47th Annual Meeting of APS-DPP
Denver, CO, USA
QP1 39 2005/10/27

Self-Consistent Full-Wave Simulation of Wave Heating and Current drive in Tokamak Plasmas

A. Fukuyama, Y. Yamahana and A. Sonoda
Department of Nuclear Engineering, Kyoto University

Outline

- TASK: **T**ransport **A**nalysing **S**ystem for tokama**K**
- Full Wave Analysis of EC Waves in a Small Tokamak
- Integro-Differential Full Wave Analysis including FLR effects
- Present Status of Self-Consistent RF Simulation
- Summary

TASK Code

- **Transport Analysing System for TokamaK**
- **Features**
  - **A Core of Integrated Modeling Code in BPSI**
    - Modular Structure
    - Reference Data Interface
  - **Various Heating and Current Drive Scheme**
    - EC, LH, IC, AW, (NB)
  - **High Portability**
    - Most of Library Routines Included (except LAPACK and MPI)
    - Own Graphic Libraries (gsaf, gsgl)
  - **Development using CVS** (Concurrent Version System)
    - Open Source (by the end of October 2005)
  - **Parallel Processing using MPI Library**
  - **Extension to Toroidal Helical Plasmas**

Modules of TASK

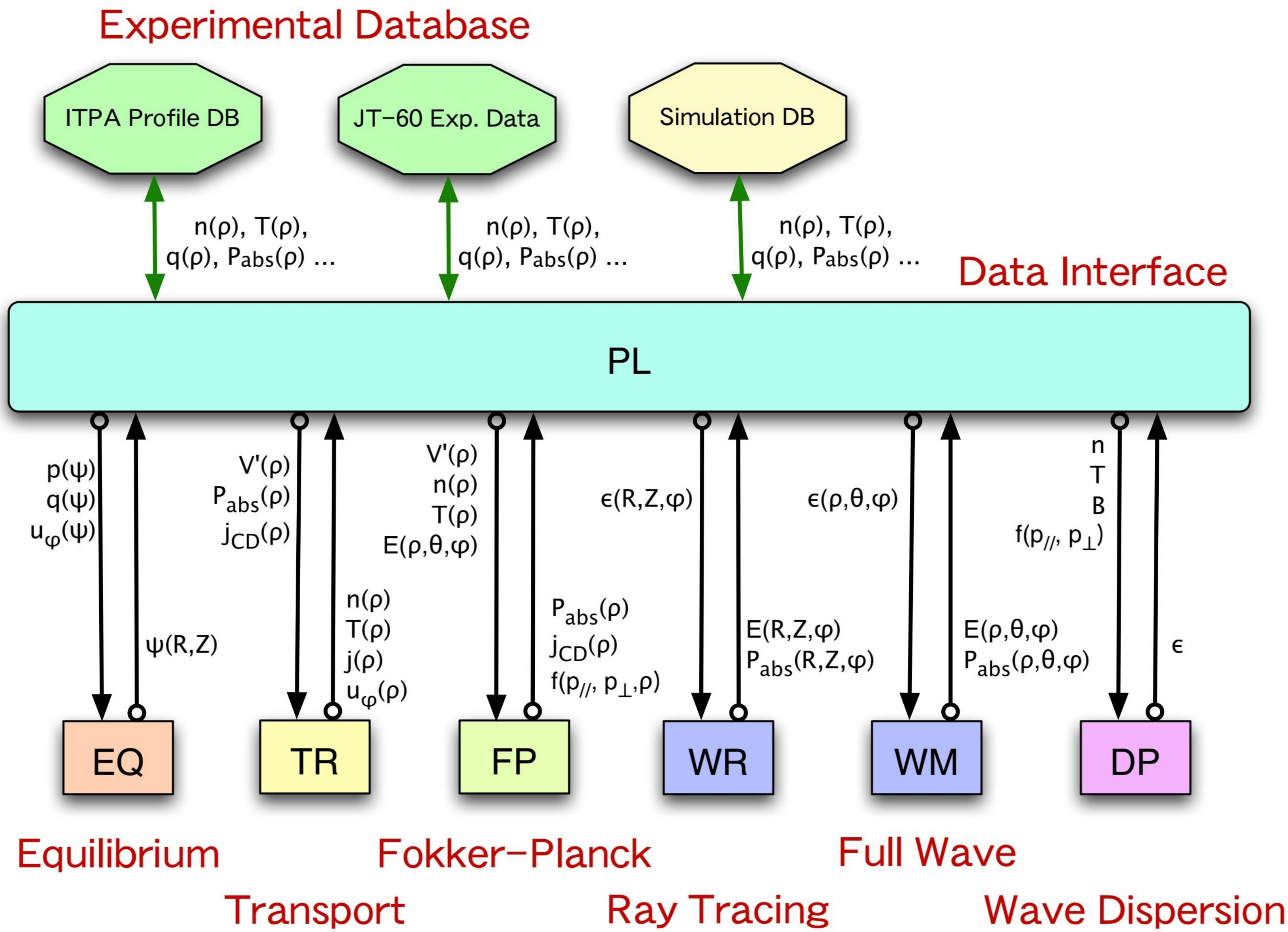
EQ	**2D Equilibrium**	Fixed boundary, Toroidal rotation
TR	**1D Transport**	Diffusive Transport, Transport models
WR	**3D Geometr. Optics**	EC, LH: Ray tracing, Beam tracing
WM	**3D Full Wave**	IC, AW: Antenna excitation, Eigen mode
FP	**3D Fokker-Planck**	Relativistic, Bounce-averaged
DP	**Wave Dispersion**	Local dielectric tensor, Arbitrary $f(\boldsymbol{v})$
PL	**Data Interface**	Data conversion, Profile database
LIB	**Libraries**	

Associated Libraries

GSAF	2D Graphic library for X Window and EPS
GSGL	3D Graphic library using OpenGL

All developed in Kyoto U

Modular Structure of TASK



Under Development

- **New Modules**
  - `EX`: 2D equilibrium with free boundary
  - `TX`: Transport analysis based on flux-averaged fluid equation
  - `WA`: Global linear stability analysis
  - `WI`: Integro-differential wave analysis (FLR, $\boldsymbol{k} \cdot \nabla B \neq 0$)
- **Extension to 3D Helical System**
  - **3D Data Structure**
  - **3D Equilibrium**: VMEC, HINT
  - **Wave Analysis**: Already 3D
  - **Transport Analysis**: New transport model
- **Open Source**: **http://bpsi.nucleng.kyoto-u.ac.jp/task/**

