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# Full Wave Analysis of RF Waves in Confined Plasmas

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## Contents

# Wave Analysis in Confined Plasmas

- **Geometrical Optics**: EC, LH
  - **Ray tracing**
  - **Beam tracing**
- **Full Wave Optics**: IC, AW, (LH, EC)
  - **Time evolution**
  - **Stationary propagation** $\Leftarrow$

- **Full wave analysis in toroidal plasmas**: TASK/WM
- **Full wave analysis in linear plasmas**: PAF/WF **(see Y. Yamaguchi)**
- **Integro-differential wave analysis**: WI

# Modeling of Full Wave Analysis

- **Numerical Method**:
  - Multi layer method (MLM): 1D
  - Finite difference method (FDM)
  - Finite element method (FEM)
  - Toroidal and poloidal mode expansion + FDM/FEM
  - Three dimensional mode expansion
- **Kinetic effects**:
  - **Parallel to $\boldsymbol{B}$**: Landau and cyclotron damping
    — Plasma dispersion function (Local)
    — Toroidal plasma dispersion function (Variation of $B$)
    — Integral representation (in real space)
  - **Perpendicular to $\boldsymbol{B}$**: FLR, Finite drift orbit size
    — Evaluate $k_\perp$ from cold plasma dispersion
    — Second-order differential operator
    — Spectral method (in $\boldsymbol{k}$ space)
    — Integral representation (in real space)

# TASK Code

- **Transport Analysing System for TokamaK**
- **Features**
  - **Core of Integrated Modelling Code**
    - — Modular structure
    - — Reference data interface
  - **Various Heating and Current Drive Scheme**
    - — EC, LH, IC, AW, (NB)
  - **High Portability**
    - — Most of library routine included
    - — Graphic libraries (gsaf, gsgl)
    - — Optional: MPI, LAPACK
  - **Development using CVS**
  - **To Be Open Source**
  - **Extension to Toroidal Helical Plasmas**

# Modules of TASK code

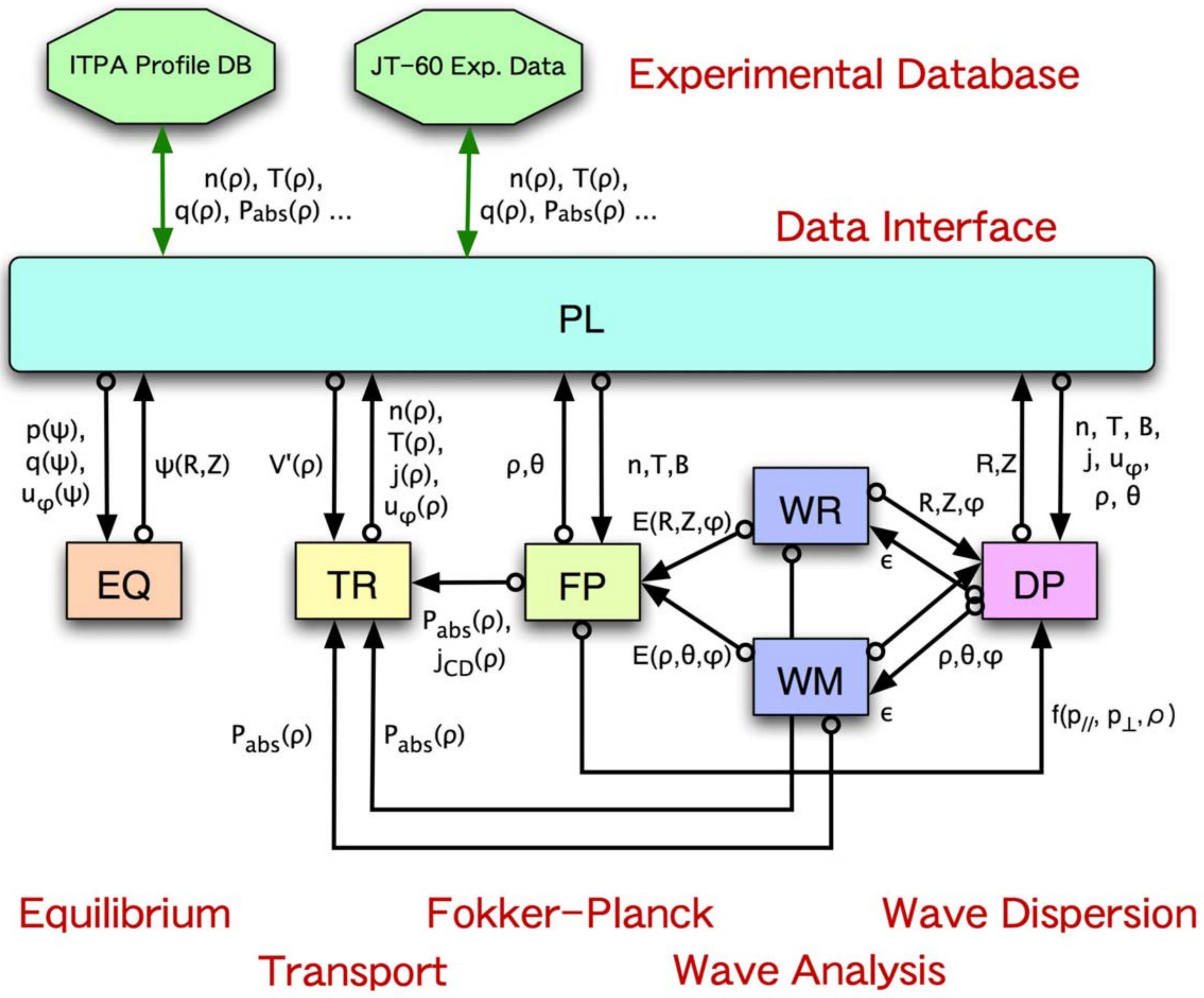
## Present Modules

| | | |
|---|---|---|
| EQ | 2D Equilibrium | Fixed boundary |
| TR | 1D Transport | Diffusive model |
| FP | 3D Fokker-Planck | Bounce averaged |
| WR | Ray/Beam Tracing | EC, LH |
| WM | Full Wave Analysis | IC,AE |
| DP | Wave Dispersion | Various models |
| PL | Data Conversion | Profile database |
| LIB | Common Library | |

## Under Development

| | | |
|---|---|---|
| TX | 1D Transport | Dynamical model |
| WA | Full Wave | Stability |
| EX | 2D Equilibrium | Free boundary |

# Present Structure of TASK



# Wave Dispersion Analysis : TASK/DP

- **Various Models of Dispersion Tensor $\overleftrightarrow{\epsilon}(\omega, \boldsymbol{k}; \boldsymbol{r})$:**
  - **Resistive MHD model**
  - **Collisional cold plasma model**
  - **Collisional warm plasma model**
  - **Kinetic plasma model (Maxwellian, non-relativistic)**
  - **Kinetic plasma model (Arbitrary $f(\boldsymbol{v})$, non-relativistic)**
  - **Kinetic plasma model (Arbitrary $f(\boldsymbol{v})$, relativistic)**
  - **Gyro-kinetic plasma model (Maxwellian, non-relativistic)**
  - **Gyro-kinetic plasma model (Arbitrary $f(\boldsymbol{v})$, non-relativistic)**
- **Arbitrary $f(\boldsymbol{v})$:**
  - **Relativistic Maxwellian**
  - **Output of TASK/FP**

# Fokker-Planck Analysis : TASK/FP

- **Fokker-Planck equation** for **velocity distribution function** $f(p_{\|}, p_{\perp}, \psi, t)$

$$\frac{\partial f}{\partial t} = E(f) + C(f) + Q(f) + L(f)$$

  - $E(f)$: Acceleration term due to DC electric field
  - $C(f)$: Coulomb collision term
  - $Q(f)$: Quasi-linear term due to wave-particle resonance
  - $L(f)$: Spatial diffusion term

- **Bounce-averaged**: Trapped particle effect, zero banana width

- **Relativistic**: momentum $\boldsymbol{p}$, weakly relativistic collision term

- **Nonlinear collision**: momentum conservation, energy conservation

- **Three-dimensional**: spatial diffusion (classical, neoclassical, turbulence)

# Full wave analysis: TASK/WM

- **magnetic surface coordinate**: $(\psi, \theta, \varphi)$
- Boundary-value problem of **Maxwell's equation**

$$\nabla \times \nabla \times \boldsymbol{E} = \frac{\omega^2}{c^2} \overleftrightarrow{\epsilon} \cdot \boldsymbol{E} + \mathrm{i}\, \omega \mu_0 \boldsymbol{j}_{\mathrm{ext}}$$

