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Integrated Analysis of Alfvén Eigenmode in Toroidal Plasmas

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Outline

- Motivation
- TASK: **T**ransport **A**nalysing **S**ystem for tokama**K**
- Analysis of Alfvén eigenmodes in ITER plasmas
- Present status of self-consistent full wave analysis
- Summary

Motivation

- **Stability of Alfvén eigenmode is sensitive to**
  - **Destabilization mechanisms**:
    — Spatial and velocity distribution of energetic particles
  - **Stabilization mechanisms**:
    — Mode structure (q profile)
    — Distribution of bulk particles
- **Integrated analysis is required for consistent analysis**:
  - Coupling with transport analysis
  - Interaction with velocity distribution
  - Finite orbit size effects

TASK Code

- **Transport Analysing System for TokamaK**
- **Features**
  - **A Core of Integrated Modeling Code in BPSI**
    — Modular Structure
    — Reference Data Interface
  - **Various Heating and Current Drive Scheme**
    — EC, LH, IC, AW, (NB)
  - **High Portability**
    — Most of Library Routines Included (except LAPACK and MPI)
    — Own Graphic Libraries (gsaf, gsgl)
  - **Development using CVS** (Concurrent Version System)
    — Open Source (Pre-release at the end of October 2005)
  - **Parallel Processing using MPI Library**
  - **Extension to Toroidal Helical Plasmas**

Modules of TASK

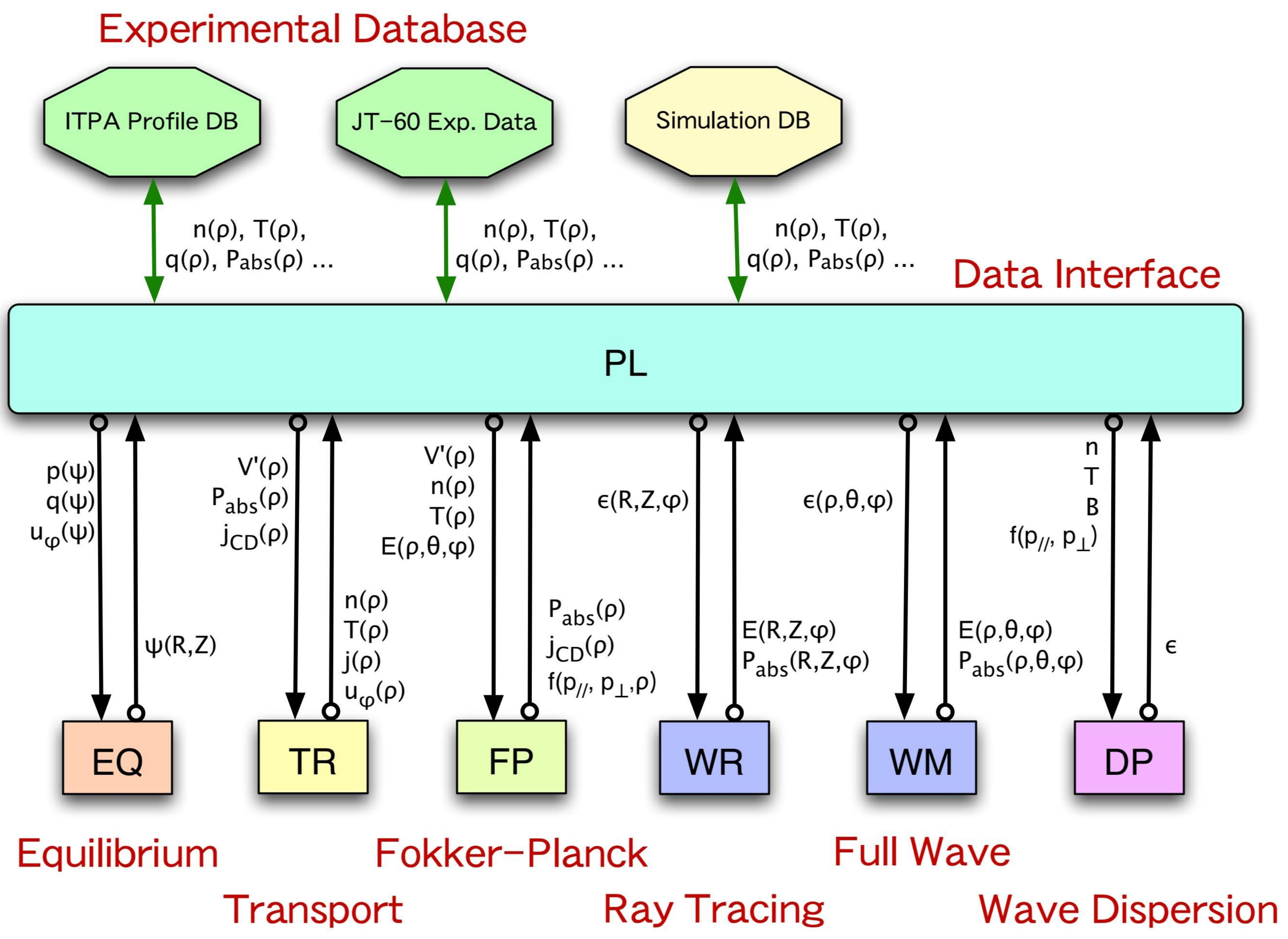
EQ	**2D Equilibrium**	Fixed boundary, Toroidal rotation
TR	**1D Transport**	Diffusive Transport, Transport models
WR	**3D Geometr. Optics**	EC, LH: Ray tracing, Beam tracing
WM	**3D Full Wave**	IC, AW: Antenna excitation, Eigen mode
FP	**3D Fokker-Planck**	Relativistic, Bounce-averaged
DP	**Wave Dispersion**	Local dielectric tensor, Arbitrary $f(\boldsymbol{v})$
PL	**Data Interface**	Data conversion, Profile database
LIB	**Libraries**	