Wave Dispersion Analysis : TASK/DP

- **Various Models of Dielectric Tensor** $\overleftrightarrow{\epsilon}(\omega, \boldsymbol{k}; \boldsymbol{r})$:
  - **Resistive MHD** model
  - **Collisional cold** plasma model
  - **Collisional warm** plasma model
  - **Kinetic plasma** model (**Maxwellian**, non-relativistic)
  - **Kinetic plasma** model (**Arbitrary** $f(\boldsymbol{v})$, relativistic)
  - **Gyro-kinetic plasma** model (Maxwellian)
- **Numerical Integration in momentum space**: **Arbitrary** $f(\boldsymbol{v})$
  - Relativistic Maxwellian
  - Output of TASK/FP: Fokker-Planck code

Full wave analysis: TASK/WM

- **magnetic surface coordinate**: (ψ, θ, φ)
- Boundary-value problem of **Maxwell's equation**

$$\nabla \times \nabla \times \boldsymbol{E} = \frac{\omega^2}{c^2} \overleftrightarrow{\epsilon} \cdot \boldsymbol{E} + \mathrm{i}\, \omega \mu_0 \boldsymbol{j}_{\mathrm{ext}}$$

- Kinetic **dielectric tensor**: $\overleftrightarrow{\epsilon}$
  - **Wave-particle resonance**: $Z[(\omega - n\omega_{\mathrm{c}})/k_{\|} v_{\mathrm{th}}]$
  - **Fast ion: Drift-kinetic**

$$\left[\frac{\partial}{\partial t} + v_{\|} \nabla_{\|} + (\boldsymbol{v}_{\mathrm{d}} + \boldsymbol{v}_{\mathrm{E}}) \cdot \nabla + \frac{e_\alpha}{m_\alpha} (v_{\|} E_{\|} + \boldsymbol{v}_{\mathrm{d}} \cdot \boldsymbol{E}) \frac{\partial}{\partial \varepsilon} \right] f_\alpha = 0$$

- Poloidal and toroidal **mode expansion**
  - **Accurate estimation of $k_{\|}$**
- Eigenmode analysis: **Complex eigen frequency** which maximize wave amplitude for fixed excitation proportional to electron density

Fokker-Planck Analysis : TASK/FP

- **Fokker-Planck equation**
  for **velocity distribution function** $f(p_{\|}, p_{\perp}, \psi, t)$

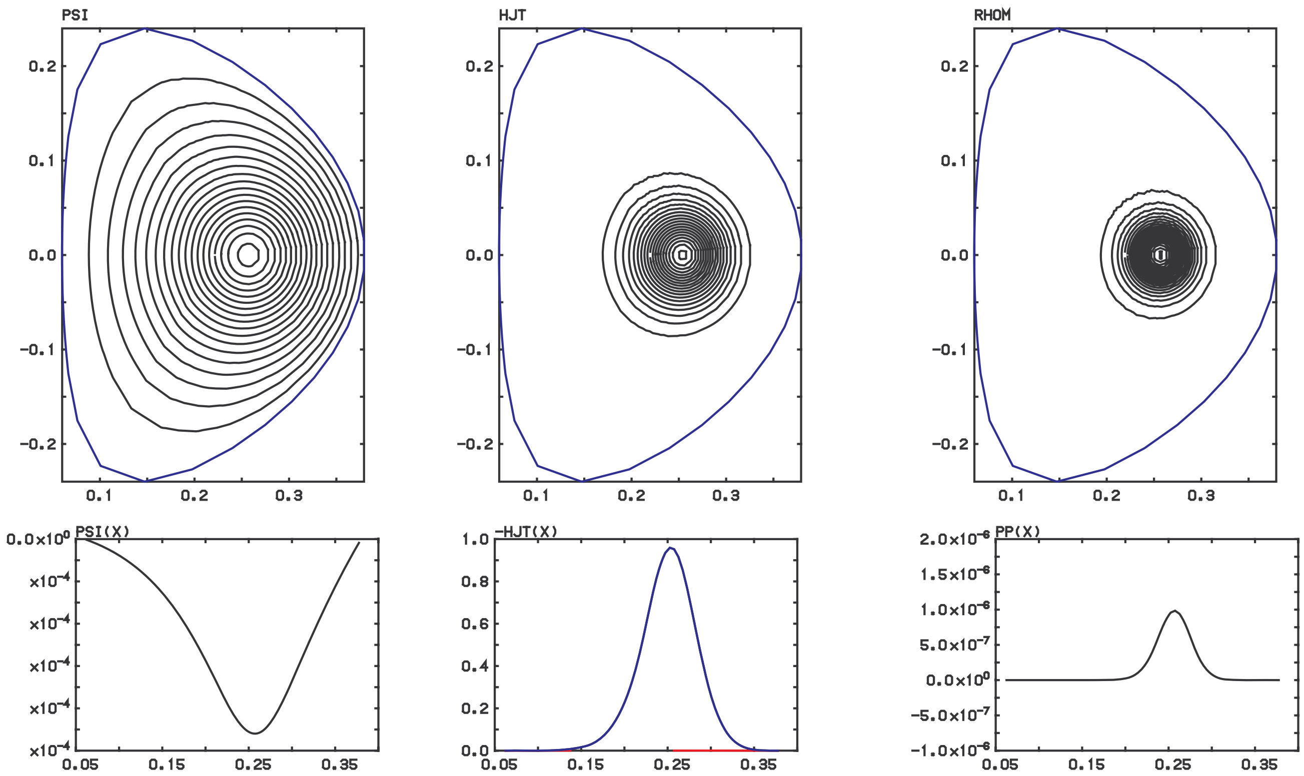
$$\frac{\partial f}{\partial t} = E(f) + C(f) + Q(f) + L(f)$$

  - $E(f)$: Acceleration term due to DC electric field
  - $C(f)$: Coulomb collision term
  - $Q(f)$: Quasi-linear term due to wave-particle resonance
  - $L(f)$: Spatial diffusion term
- **Bounce-averaged**: Trapped particle effect, zero banana width
- **Relativistic**: momentum $\boldsymbol{p}$, weakly relativistic collision term
- **Nonlinear collision**: momentum or energy conservation
- **Three-dimensional**: spatial diffusion (neoclassical, turbulent)