- Kinetic **dielectric tensor**: $\overleftrightarrow{\epsilon}$
  - **Wave-particle resonance**: $Z[(\omega - n\omega_{\mathrm{c}})/k_{\|} v_{\mathrm{th}}]$
  - **Fast ion: Drift-kinetic**

$$\left[ \frac{\partial}{\partial t} + v_{\|} \nabla_{\|} + (\boldsymbol{v}_{\mathrm{d}} + \boldsymbol{v}_{\mathrm{E}}) \cdot \nabla + \frac{e_\alpha}{m_\alpha} (v_{\|} E_{\|} + \boldsymbol{v}_{\mathrm{d}} \cdot \boldsymbol{E}) \frac{\partial}{\partial \varepsilon} \right] f_\alpha = 0$$

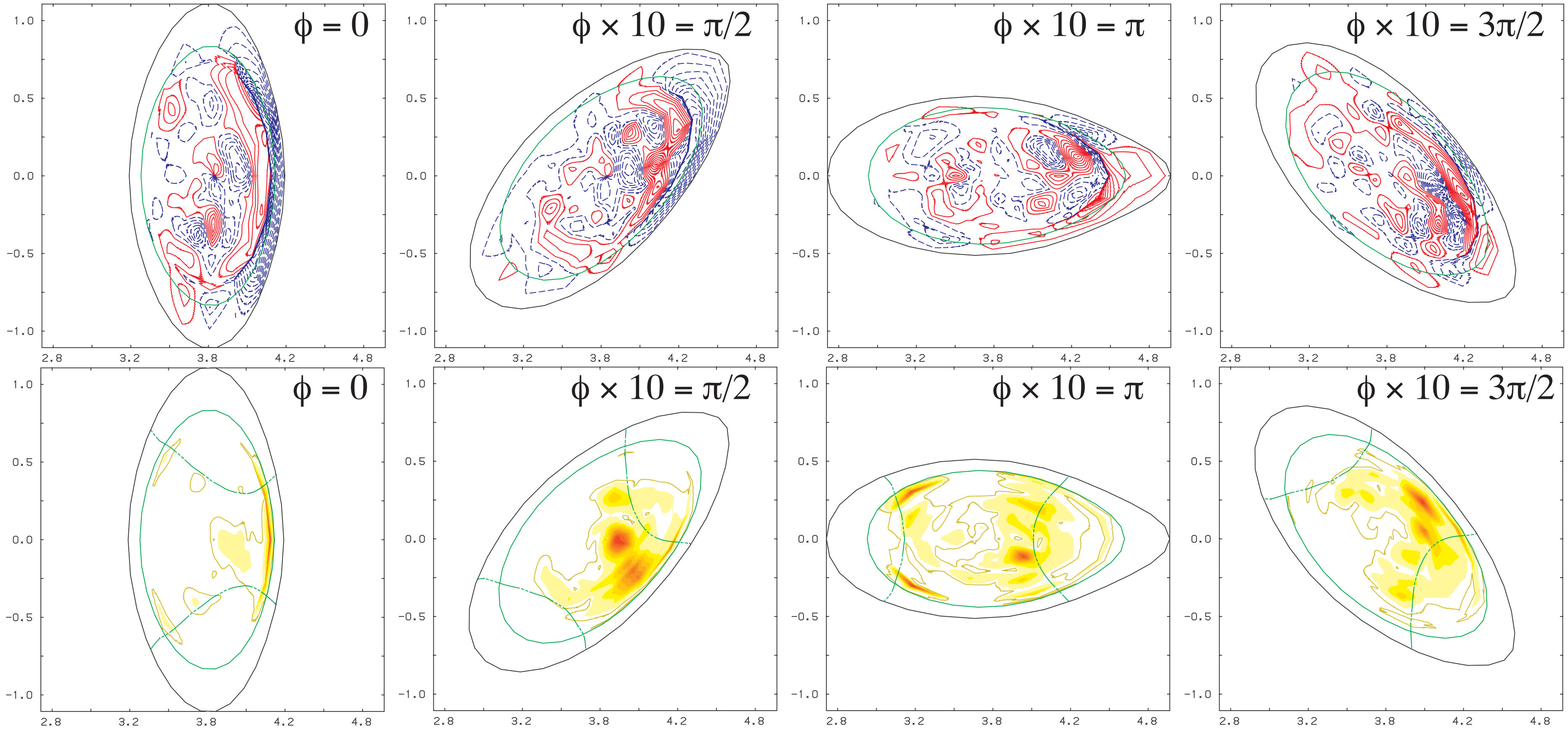
- Poloidal and toroidal **mode expansion**
  - **Accurate estimation of $k_{\|}$**
- Eigenmode analysis: **Complex eigen frequency** which maximize wave amplitude for fixed excitation proportional to electron density

# Typical Poloidal Profile

**LHD** ($B_0 = 3\,\mathrm{T}$, $R_0 = 3.8\,\mathrm{m}$)

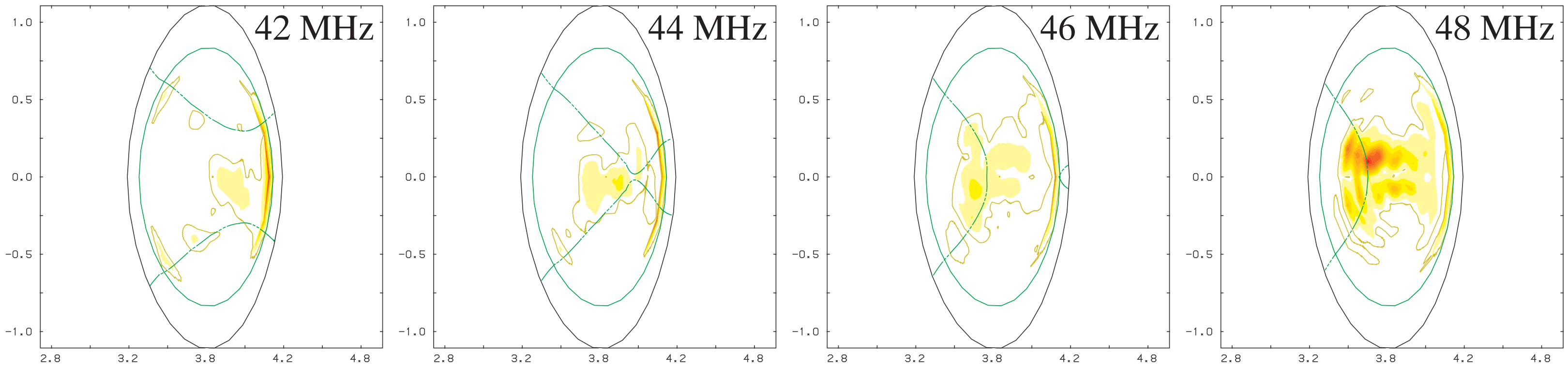
$f = 42\,\mathrm{MHz}$, $n_{\phi 0} = 20$, $n_{e0} = 3 \times 10^{19}\,\mathrm{m}^{-3}$, $n_\mathrm{H}/(n_\mathrm{He} + n_\mathrm{H}) = 0.235$,

$N_{r\max} = 100$, $N_{\theta\max} = 16$ ($m = -7 \ldots 7$), $N_{\phi\max} = 4$ ($n = 10, 20, 30$)