Associated Libraries

GSAF	2D Graphic library for X Window and EPS
GSGL	3D Graphic library using OpenGL

All developed in Kyoto U

Modular Structure of TASK



Under Development

- **New Modules**
  - **EX**: 2D equilibrium with free boundary
  - **TX**: Transport analysis based on flux-averaged fluid equation
  - **WA**: Global linear stability analysis
  - **WI**: Integro-differential wave analysis (FLR, $\boldsymbol{k} \cdot \nabla B \neq 0$)
- **Extension to 3D Helical System**
  - **3D Data Structure**
  - **3D Equilibrium**: VMEC, HINT
  - **Wave Analysis**: Already 3D
  - **Transport Analysis**: New transport model
- **Open Source**:
  - **http://bpsi.nucleng.kyoto-u.ac.jp/task/**

Wave Dispersion Analysis : TASK/DP

- **Various Models of Dielectric Tensor** $\overleftrightarrow{\epsilon}(\omega, \boldsymbol{k}; \boldsymbol{r})$:
  - **Resistive MHD** model
  - **Collisional cold** plasma model
  - **Collisional warm** plasma model
  - **Kinetic plasma** model (**Maxwellian**, non-relativistic)
  - **Kinetic plasma** model (**Arbitrary** $f(\boldsymbol{v})$, relativistic)
  - **Gyro-kinetic plasma** model (**Maxwellian**, no magnetic drift)
  - \- Gyro-kinetic plasma model (Arbitrary $f(\boldsymbol{v})$, non-relativistic)

- **Numerical Integration in momentum space**: **Arbitrary** $f(\boldsymbol{v})$
  - Relativistic Maxwellian
  - Velocity dependent magnetic drift terms
  - Output of TASK/FP: Fokker-Planck code

Full wave analysis: TASK/WM

- **magnetic surface coordinate**: (ψ, θ, φ)
- Boundary-value problem of **Maxwell's equation**

$$\nabla \times \nabla \times \boldsymbol{E} = \frac{\omega^2}{c^2} \overleftrightarrow{\epsilon} \cdot \boldsymbol{E} + \mathrm{i}\,\omega\mu_0 \boldsymbol{j}_{\mathrm{ext}}$$