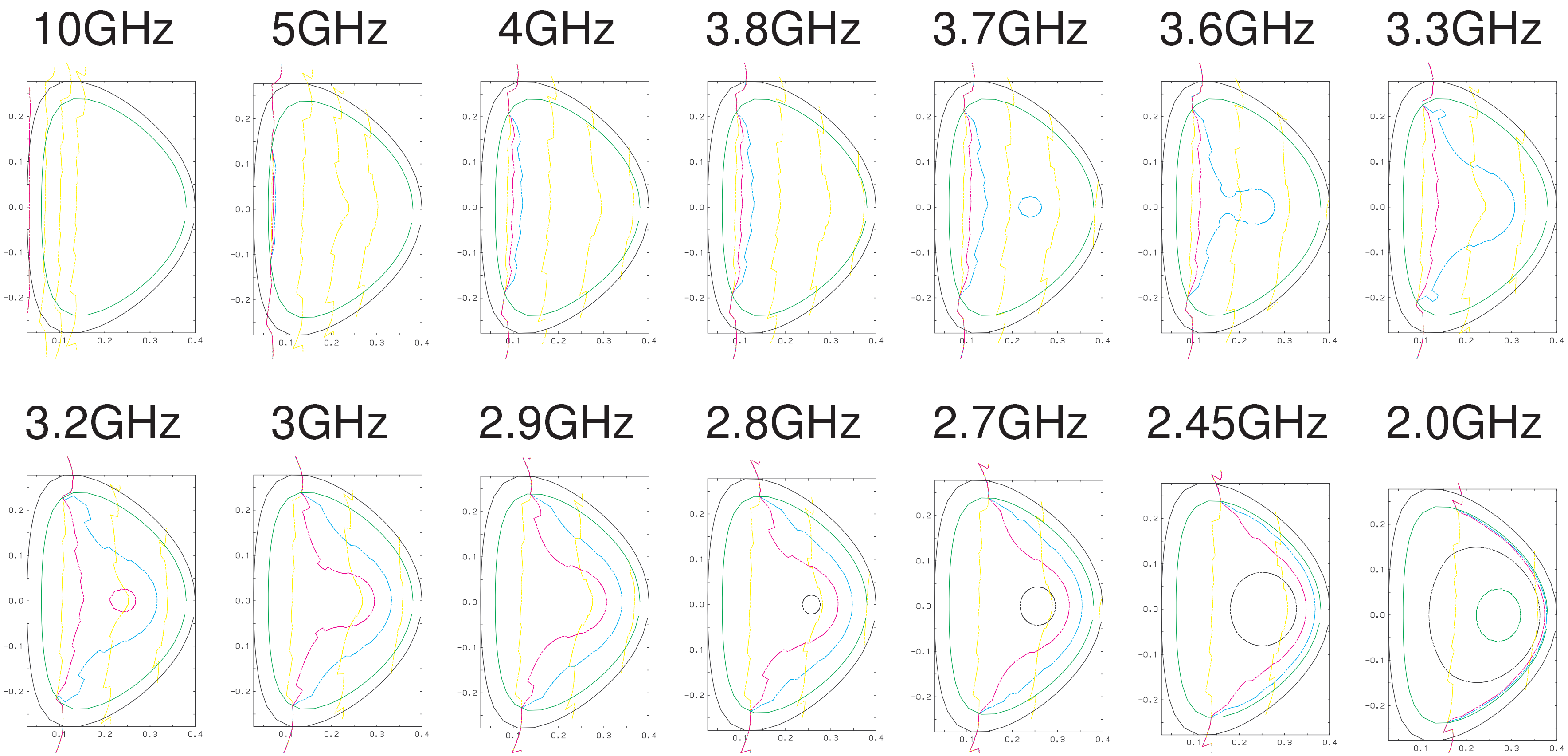
Full Wave Analysis of ECH in Small-Size ST (1)

- **Small-size spherical tokamak: LATE** (Kyoto University)
  - **T. Maekawa et al., Proc. 20th IAEA Fusion Energy Conf., IAEA-CN-116/EX/P4-27 (Vilamoura, Portuga, 2004)**
  - $R = 0.22\,\mathrm{m}$, $a = 0.16\,\mathrm{m}$, $B_0 = 0.0552\,\mathrm{T}$, $I_p = 6.25\,\mathrm{kA}$, $\kappa = 1.5$

Poloidal Flux Current Density Number Density



Full Wave Analysis of ECH in Small-Size ST (2)

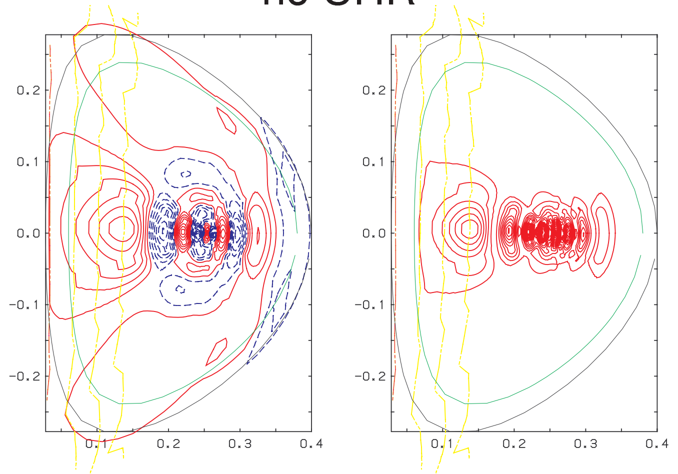


Line color	Large Location
Yellow	n-th EC harmonics
Violet	Upper hybrid resonance
Blue	RHS cyclotron cutoff
Green	LHS cyclotron cutoff
Black	Plasma cutoff / resonance