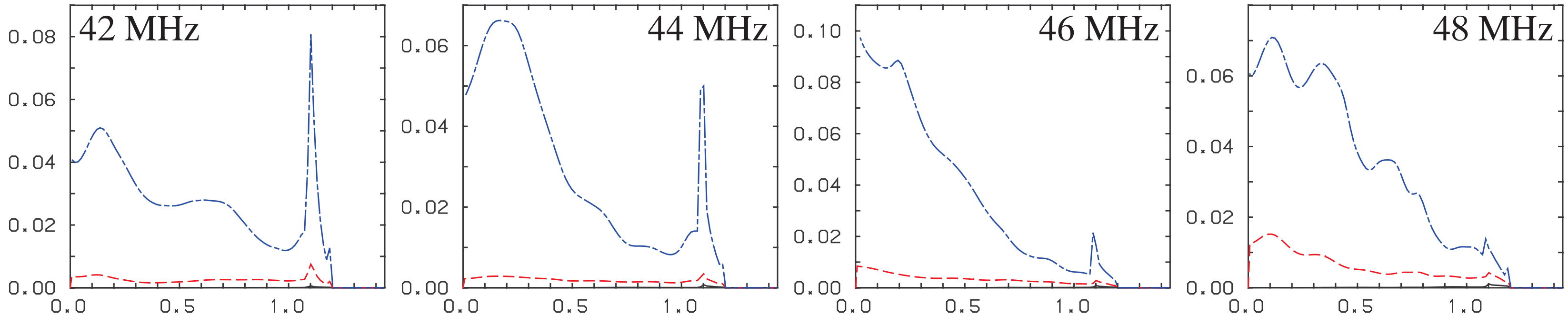


# Frequency Dependence

- **Power deposition profile**

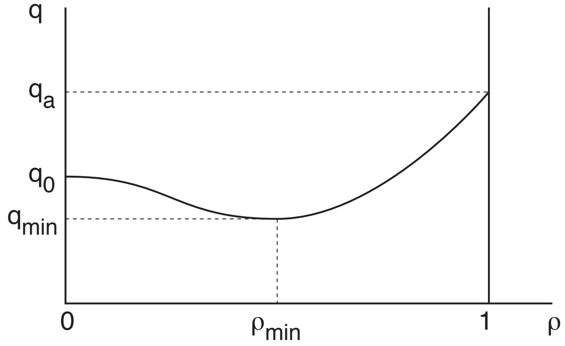



- **Radial deposition profile**



# Analysis of TAE in Reversed Shear Configuration

## Assumed $q$ profile



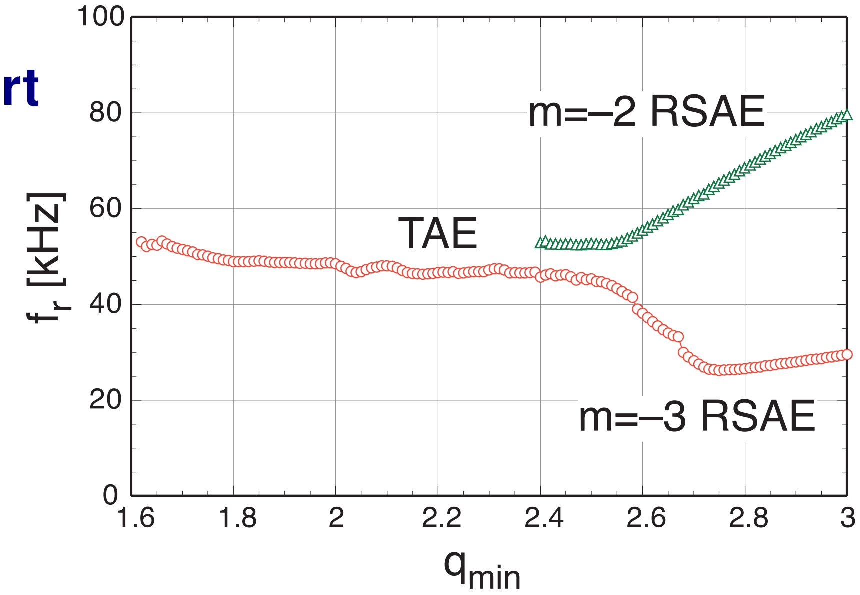

## Plasma Parameters

| | |
|---|---|
| $R_0$ | 3 m |
| $a$ | 1 m |
| $B_0$ | 3 T |
| $n_e(0)$ | $10^{20}\,\mathrm{m}^{-3}$ |
| $T(0)$ | 3 keV |
| $q(0)$ | 3 |
| $q(a)$ | 5 |
| $\rho_{\min}$ | 0.5 |
| $n$ | 1 |

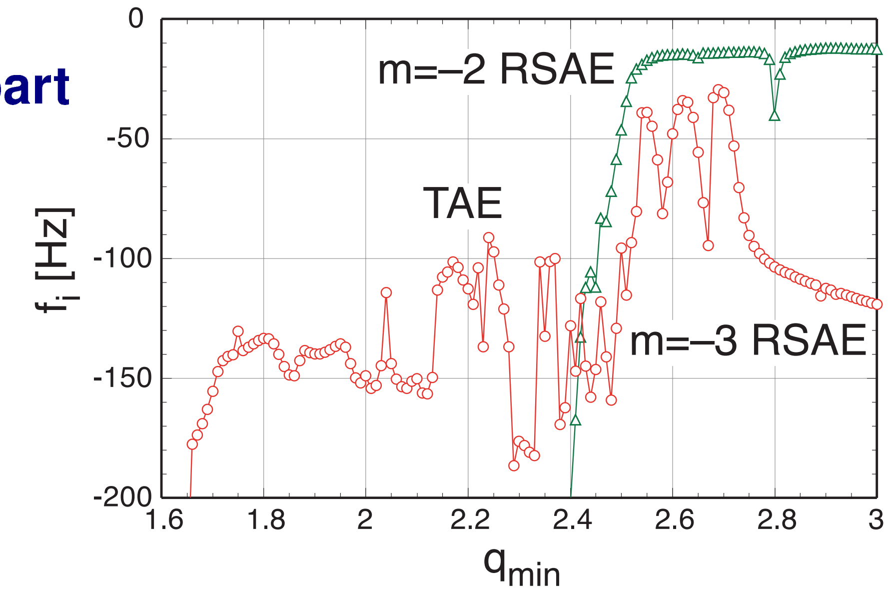
Flat density profile

## $q_{\min}$ Dependence of Eigenmode Frequency

**Real part**




**Imag part**



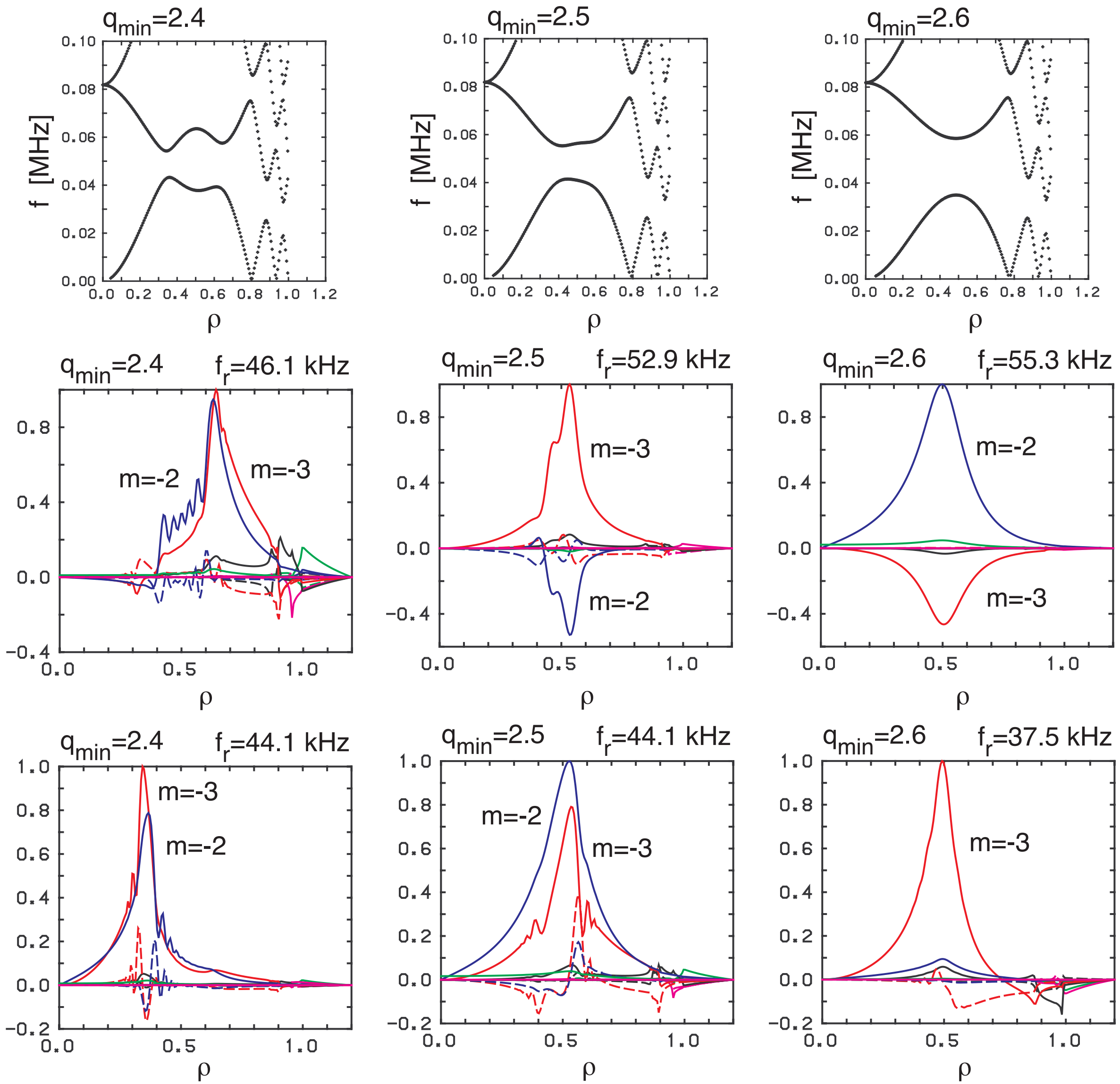
# Eigenmode Structure in Reversed Shear Configuration