- Kinetic **dielectric tensor**: $\overleftrightarrow{\epsilon}$
  - **Wave-particle resonance**: $Z[(\omega - n\omega_{\mathrm{c}})/k_{\|}v_{\mathrm{th}}]$
  - **Fast ion: Drift-kinetic**

$$\left[\frac{\partial}{\partial t} + v_{\|}\nabla_{\|} + (\boldsymbol{v}_{\mathrm{d}} + \boldsymbol{v}_{\mathrm{E}}) \cdot \nabla + \frac{e_\alpha}{m_\alpha}(v_{\|}E_{\|} + \boldsymbol{v}_{\mathrm{d}} \cdot \boldsymbol{E})\frac{\partial}{\partial \varepsilon}\right] f_\alpha = 0$$

- Poloidal and toroidal **mode expansion**
  - **Accurate estimation of $k_{\|}$**
- Eigenmode analysis: **Complex eigen frequency** which maximize wave amplitude for fixed excitation proportional to electron density

Fokker-Planck Analysis : TASK/FP

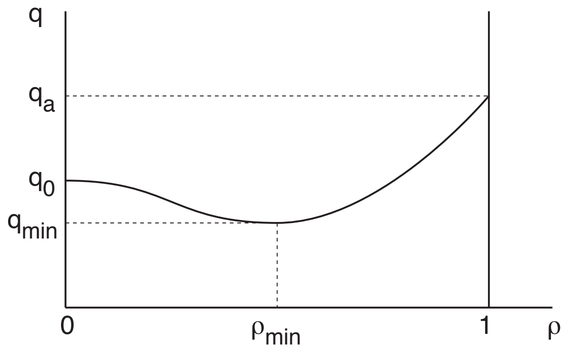
- **Fokker-Planck equation**
  for **velocity distribution function** $f(p_{||}, p_\perp, \psi, t)$

$$\frac{\partial f}{\partial t} = E(f) + C(f) + Q(f) + L(f)$$

  - $E(f)$: Acceleration term due to DC electric field
  - $C(f)$: Coulomb collision term
  - $Q(f)$: Quasi-linear term due to wave-particle resonance
  - $L(f)$: Spatial diffusion term
- **Bounce-averaged**: Trapped particle effect, zero banana width
- **Relativistic**: momentum $\boldsymbol{p}$, weakly relativistic collision term
- **Nonlinear collision**: momentum or energy conservation
- **Three-dimensional**: spatial diffusion (neoclassical, turbulent)

Analysis of TAE in Reversed Shear Configuration

Assumed q profile



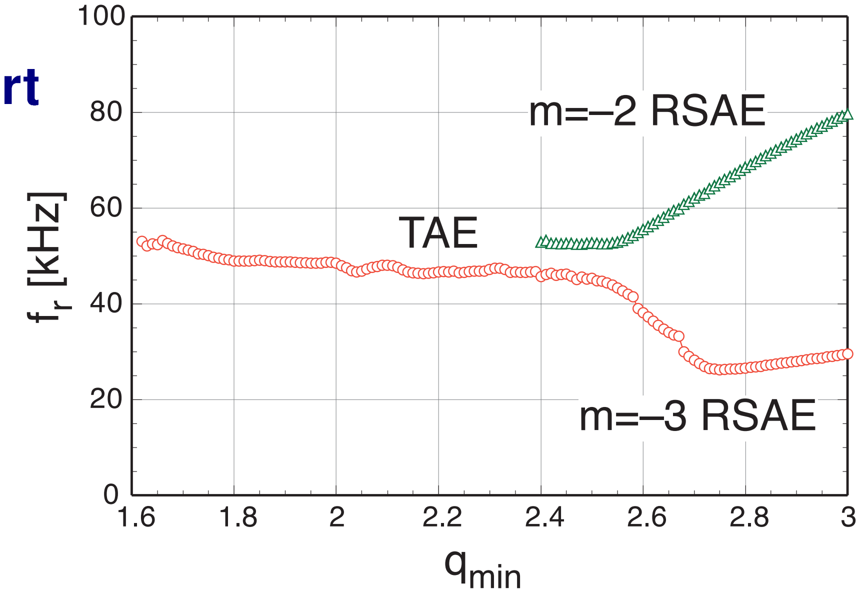
Plasma Parameters

R_0	3 m
a	1 m
B_0	3 T
$n_e(0)$	$10^{20}\,\mathrm{m}^{-3}$
$T(0)$	3 keV
$q(0)$	3
$q(a)$	5
$\rho_{\rm min}$	0.5
n	1