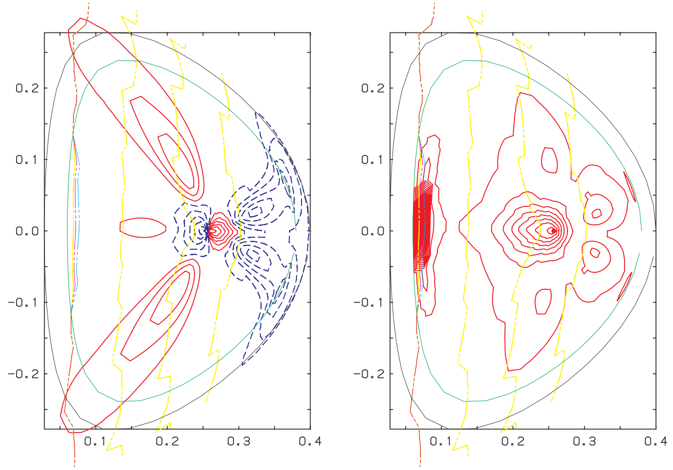
Full Wave Analysis of ECH in Small-Size ST (3)

- **Standard density:** $10^{17}\,\mathrm{m}^{-3}$: **Extra-ordinary wave excitation**

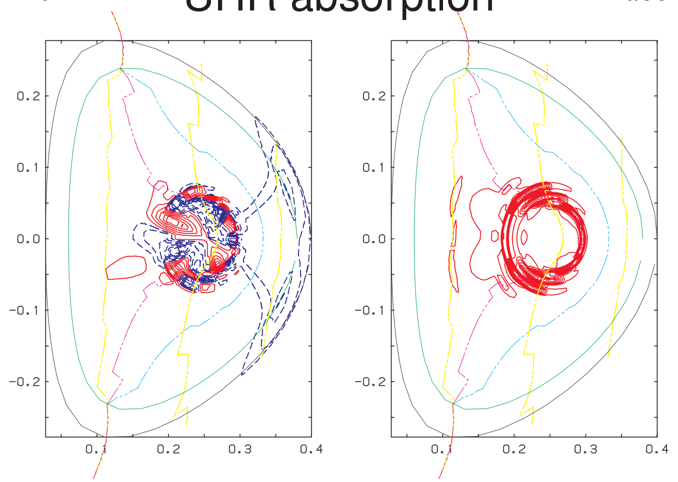
10GHz
no UHR



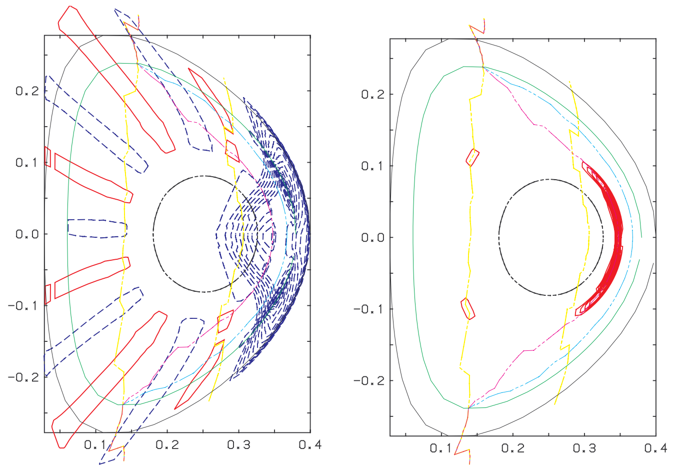
5GHz
UHR on HFS



3GHz
UHR absorption

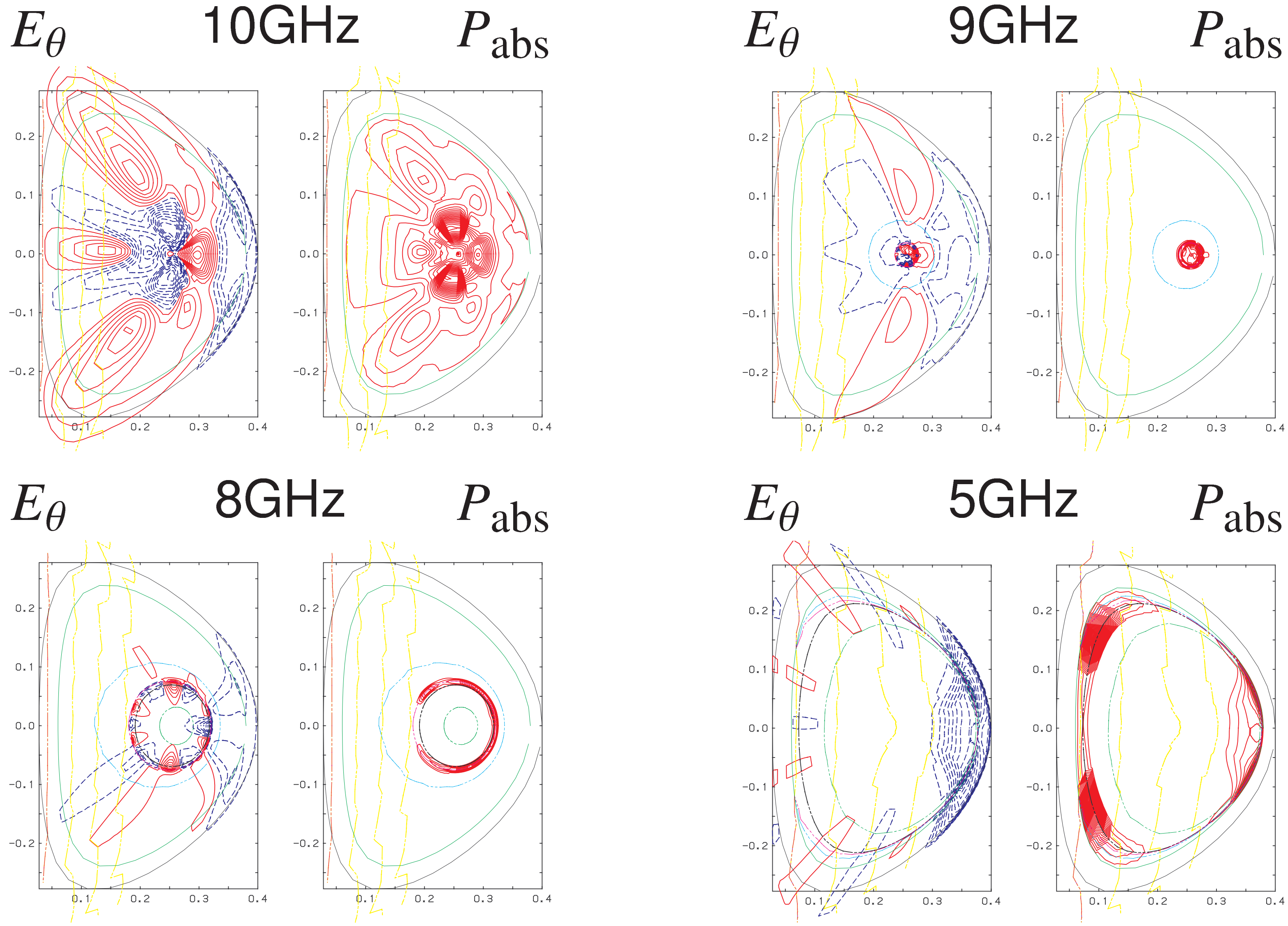


2.45GHz
UHR on LFS



Full Wave Analysis of ECH in Small-Size ST (4)