$q_{\rm min} = 2.4$ | $q_{\rm min} = 2.5$ | $q_{\rm min} = 2.6$

**Alfvén resonance**

**Higher freq.**

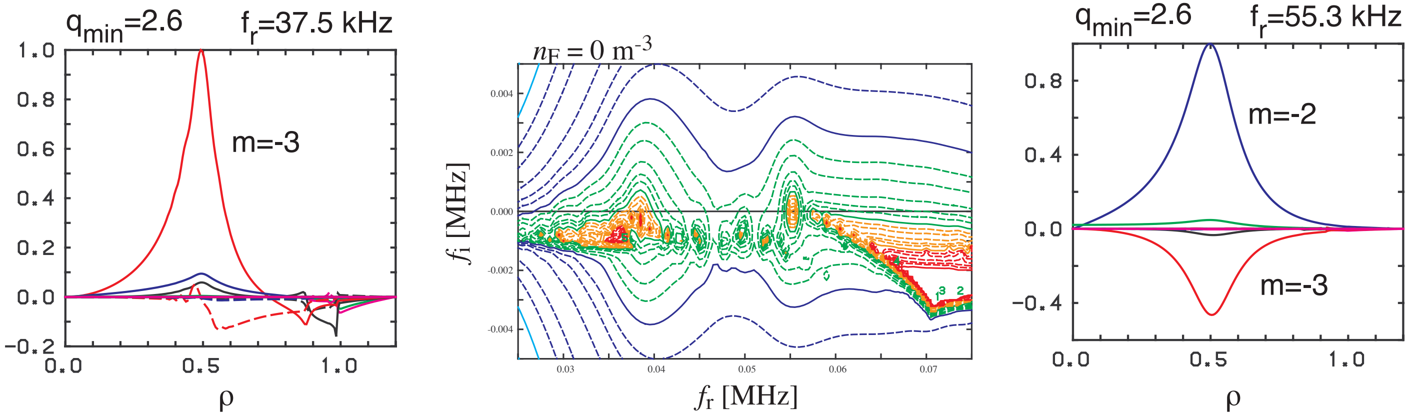
**Lower freq.**




**TAEs** | **Double TAE** | **RSAE**

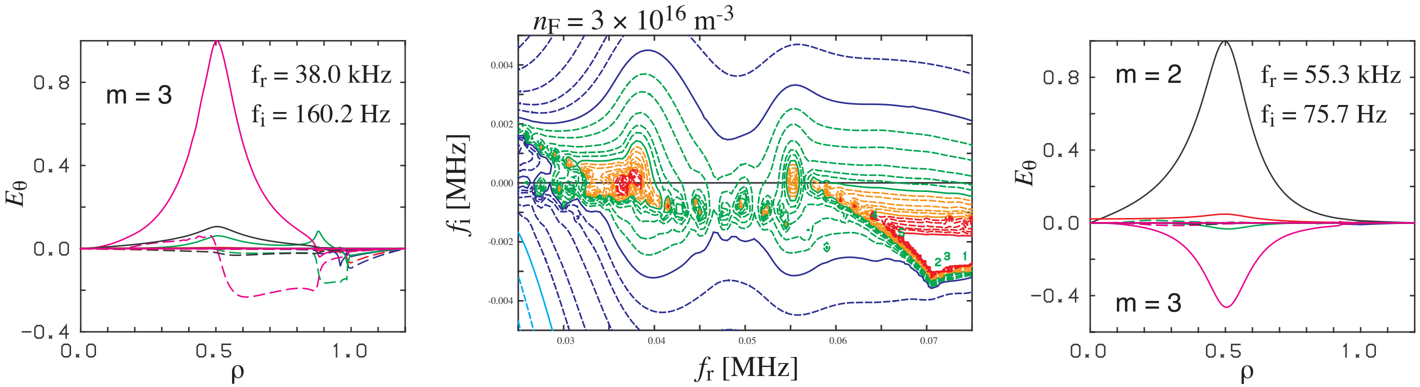
# Excitation by Energetic Particles ($q_{min} = 2.6$)

- Without EP



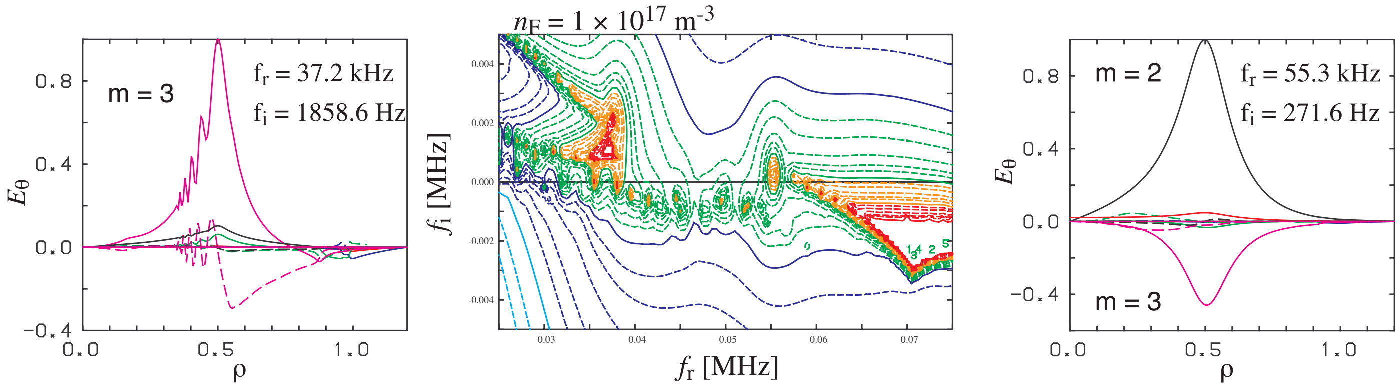

- With EP

  $3 \times 10^{16}\,\mathrm{m}^{-3}$
  360 keV
  0.5 m




- With EP

  $1 \times 10^{17}\,\mathrm{m}^{-3}$
  360 keV
  0.5 m



# RF Plasma Production Analysis Code

- **Self-Consistent Analysis of RF Plasma Production**
  - **Arbitrary Device Configuration**
  - **Realistic RF Excitation**
  - **Fluid-Particle Hybrid Model**
  - **Fast Computation**
- **PAF**: Plasma Analysis with **Finite element method**
  - **WF**: Wave field solver (3D): **this presentation**
  - **MF**: Time dependent Maxwell equation solver (2D)
  - TF: Diffusive transport model (2D)
  - **FF**: Dynamic fluid model (2D)
  - PF: Particle-in-(triangle) cell model (2D)
  - MG: Mesh generator
  - **MX**: Parallelized matrix solver

**XX**: under development

# PAF/WF: Wave Analysis Code

- **Steady-state electric field**: $\tilde{\boldsymbol{E}}(\boldsymbol{r}, t) = \boldsymbol{E}(\boldsymbol{r})\,\mathrm{e}^{-\mathrm{i}\,\omega t}$
- **Maxwell's equation**:

$$\nabla \times \nabla \times \boldsymbol{E} - \frac{\omega^2}{c^2}\overleftrightarrow{\epsilon} \cdot \boldsymbol{E} = \mathrm{i}\,\omega\mu_0 \boldsymbol{j}_{\mathrm{ext}}$$

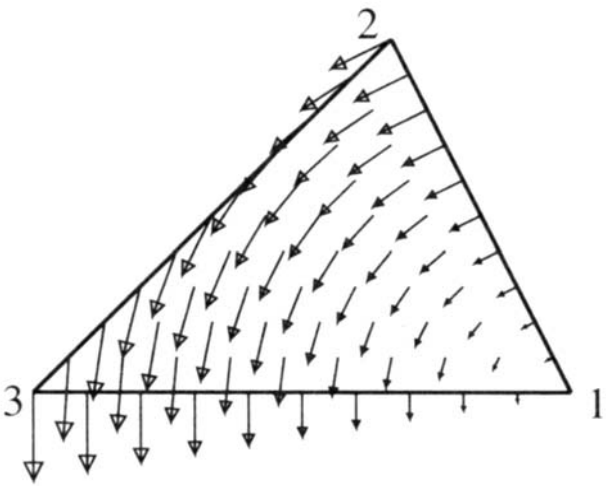
  - $\overleftrightarrow{\epsilon}$: Dielectric tensor (Cold plasma including neutral collision)

- **Excitation**:
  - Antenna (Given current profile)
  - Waveguide (Co-axial, circular, rectangular)
  - Electrode

- **Numerical Method**
  - Finite element method with tetrahedron elements
  - Variables: Line integral of $\boldsymbol{E}$ along the ridges of tetrahedrons