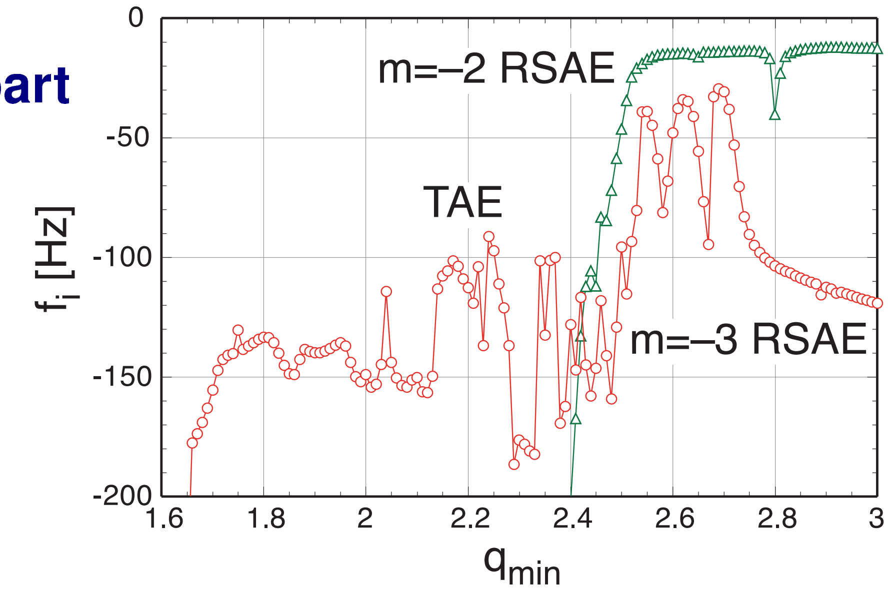
Flat density profile

$q_{\rm min}$ Dependence of Eigenmode Frequency

Real part



Imag part



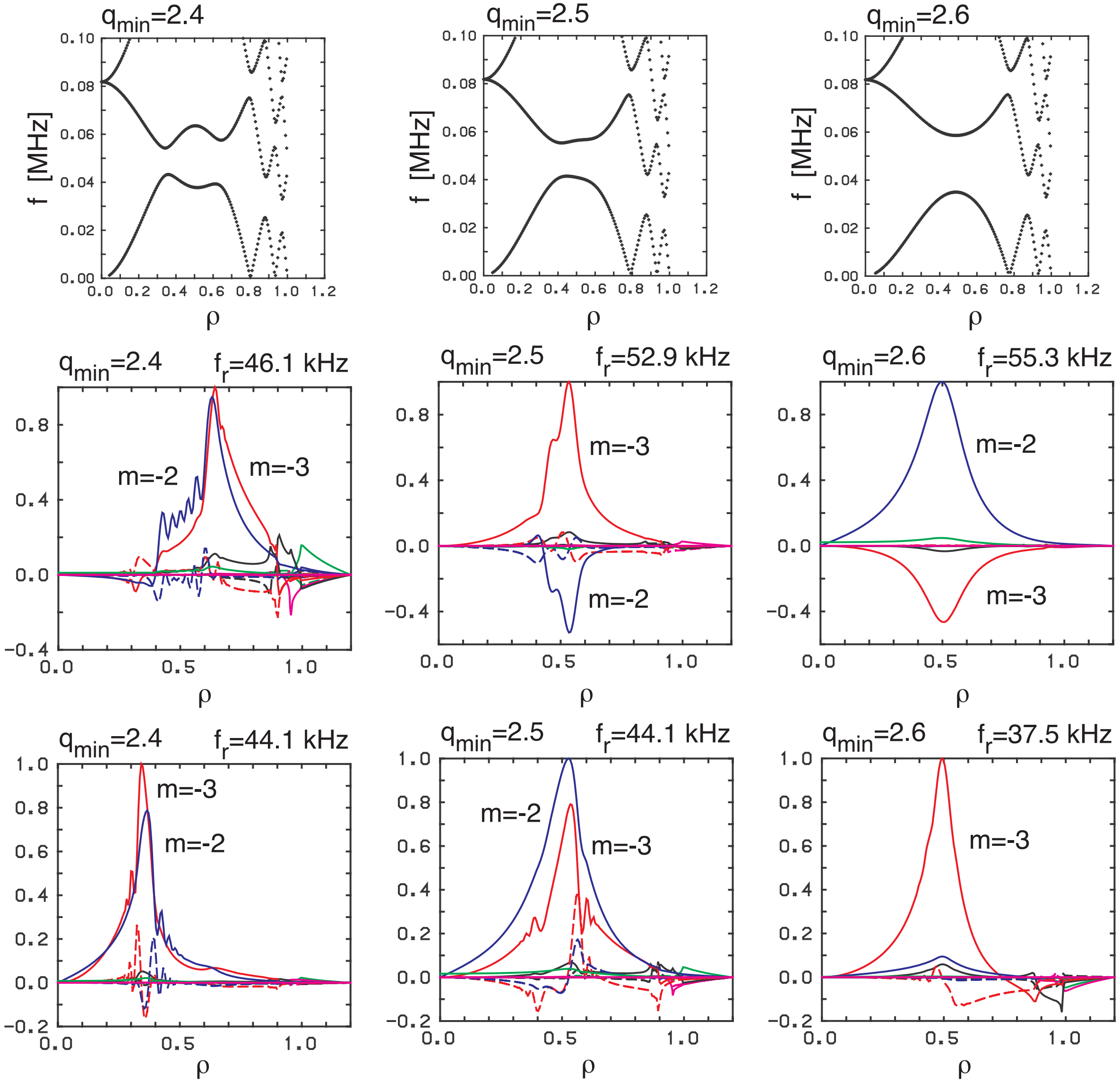