- **High density:** $10^{18}\,\mathrm{m}^{-3}$: **Extra-ordinary wave excitation**



Integro-Differential Full Wave Analysis

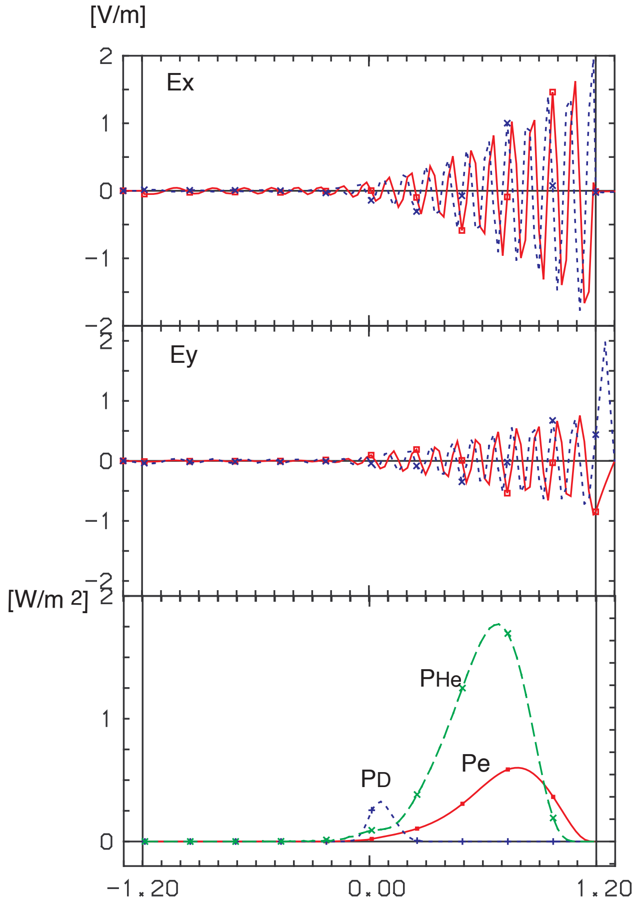
- **Purpose**
  - **FLR effects**
    - — **Existence of energetic particles**
    - — **Cyclotron higher harmonics**
  - **Inhomogeneous k along k**
    - — Landau damping in inhomogeneous plasmas
    - — Cyclotron resonance along the field line

- **Present analysis: FLR effects**
  - **ICRF**: Analysis of absorption by α particles
  - **ECRF**: Analysis of electron Bernstein waves

1D Full Wave Analysis including FLR Effects

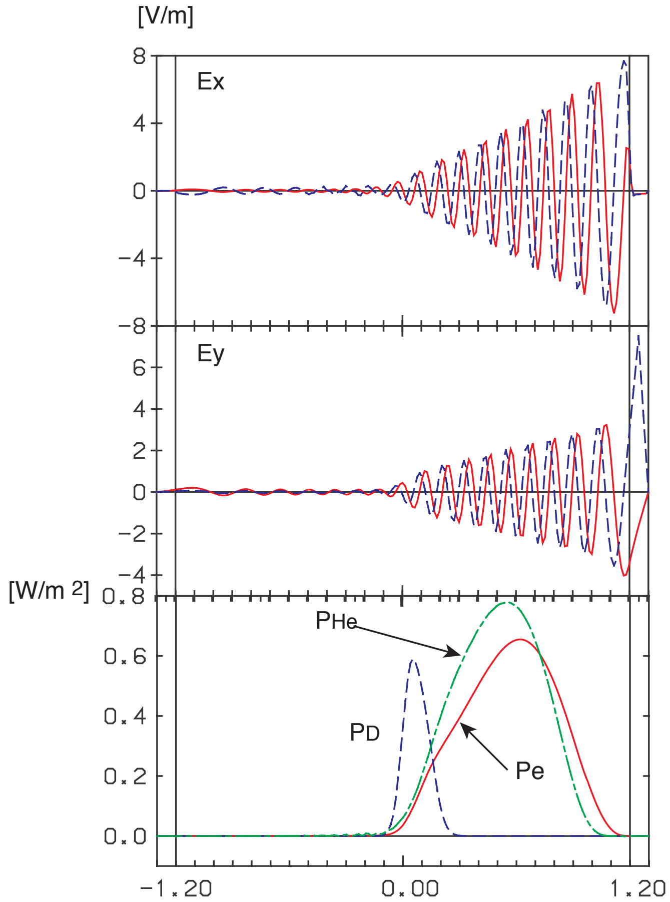
- **Absorption of ICRF wave by α particles:**
  O. Sauter, J. Vaclavik (1992), Y. Uetani, A. Fukuyama (1998)

Differential analysis up to $k_\perp^2 \rho_H^2$



Overestimate α absorption

Integro-Differential Analysis



2D Formulation in Tokamaks (1)

- **Induced current**

$$\begin{aligned} \boldsymbol{J}_{\text{ind}}(\boldsymbol{r},t) &= \int \mathrm{d}\boldsymbol{v} q \boldsymbol{v} f(\boldsymbol{r},\boldsymbol{v},t) \\ &= -\frac{q}{m}\int \mathrm{d}\boldsymbol{v} q\boldsymbol{v} \int_0^\infty \mathrm{d}\tau \left[\boldsymbol{E}(\boldsymbol{r}',t-\tau) + \boldsymbol{v}'\times\boldsymbol{B}(\boldsymbol{r}',t-\tau)\right]\cdot\frac{\partial f_0(\boldsymbol{r}',\boldsymbol{v}')}{\partial \boldsymbol{v}'} \end{aligned}$$