# FEM Analysis of Maxwell's Equation

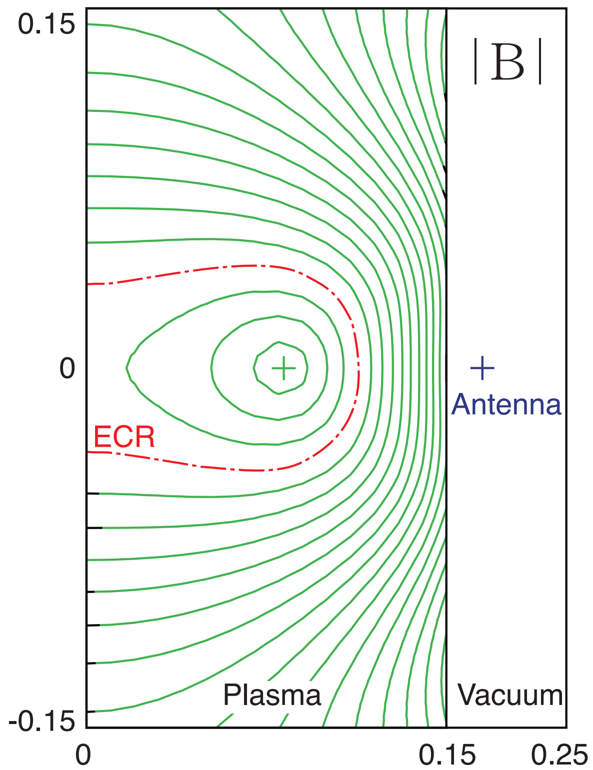
- **Weak form of Maxwell's equation may introduce unphysical spurious solutions**
- **Choice of variables**
  - **Electric field $E$ on nodes**: problems in low plasma density region
  - **Potentials $A$ and $\phi$ on nodes**: problems in vacuum region
  - **Electric field $E$ on sides (ridges)**: OK
  - **Potentials $A$ and $\phi$ on sides (ridges)**: OK, better convergence in matrix solver, but more variables

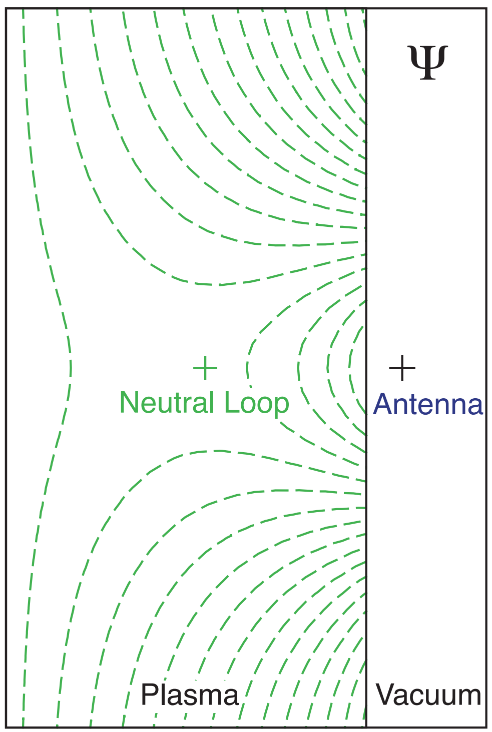



- **FEM using ridge variables completely suppress unphysical spurious modes.**

# 2D Example: Neutral Loop Discharge Plasma

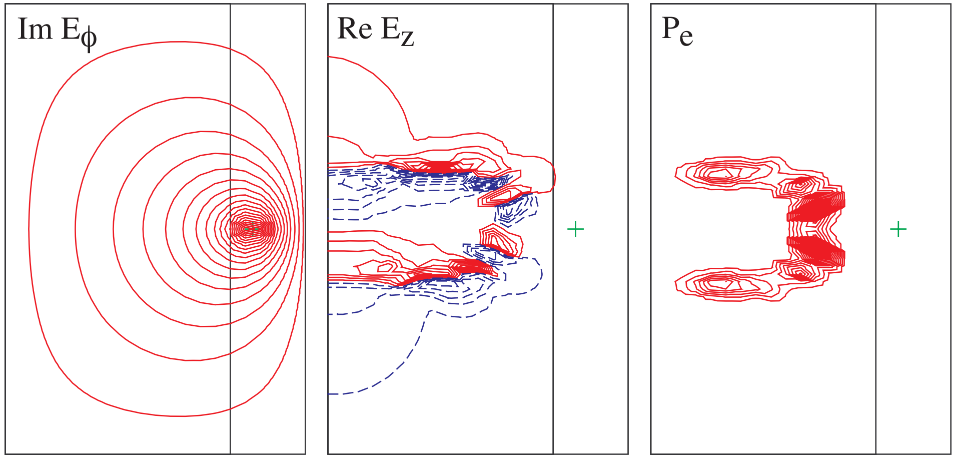
- **Static magnetic field** : 3 loop coils (r= 0.3 m)
- **RF field** : Loop antenna (r= 0.165 m, f = 13.56 MHz)
- **Plasma** : Cylindrical plasma (r= 0.15 m, parabolic density profile)



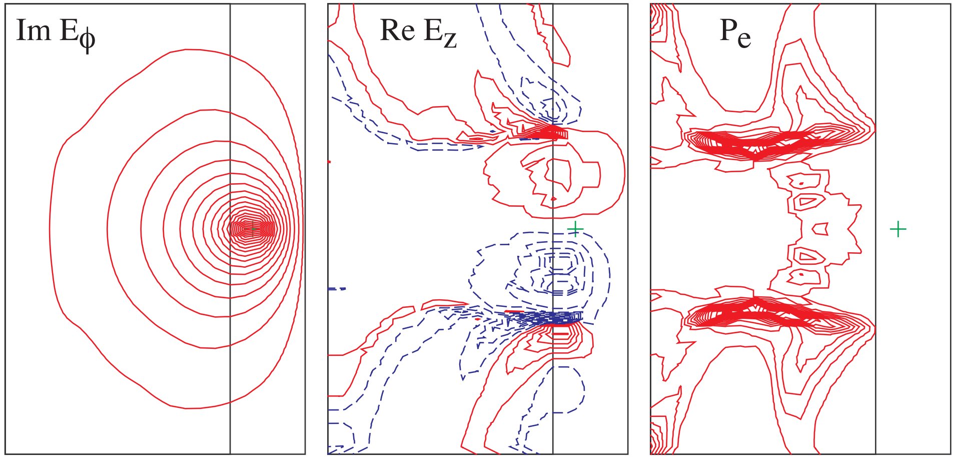



# Wave Structures in Low and High Density Plasmas

- **Low density plasma** $(10^{12}\,\mathrm{m}^{-3})$ . . . . . . . . . . . . . . . . . . Absorption near the ECR layer

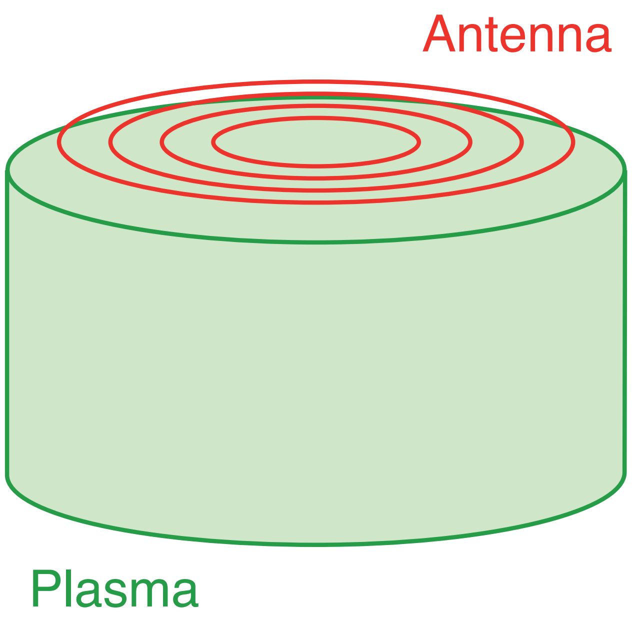


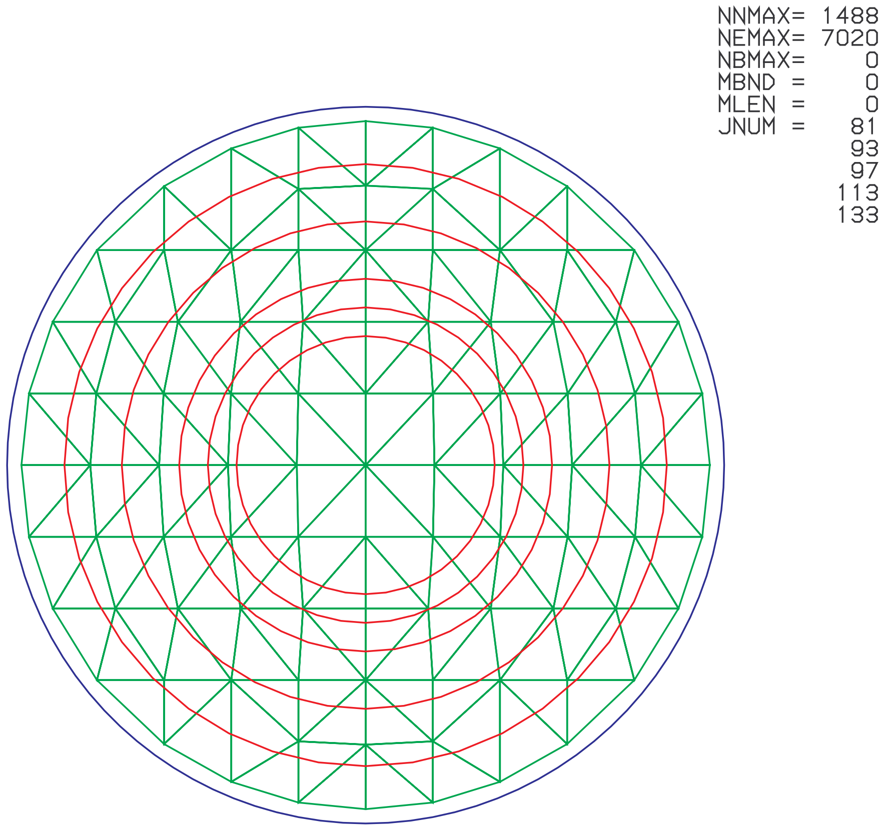

- **High density plasma** $(10^{16}\,\mathrm{m}^{-3})$ . . . . . . . . . . . . . . . . . . Absorption near the LHR layer



