidal ITG mode, while electron thermal diffusivity X_{e} stabilizes it.

EM Limits

- **Electromagnetic Limit**: $\psi_1 \sim \phi_1$

$$\frac{m_\mathrm{i}}{m_\mathrm{e}}\frac{v_\mathrm{Ti}^2}{\omega^2}\frac{m^2 f^2}{r^2}\frac{c^2}{\omega_\mathrm{pe}^2}\nabla_\parallel^2\phi_1 + \left(1 + iM_{\mathrm{i}\perp} - \frac{\omega_{*\mathrm{pi}}}{\omega}\right)\frac{m^2 f^2}{r^2}\rho_\mathrm{i}^2\phi_1$$
$$+\frac{m}{r}\frac{v_\mathrm{Ti}\rho}{\omega R}\left(\frac{\omega_{*\mathrm{pe}}}{\omega} - \frac{1}{1+iX_\mathrm{i}}\frac{\omega_{*\mathrm{pi}}}{\omega}\right)H(\xi)\,\phi_1 = 0$$

- **Linear case: Kinetic ballooning mode**

$$\frac{m_\mathrm{i}}{m_\mathrm{e}}\frac{v_\mathrm{Ti}^2}{\omega^2}\frac{m^2 f^2}{r^2}\frac{c^2}{\omega_\mathrm{pe}^2}\nabla_\parallel^2\phi_1 + \left(1 - \frac{\omega_{*\mathrm{pi}}}{\omega}\right)\frac{m^2 f^2}{r^2}\rho_\mathrm{i}^2\phi_1 + \frac{m}{r}\frac{v_\mathrm{Ti}\rho}{\omega R}\left(\frac{\omega_{*\mathrm{pe}}}{\omega} - \frac{\omega_{*\mathrm{pi}}}{\omega}\right)H(\xi)\,\phi_1 = 0$$

- Compressibility nullify the destabilization by current diffusivity (electron parallel viscosity) and resistivity.

Ion-Parallel-Viscosity Driven Toroidal ITG Mode

- **Low frequency limit of ES case**:
- **Marginal stability condition**: $\omega = i\gamma = 0$

$$\frac{1}{\mu_{i\|}}\frac{d^2\phi_1}{d\xi^2} - \frac{\chi_e T_{i0}}{\Gamma_e T_{e0}}\frac{q^2R^2}{v_{Ti}^2}\frac{m^4 f^4(\xi)}{r^4}\phi_1 + \frac{\alpha_i}{\chi_i}\frac{c^2}{\omega_{pi}^2}\frac{m^2}{r^2}H(\xi)\,\phi_1 = 0$$

- Introducing A and B,

$$\frac{d^2\phi_1}{d\xi^2} - Am^4[1 + (s-\alpha)^2\xi^2]^2\phi_1 + Bm^2[1 + \kappa + (s - \alpha - 1/2)\xi^2]\phi_1 = 0$$

- **Eigenvalue Problem**:

$$[Bm^2(1+\kappa) - Am^4]^2 = (2\ell + 1)^2[Bm^2(\alpha - s + 1/2) + 2Am^4(s-\alpha)^2]$$

- **Lowest Eigenvalue**:

$$N^2 \equiv \frac{Am^4}{Bm^2} = \frac{\chi_i\chi_e}{\alpha_i}\frac{T_{i0}}{\Gamma_e T_{e0}}\frac{q^2R^2}{v_{Ti}^2 r^2}\frac{\omega_{pi}^2}{c^2}m^2 = F\,m^2$$

then

$$\frac{B}{F} = \frac{\alpha_i^2\mu_{i\|}}{\chi_i^2\chi_e}\frac{\Gamma_e T_{e0}}{T_{i0}}\frac{v_{Ti}^2}{q^2R^2}\frac{c^4}{\omega_{pi}^4} = \frac{(\alpha - s + 1/2) + 2N^2(\alpha - s)^2}{N^2(1 + \kappa - N^2)^2}$$

Transport Coefficients of IPV Toroidal ITG

- The RHS $\dfrac{(\alpha - s + 1/2) + 2N^2(\alpha - s)^2}{N^2(1+\kappa - N^2)^2}$ has minimum approximately for $N^2 \sim \dfrac{1+\kappa}{2}$
- **Transport coefficients are given by**

$$\chi_{\mathrm{e}} \sim \chi_{\mathrm{i}} \sim \mu_{\mathrm{i}\|} = \alpha_{\mathrm{i}} \sqrt{\frac{\Gamma_{\mathrm{e}} T_{\mathrm{e}}}{T_{\mathrm{i}}}} \frac{c^2}{\omega_{\mathrm{pi}}^2} \frac{v_{\mathrm{Ti}}}{qR} g(s, \alpha, \kappa)$$

$$g(s, \alpha, \kappa) \sim \sqrt{\frac{(1+\kappa)^3}{8[\alpha - s + 1/2 + (1+\kappa)(\alpha - s)^2]}}$$

- **Comparison with CDBM model**
  - α dependence:

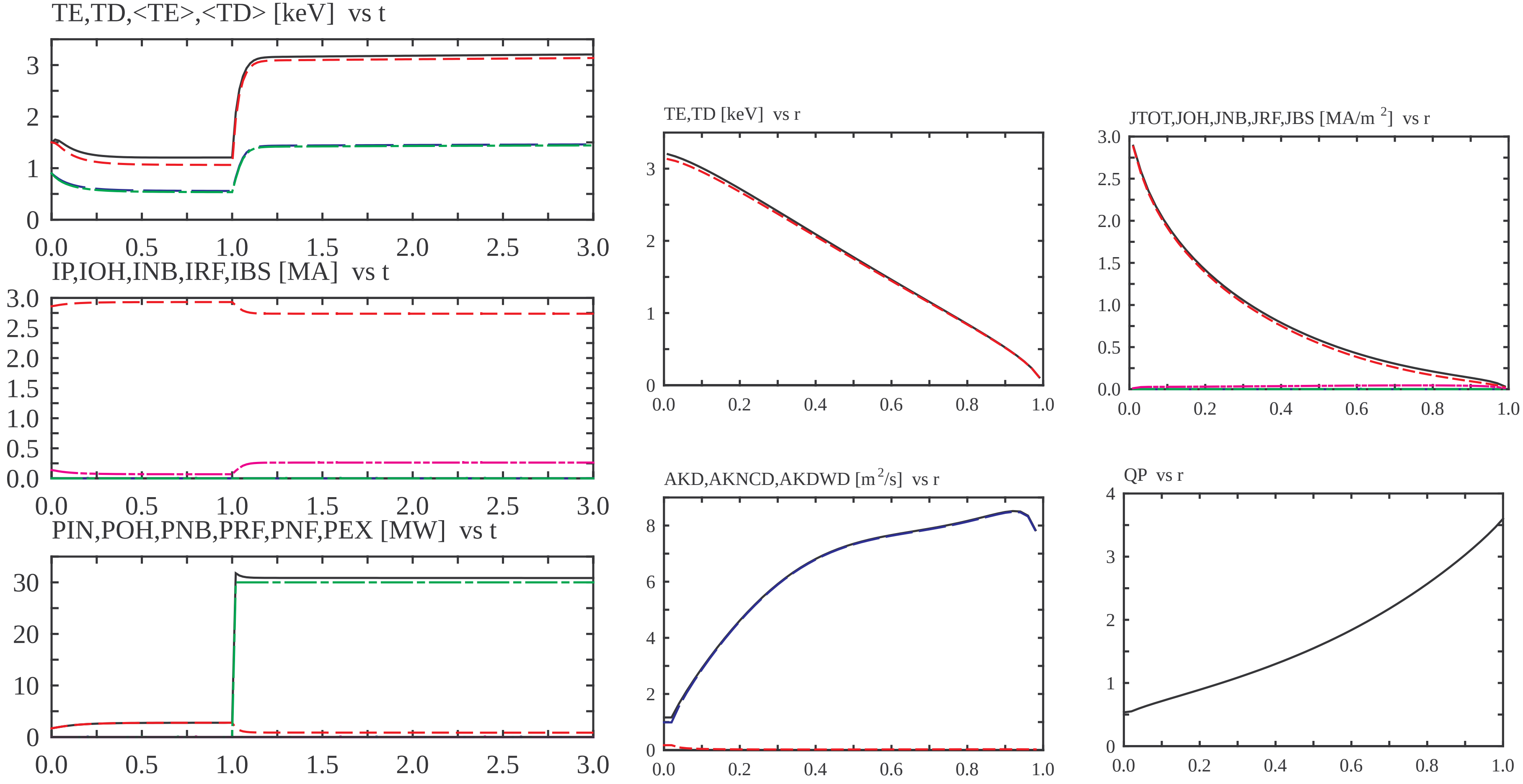
$$\alpha^{3/2} \rightarrow \alpha_{\mathrm{i}}$$

  - Difference of parameter dependence

$$\frac{\chi_{\mathrm{IPV}}}{\chi_{\mathrm{CDBM}}} = \frac{v_{\mathrm{Ti}}}{v_{\mathrm{A}}} \frac{m_{\mathrm{i}}}{m_{\mathrm{e}}} = \sqrt{\beta_{\mathrm{i}}} \frac{m_{\mathrm{i}}}{m_{\mathrm{e}}}$$