- **Wave electric field**

$$\boldsymbol{E}(\boldsymbol{r},t) = \sum_{MN} \boldsymbol{E}_{MN}(\boldsymbol{r}) \exp\left\{iM\theta + iN\phi - i\omega t\right\}$$

- **Equilibrium distribution function**

$$f_0(r_0,\boldsymbol{v}) = n_0(r_0)\left(\frac{m}{2\pi T_\perp(r_0)}\right)^{\frac{3}{2}}\left(\frac{T_\perp(r_0)}{T_\parallel(r_0)})\right)\exp\left\{-\frac{mv_\perp^2}{2T_\perp(r_0)} - \frac{mv_\parallel^2}{2T_\parallel(r_0)}\right\}$$

2D Formulation in Tokamaks (2)

- **Induced current**

$$\boldsymbol{J}_{\rm ind}(\boldsymbol{r},t_0) = \int {\rm d}v_\parallel \int {\rm d}v_\perp \int {\rm d}\psi \int_0^\infty {\rm d}\tau(-q^2)\exp i\omega\tau$$

$$\cdot\ n_0\left(\frac{m}{2\pi T_\perp}\right)^{\frac{3}{2}}\left(\frac{T_\perp}{T_\parallel}\right)^{\frac{1}{2}}\exp\left\{-\frac{mv_\perp^2}{2T_\perp}-\frac{mv_\parallel^2}{2T_\parallel}\right\}\overleftrightarrow{G}_0$$

$$\cdot\ \sum_{M,N}\begin{pmatrix}E_{r'MN}\\ E_{\theta'MN}\\ E_{\phi'MN}\end{pmatrix}\exp i\left[M\theta+N\phi-\omega t_0-k_\parallel v_\parallel\tau+\omega\tau\right]$$

$$\times \sum_{n_1,n_2=-\infty}^{\infty} J_{n_1}\left(\frac{k_\perp v_\perp}{\Omega}\right)J_{n_2}\left(\frac{k_\perp v_\perp}{\Omega}\right)\exp i\left\{n_1(\Omega\tau-\theta_0+\psi)-n_2(-\theta_0+\psi)\right\}$$

$$\overleftrightarrow{G}_0 = \begin{pmatrix} v_\perp^2\sin(-\theta_0+\psi)\\ -v_\perp^2\cos(-\theta_0+\psi)\\ v_\parallel v_\perp\end{pmatrix}\cdot\begin{pmatrix} -\dfrac{v_\perp}{T_\perp}\sin(\Omega\tau-\theta_0+\psi)+v_\perp v_\parallel\left(\dfrac{1}{T_\perp}-\dfrac{1}{T_\parallel}\right)\dfrac{N}{R\omega}\sin(\Omega\tau-\theta_0+\psi)\\ \dfrac{v_\perp}{T_\perp}\cos(\Omega\tau-\theta_0+\psi)-v_\perp v_\parallel\left(\dfrac{1}{T_\perp}-\dfrac{1}{T_\parallel}\right)\dfrac{N}{R\omega}\cos(\Omega\tau-\theta_0+\psi)\\ -\dfrac{v_\parallel}{T_\parallel}+v_\perp v_\parallel\left(\dfrac{1}{T_\perp}-\dfrac{1}{T_\parallel}\right)\left\{\dfrac{M}{r_0\omega}\cos(\Omega\tau-\theta_0+\psi)+\dfrac{i}{\omega}\sin(\Omega\tau-\theta_0+\psi)\dfrac{\partial}{\partial r'}\right\}\end{pmatrix}$$

2D Formulation in Tokamaks (3)

- **Variable transformation**
  - Velocity $(v_\perp, \psi) \rightarrow$ Position (r', r_0)

$$\begin{cases} r = r_0 + \dfrac{v_\perp}{\Omega}\cos(-\theta_0 + \psi) \\ r' = r_0 + \dfrac{v_\perp}{\Omega}\cos(\Omega\tau - \theta_0 + \psi) \end{cases}$$

  - Jacobian: $J = \dfrac{\partial(v_\perp, \psi)}{\partial(r', r_0)} = -\dfrac{\Omega^2}{v_\perp \sin\Omega\tau}$
  - Then

$$\int_0^\infty \mathrm{d}v_\perp \int_0^{2\pi} \mathrm{d}\psi = \int_0^R \mathrm{d}r' \int_0^R \mathrm{d}r_0\, |J| = \int_0^R \mathrm{d}r' \int_0^R \mathrm{d}r_0 \frac{\Omega^2}{v_\perp |\sin\Omega\tau|}$$

- **Fourier expansion with respect to $\Omega\tau$**
- **Time integral with respect to τ**
- **Velocity integral with respect to $v_\parallel$**

2D Formulation in Tokamaks (4)

- **Finally we obtain:**

$$\boldsymbol{J}_{\text{ind}}(\boldsymbol{r},t) = \sum_{MN} \int_0^R \mathrm{d}r' \overleftrightarrow{\sigma}_{MN}(r,r')\boldsymbol{E}_{MN}(r') \exp i\{M\theta + N\phi - \omega t\}$$

$$\begin{aligned}
\overleftrightarrow{\sigma}_{MN}(r,r') &= -\sum_{n_1 n_2}\sum_{\ell=-\infty}^{\infty}\int_0^R \mathrm{d}r_0\, q^2\Omega^2 n_0 \left(\frac{1}{2\pi}\right)^{\frac{5}{2}} \frac{i}{v_{T\perp}^2} \\
&\times \int_0^{2\pi} \mathrm{d}\lambda \frac{1}{|\sin\lambda|} \exp\left\{-\frac{R_v^2\Omega^2}{2v_{T\perp}^2}\right\} \overleftrightarrow{G}_2 J_{n_1}(k_\perp R_v) J_{n_2}(k_\perp R_v) \\
&\times \exp(-in_1\lambda)\left\{\frac{r - r_0 + iR_m}{R_v}\right\}^{n_1-n_2} \mathrm{e}^{i\ell\lambda}
\end{aligned}$$

$$\begin{aligned}
G_{rr} &= A_1 R_v^2 \\
G_{r\theta} &= -(r'-r_0)A_1 R_m \\
G_{r\phi} &= R_m\left[A_2 + A_3\left\{\frac{M}{r'}(r'-r_0) + iR_p\frac{\partial}{\partial r'}\right\}\right] \\
G_{\theta r} &= -(r-r_0)A_1 R_p
\end{aligned}$$