# 3D Full Wave Analysis of ICP

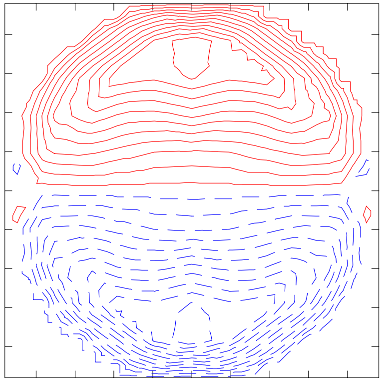
- **Cylindrical Plasma**
  - Diameter=0.48 m
  - Height=0.3 m
- **RF Excited by Multiple Loops**
  - Frequency=13.56 MHz



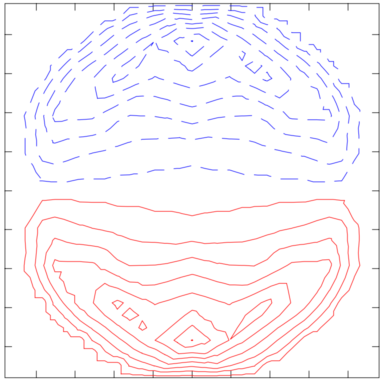



# Wave Electric Field and Absorbed Power: $z = z_{\mathrm{ant}}$

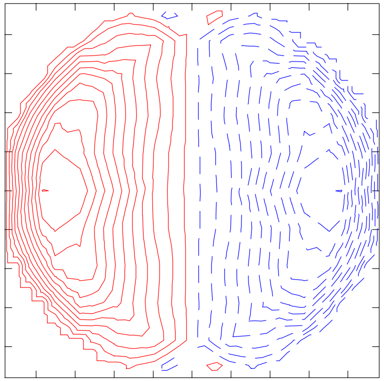
```
RF   = 13.560    BB   =   0.000    M= 2 0 0 2 6 0 IJ   AJ   PHASE       R            X
NNMAX=   1488    XYZ MAX=   0.240   0.240   0.300  1   0.3     0.0  1.413E-02   1.287E+00
NEMAX=   7020    XYZ MIN= -0.240 -0.240   0.000  2   0.3     0.0  2.153E-02   1.967E+00
 NK  NM  PABS       NK  NM  PABS                   3   0.3     0.0  2.927E-02   2.498E+00
  1   0   3.02E-01                                 4   1.0     0.0  1.339E-01   1.425E+01
 NS      PA       PZ       PN       PZCL     PABS  5   1.0     0.0  1.029E-01   1.362E+01
  1  5.49E-04  -1.  5.00E-03  0.03  3.02E-01
  2  3.99E+01   1.  5.00E-03  0.00  1.38E-07
```



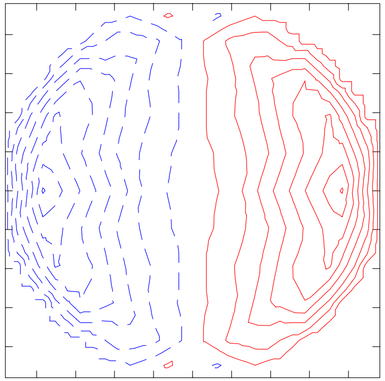
EXR(XY) Z=0.16
MAX= 1.43E-01
MIN=-1.43E-01
STP= 1.25E-02



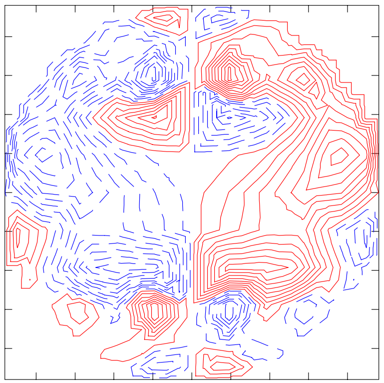
EXI(XY) Z=0.16
MAX= 7.55E+00
MIN=-7.55E+00
STP= 1.00E+00



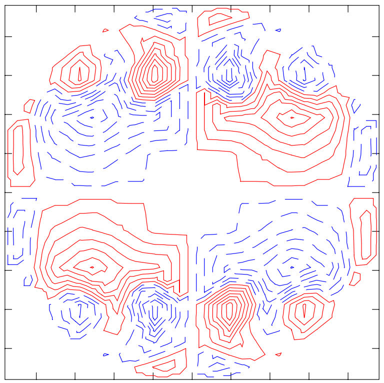
EYR(XY) Z=0.16
MAX= 1.44E-01
MIN=-1.44E-01
STP= 1.25E-02



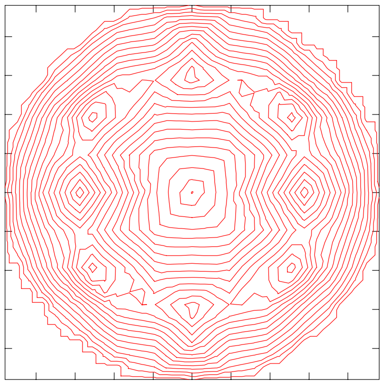
EYI(XY) Z=0.16
MAX= 7.62E+00
MIN=-7.62E+00
STP= 1.00E+00



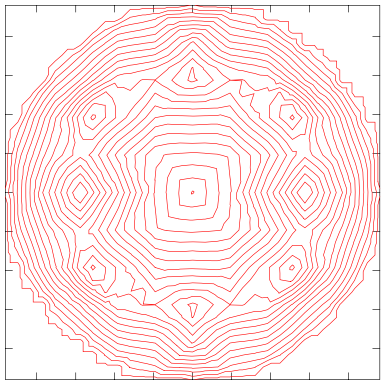
EZR(XY) Z=0.16
MAX= 2.47E-02
MIN=-2.47E-02
STP= 2.50E-03



EZI(XY) Z=0.16
MAX= 1.79E+00
MIN=-1.79E+00
STP= 2.50E-01



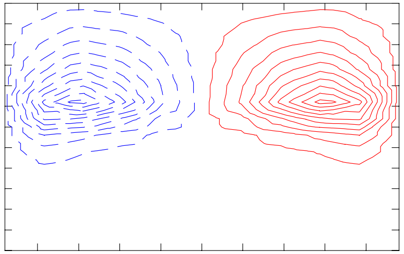
P1C(XY) Z=0.16
MAX= 8.61E+01
MIN= 0.00E+00
STP= 5.00E+00



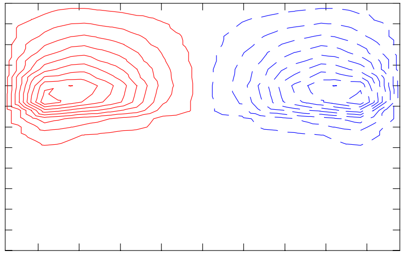
P2C(XY) Z=0.16
MAX= 3.94E-05
MIN= 0.00E+00
STP= 2.50E-06

# Wave Electric Field and Absorbed Power: $x = 0$

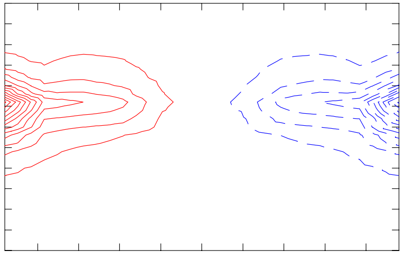
```
RF   = 13.560   BB   =   0.000   M= 2 0 0 2 6 0 IJ   AJ   PHASE       R           X
NNMAX=   1488   XYZ MAX=   0.240   0.240   0.300  1   0.3    0.0  1.413E-02  1.287E+00
NEMAX=   7020   XYZ MIN= -0.240 -0.240   0.000  2   0.3    0.0  2.153E-02  1.967E+00
 NK  NM PABS         NK  NM PABS                 3   0.3    0.0  2.927E-02  2.498E+00
  1   0   3.02E-01                               4   1.0    0.0  1.339E-01  1.425E+01
 NS     PA       PZ      PN       PZCL     PABS     5   1.0    0.0  1.029E-01  1.362E+01
  1  5.49E-04  -1.   5.00E-03  0.03  3.02E-01
  2  3.99E+01   1.   5.00E-03  0.00  1.38E-07
```