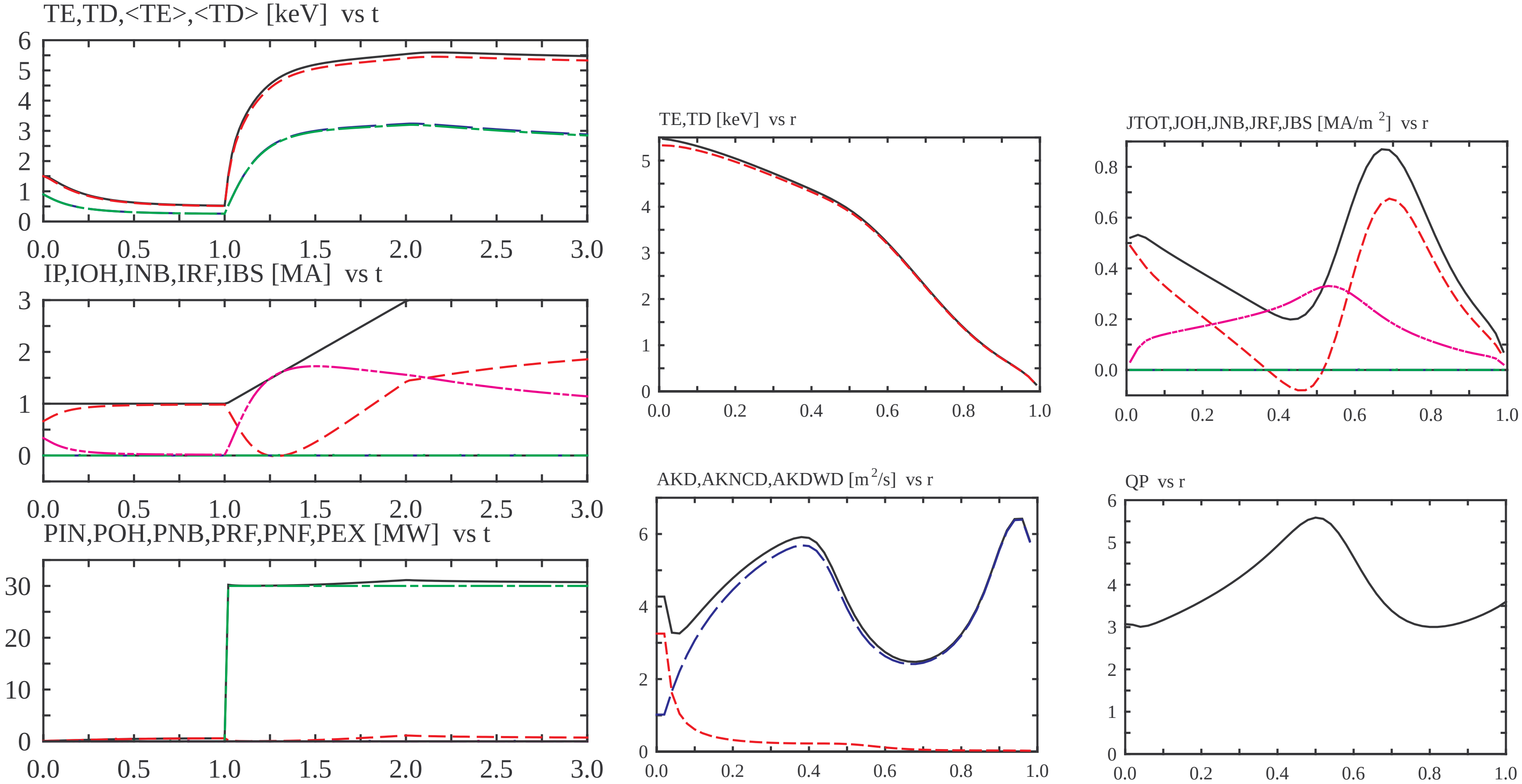
Preliminary Result of Transport Simulation (1)

- **Coefficient**: $\chi = \chi_{\mathrm{NC}} + 0.3\,\chi_{IPVBM}$
- **Device Parameters**: $R_0 = 3\,\mathrm{m}$, $a = 1.2\,\mathrm{m}$, $\kappa_e = 1.5$, $B_0 = 3\,\mathrm{T}$
- **L-mode**: $I_{\mathrm{p}} = 3\,\mathrm{MA}$



Preliminary Result of Transport Simulation (2)

- **Coefficient**: $\chi = \chi_{\mathrm{NC}} + 0.3\,\chi_{IPVBM}$
- **Device Parameters**: $R_0 = 3\,\mathrm{m}$, $a = 1.2\,\mathrm{m}$, $\kappa_e = 1.5$, $B_0 = 3\,\mathrm{T}$
- **Reversed shear**: $I_{\mathrm{p}} = 1 \rightarrow 3\,\mathrm{MA}$



Summary

- **We have derived a set of reduced two-fluid equations in both slab and toroidal configurations and solved numerically and analytically.**

- **Linear analysis in a toroidal configuration describes the toroidal ITG mode for low pressure gradient and kinetic ballooning mode for high pressure gradient.**

- **Based on the theory of self-sustained turbulence, we have obtained an approximate formula of the transport coefficients from the marginal stability condition for electrostatic case (low pressure gradient case). This mode is destabilized by the ion-parallel-viscosity and stabilized by the electron thermal diffusivity.**

- **Preliminary result of transport simulation suggests that the internal transport barrier formation is more difficult for this approximate formula of the ion-parallel-viscosity (IPV) driven mode model than the CDBM model.**

- **To Dos**
  - Derivation of more general formula of the IPV mode
  - Comparison with experimental observations and simulation results