$$G_{\theta\theta} = A_1(r - r_0)(r' - r_0)$$

$$G_{\theta\phi} = -(r - r_0)\left[A_2 + A_3\left\{\frac{M}{r'}(r' - r_0) + iR_p\frac{\partial}{\partial r'}\right\}\right]$$

$$G_{\phi r} = \left\{A_2 + \frac{1}{k_{\|}}\frac{\omega - \ell\Omega}{\Omega}\frac{N}{R}A_3\right\}R_p$$

$$G_{\phi\theta} = -\left\{A_2 + \frac{1}{k_{\|}}\frac{\omega - \ell\Omega}{\Omega}\frac{N}{R}A_3\right\}(r' - r_0)$$

$$G_{\phi\phi} = \frac{1}{k_{\|}}\frac{\omega - \ell\Omega}{\Omega}\left[A_2 + A_3\left\{\frac{M}{r'}(r' - r_0) + iR_p\frac{\partial}{\partial r'}\right\}\right]$$

where A_1, A_2, A_3, R_v. R_p. R_m are defined by

$$A_1 \equiv \frac{\Omega^2}{T_\perp}\frac{\sqrt{\pi}}{k_{\|}v_{T_{\|}}}Z(\eta_\ell) + \sqrt{\frac{\pi}{2}}\frac{\Omega^2}{k_{\|}}Z'(\eta_\ell)\left(\frac{1}{T_\perp} - \frac{1}{T_{\|}}\right)\frac{N}{R\omega}$$

$$A_2 \equiv -\sqrt{\frac{\pi}{2}}\frac{\Omega}{k_{\|}T_{\|}}Z'(\eta_\ell)$$

$$A_3 \equiv \sqrt{\frac{\pi}{2}}\frac{\Omega^2}{k_{\|}\omega}Z'(\eta_\ell)\left(\frac{1}{T_\perp} - \frac{1}{T_{\|}}\right)$$

$$R_v^2 = \left(\frac{r + r'}{2} - r_0\right)^2\frac{1}{\cos^2\frac{1}{2}\lambda} + \left(\frac{r - r'}{2}\right)^2\frac{1}{\sin^2\frac{1}{2}\lambda}$$

$$R_p = \frac{r - r'}{2}\frac{1}{\tan\frac{1}{2}\lambda} + \left(\frac{r + r'}{2} - r_0\right)\tan\frac{1}{2}\lambda$$

$$R_m = \frac{r - r'}{2}\frac{1}{\tan\frac{1}{2}\lambda} - \left(\frac{r + r'}{2} - r_0\right)\tan\frac{1}{2}\lambda$$

and

$$\eta_\ell \equiv \frac{\omega - \ell\Omega}{k_{\|}v_{\mathrm{T}\|}}, \quad k_{\|} \equiv \frac{M}{r_0}\frac{B_\theta}{B} + \frac{N}{R}\frac{B_\phi}{B}, \quad k_\perp \equiv \frac{M}{r_0}\frac{B_\phi}{B} - \frac{N}{R}\frac{B_\theta}{B}$$

Code development using FEM is under way

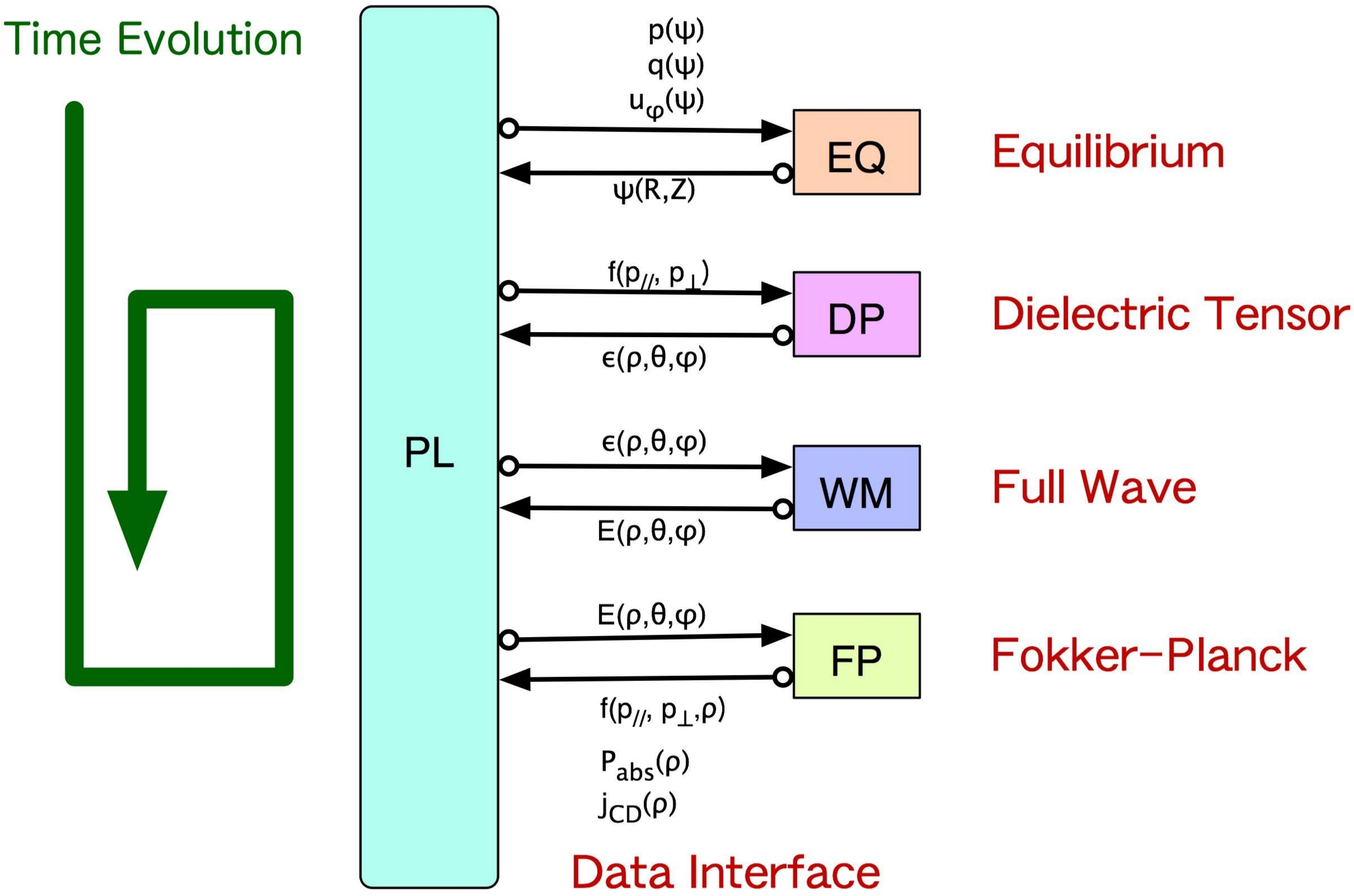
Self-Consistent Full Wave Analysis (1)