Eigenmode Structure

$q_{\min} = 2.4$ $q_{\min} = 2.5$ $q_{\min} = 2.6$

Alfvén resonance

Higher freq.

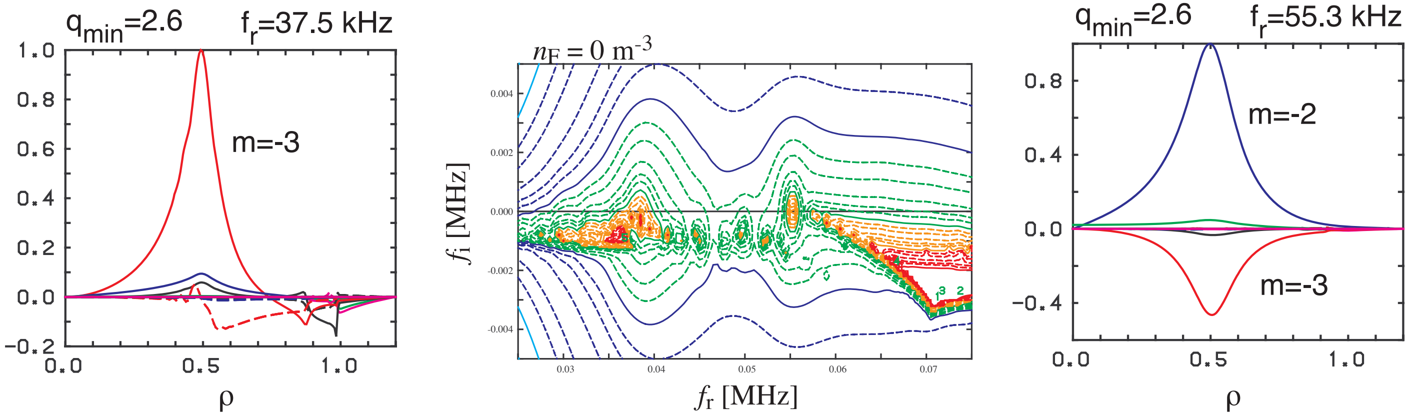
Lower freq.