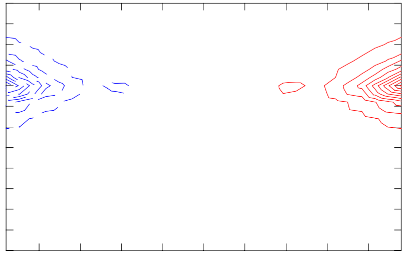
EXR(YZ) X=0.0
MAX= 2.47E-01
MIN=-2.47E-01
STP= 2.50E-02



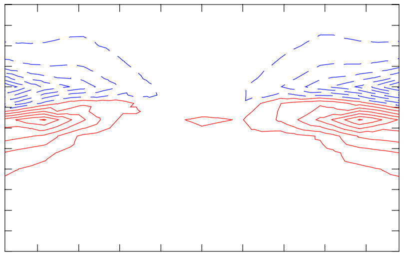
EXI(YZ) X=0.0
MAX= 2.32E+01
MIN=-2.32E+01
STP= 2.50E+00



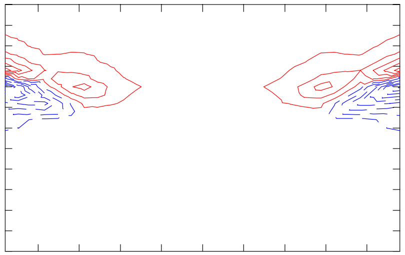
EYR(YZ) X=0.0
MAX= 5.85E-08
MIN=-5.86E-08
STP= 5.00E-09



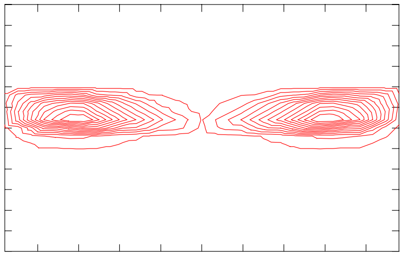
EYI(YZ) X=0.0
MAX= 4.85E-05
MIN=-4.85E-05
STP= 5.00E-06



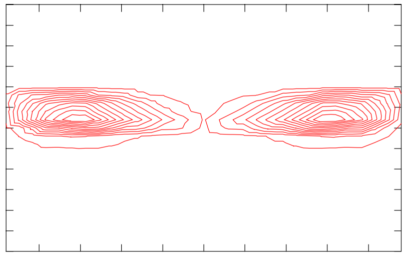
EZR(YZ) X=0.0
MAX= 6.61E-09
MIN=-7.90E-09
STP= 1.00E-09



EZI(YZ) X=0.0
MAX= 5.02E-06
MIN=-8.75E-06
STP= 1.00E-06



P1C(YZ) X=0.0
MAX= 6.95E+01
MIN=-4.59E-14
STP= 5.00E+00



P2C(YZ) X=0.0
MAX= 3.19E-05
MIN= 0.00E+00
STP= 2.50E-06

# Integro-Differential Full Wave Analysis

- **Purpose**: Wave-particle Interaction in Inhomogeneous Plasmas
- **Formulation**:
  - Particle orbit: $x = x' + v(t - t')$
  - Particle at $x(t)$ is affected by the wave field $E(x', t')$
  - Linearized Vlasov equation: $\dfrac{\mathrm{d}f(x',t')}{\mathrm{d}t} = -\dfrac{q}{m} E(x',t') \dfrac{\partial f_0(v)}{\partial v}$
  - Perturbed distribution function: $f(x,v,t) = \dfrac{qn_0}{(2\pi T/m)^{3/2} T} \displaystyle\int_{-\infty}^{t} \mathrm{d}t' v E(x',t') \exp\left(-\dfrac{mv^2}{2T}\right)$
  - Induce current: $j(x,t) = \displaystyle\int_{-\infty}^{\infty} \mathrm{d}v\, qv f(x,v,t)$
  - Substituting into $f(x,v,t)$, we obtain

$$j(x,t) = \int_{-\infty}^{\infty} \mathrm{d}x'\, K(x-x', t-t')\, E(x',t')$$

$$K(x-x', t-t') = \frac{qn_0}{(2\pi T/m)^{3/2} T} \int_{-\infty}^{t} \mathrm{d}t' \frac{x-x'}{t-t'} \exp\left[-\frac{m}{2T}\left(\frac{x-x'}{t-t'}\right)^2\right]$$

# Wave Absorption in a Plasma with Density Gradient

- **Excitation of Plasma Wave by Obliquely Incident EM Wave**
  - **Density profile**: $n(x) = n_0\,\mathrm{e}^{-\kappa x}$
  - **Electrostatic potential**: $\Phi(x) = \dfrac{\kappa T}{q}\,x$
  - **Particle motion**: Acceleration by electric field
- **basic Equation**: $\dfrac{1}{\beta^2}\boldsymbol{\nabla}\times\boldsymbol{\nabla}\times\boldsymbol{E}(x) - \displaystyle\int_{-\infty}^{\infty}\mathrm{d}x'\,\overleftrightarrow{\epsilon}(x-x')\,\boldsymbol{E}(x') = 0$

$$\overleftrightarrow{\epsilon}(x-x') = \delta(x-x')\overleftrightarrow{I}\;\mathrm{i}\,\frac{\omega_{\mathrm{p0}}^2}{\omega^2}\,\mathrm{e}^{-\kappa(x+x')}$$

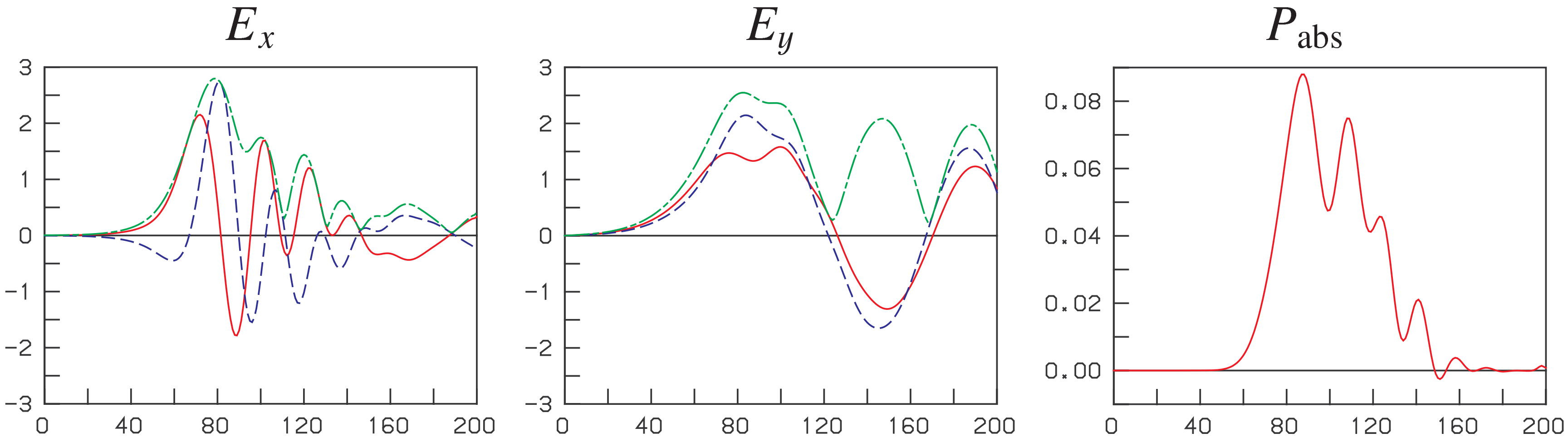
$$\times\begin{pmatrix} (x-x')^2U_{-2}-\kappa^2U_2 & -\mathrm{i}\,n_y\beta[(x-x')U_0-\kappa U_2] & 0 \\ -\mathrm{i}\,n_y\beta[(x-x')U_0+\kappa U_2] & U_0-n_y^2\beta^2U_2 & 0 \\ 0 & 0 & U_0 \end{pmatrix}$$

$$U_n = U_n\left(\omega(x-x')/\sqrt{T/m},\ \sqrt{\kappa^2+n_y^2\beta^2},\ \sqrt{T/m}\right),\quad \beta = \sqrt{T/m}/c$$