- **Deviation of velocity distribution from Maxwellian may strongly affect**
  - Power absorption of ICRF waves in the presence of energetic ions
  - Current drive efficiency of LHCD
  - NTM controllability of ECCD

- **Systematic analyses including the modification of velocity distribution by TASK code is under way.**
  - **Full wave analysis with arbitrary velocity distribution**
  - **Bounce averaged Fokker-Plank analysis**

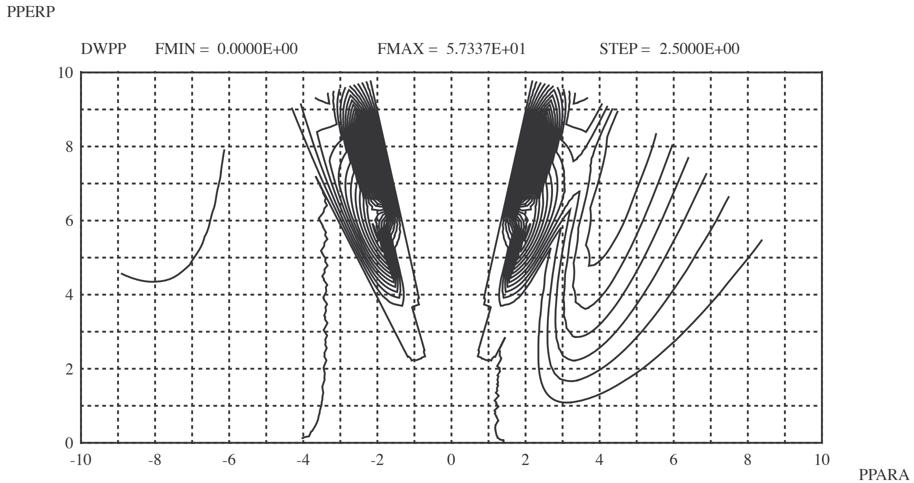
- **Upgrade of Fokker-Planck module is not completed yet.**

Self-consistent Full Wave Analysis (2)

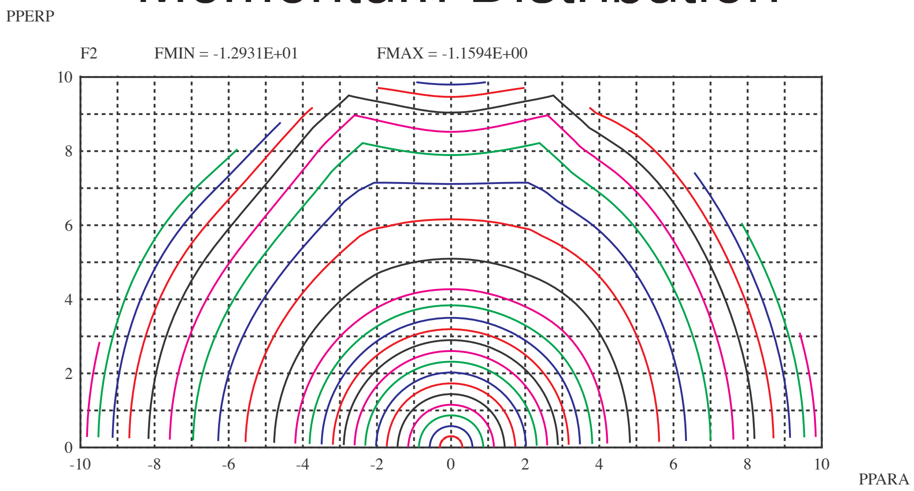


Fast Ion Tail Formation by ICRF

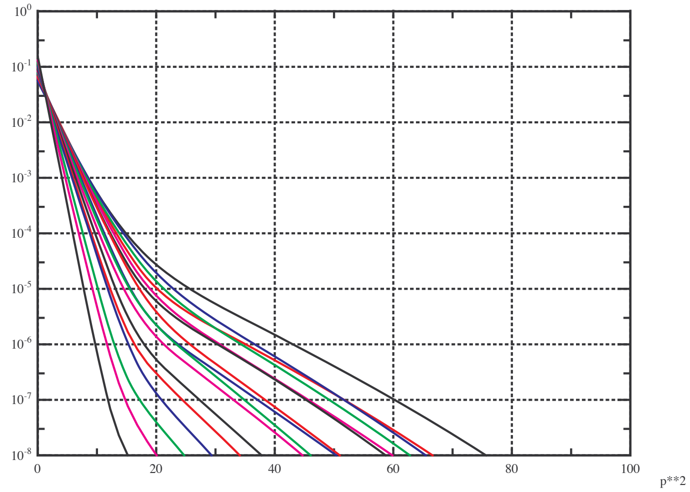
Quasi-linear Diffusion



Momentum Distribution



Tail Formation



Summary

- **Several improvement of the TASK code for full wave analysis of wave heating and current drive is under way.**
- **Full wave analysis of EC wave propagation in a small-size ST**
  - Tunneling through the cutoff layer and absorption on the upper hybrid layer were described.
  - The description of electron Bernstein waves requires to include FLR effects in TASK/WM.
- **Formulation of 2D integro-differential full wave analysis including FLR effects**
  - Formulation was extended to 2D configuration.
  - Implementation is under way.
  - Behavior of power absorption should be examined.

- **Self-consistent analysis including modification of velocity distribution**
  - Full wave analysis with arbitrary velocity distribution was completed.
  - Fokker-Planck analysis can use wave electric fields calculated by the full wave module.
  - Coupling of the full-wave module and the Fokker-Planck module is almost completed.