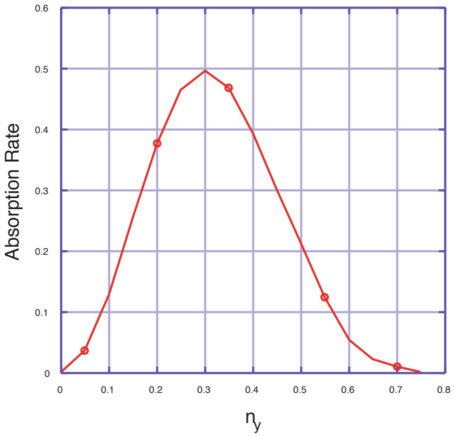
$$U_n(\xi,\eta) = \frac{1}{\sqrt{2\pi}}\int_0^{\infty}\mathrm{d}\tau\,\tau^{n-1}\exp\left[-\frac{1}{2}\frac{\xi^2}{\tau^2}-\frac{1}{2}\eta^2\tau^2+\mathrm{i}\,\tau\right]$$

# Numerical Results

- Density: $(\omega_p^2/\omega^2)_{max} = 2$, $(\omega_p^2/\omega^2)_{min} = 0$,
- Incident angle: $n_y = 0.2$
- Temperature: $\beta = 0.1$




- Absorption Rate vs. Incident Angle:



# Wave Absorption in a Plasma with Magnetic Field Gradient

- **Wave Absorption at ECR**
  - **Inhomogeneous Magnetic Field**: $B(z) = B_0\left(1 + \dfrac{x}{L}\right)$
- **Basic Equation**: $\dfrac{1}{\beta^2}\nabla\times\nabla\times\boldsymbol{E}(x) - \displaystyle\int_{-\infty}^{\infty}\mathrm{d}z'\,\overleftrightarrow{\epsilon}(z,z')\,\boldsymbol{E}(z') = 0$

$$\overleftrightarrow{\epsilon}(z,z') = \delta(z-z')\overleftrightarrow{I}\,\mathrm{i}\,\frac{\omega_{\mathrm{p0}}^2}{\omega^2}\times\begin{pmatrix} (\chi_+\chi_-)/2 & -\mathrm{i}\,(\chi_-\chi_-)/2 & 0 \\ \mathrm{i}\,(\chi_-\chi_-)/2 & (\chi_+\chi_-)/2 & 0 \\ 0 & 0 & \chi_0 \end{pmatrix}$$

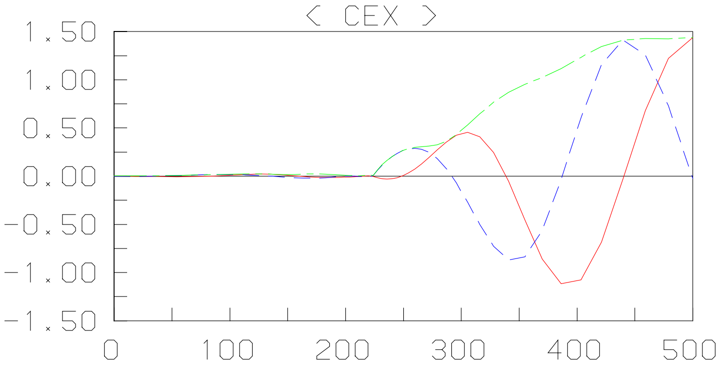
$$\chi_\pm = \frac{(1+\kappa z)^{3/2}(1+\kappa z')^{3/2}}{(1+\kappa(z+z')/2)^2}U_0(\xi_\pm)$$

$$\chi_0 = \frac{(1+\kappa z)(1+\kappa z')}{(1+\kappa(z+z')/2)}\left[\xi U_2(\xi) - \frac{\kappa^2}{2(1+\kappa(z+z')/2)^2}U_{-2}(\xi)\right]$$

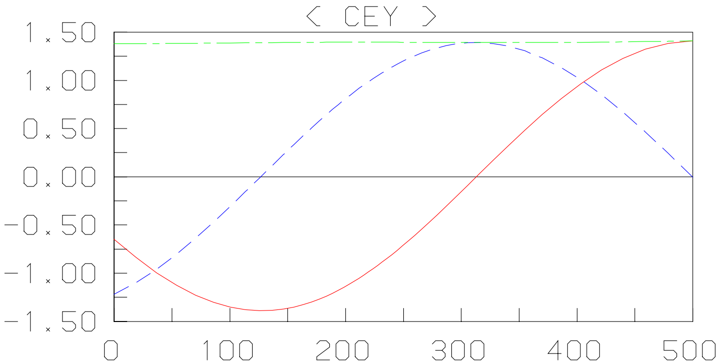
$$\xi = \frac{\omega(z-z')}{\sqrt{T/m}},\quad \xi_\pm = \frac{(\omega+\Omega)(z-z')}{\sqrt{T/m}},\quad \Omega = \frac{qB_0}{m}(1+(z+z')/2L),\quad \kappa = \frac{\sqrt{T/m}}{\omega L}$$

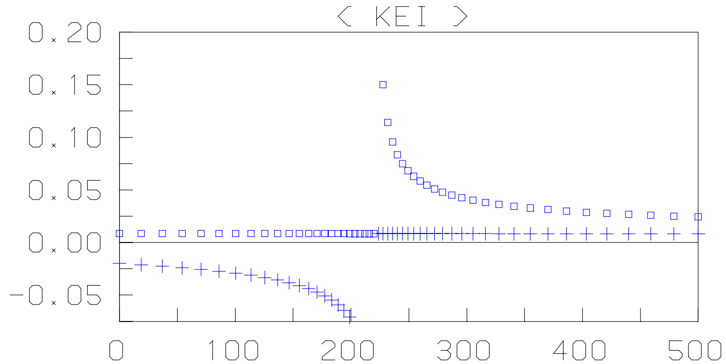
$$U_n(\xi) = \frac{1}{\sqrt{2\pi}}\int_0^\infty \frac{\mathrm{d}\theta}{\theta^{n+1}}\exp\left[-\frac{1}{2}\frac{\xi^2}{\theta^2} + \mathrm{i}\,\theta\right]$$

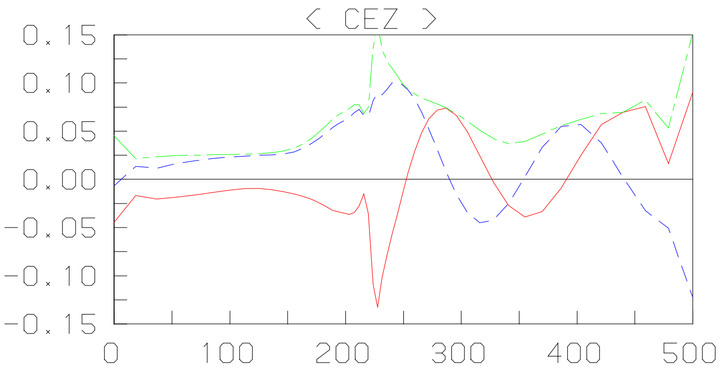
# Typical Results: L+R Injection

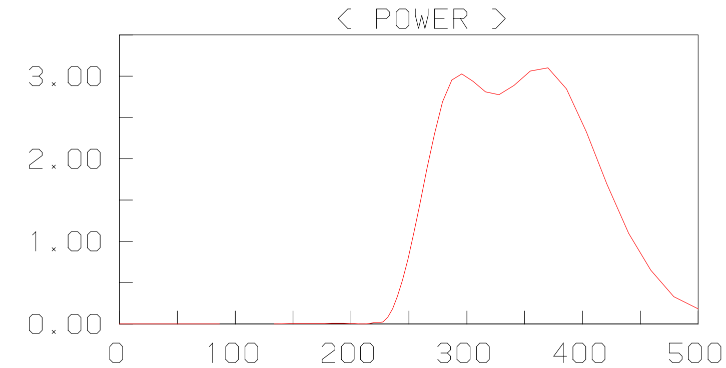



```
NZ     =        50   NW     =        50
PN0    =   0.50000   ZMAX   =  500.0000
PB0    =   0.90000   DZMAX  =    5.0000
PB1    =   0.75150   DZWID  =   50.0000
AKX    =   0.10000
BETA   =   0.01000
IN     =  0.0000 0.0000 1.0000 1.0000
REFL   =  0.9515 0.0000 0.0003 0.0002
P-IN   =  0.0000 0.0000 0.7509 0.2491
P-REF  =  0.2424 0.0000 0.0002 0.0000
P-ABS  =  0.7574  P-IN -0.2424 0.9997
```



# Summary

- **Full wave code TASK/WM has been successfully applied to describe ICRF heating and Alfvén eigenmode in toroidal plasmas, both tokamaks and helical systems.**
- **FEM full wave code PAF/WF was extended to 3D configurations. FEM using ridge variables completely suppress unphysical spurious modes.**
- **Integro-differential code successfully describe wave-particle interactions in an inhomogeneous plasma, Landau and cyclotron damping and finite gryoradius effects.**
- **Future works**
  - Improved coupling among TASK modules
  - Kinetic analysis of MHD stability by full wave code
  - 3D integlo-differential full wave analysis
  - Self-consistent simulation of RF plasma production