TAEs **Double TAE** **RSAE**

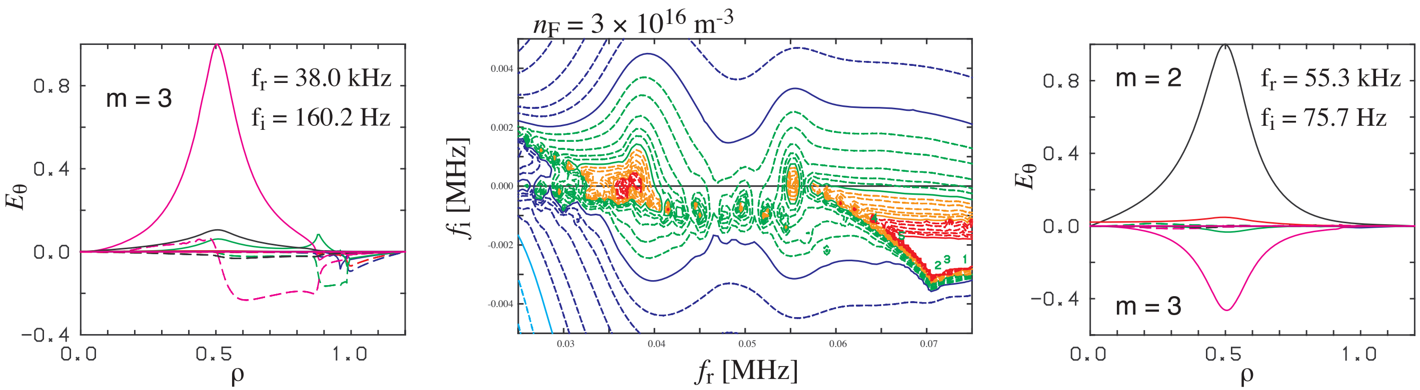
Excitation by Energetic Particles (q_{min} = 2.6)

- Without EP



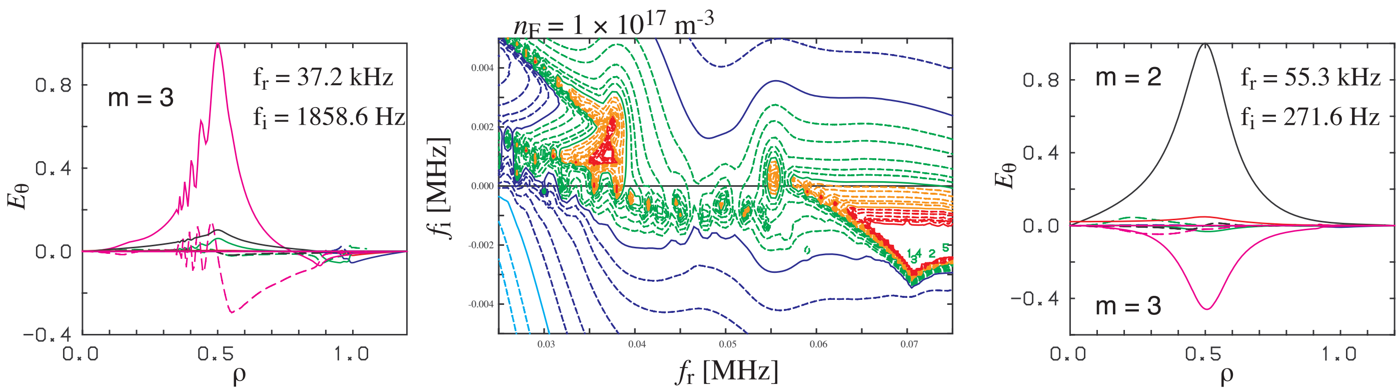
- With EP

  $3 \times 10^{16}\,\mathrm{m}^{-3}$
  360 keV
  0.5 m



- With EP

  $1 \times 10^{17}\,\mathrm{m}^{-3}$
  360 keV
  0.5 m

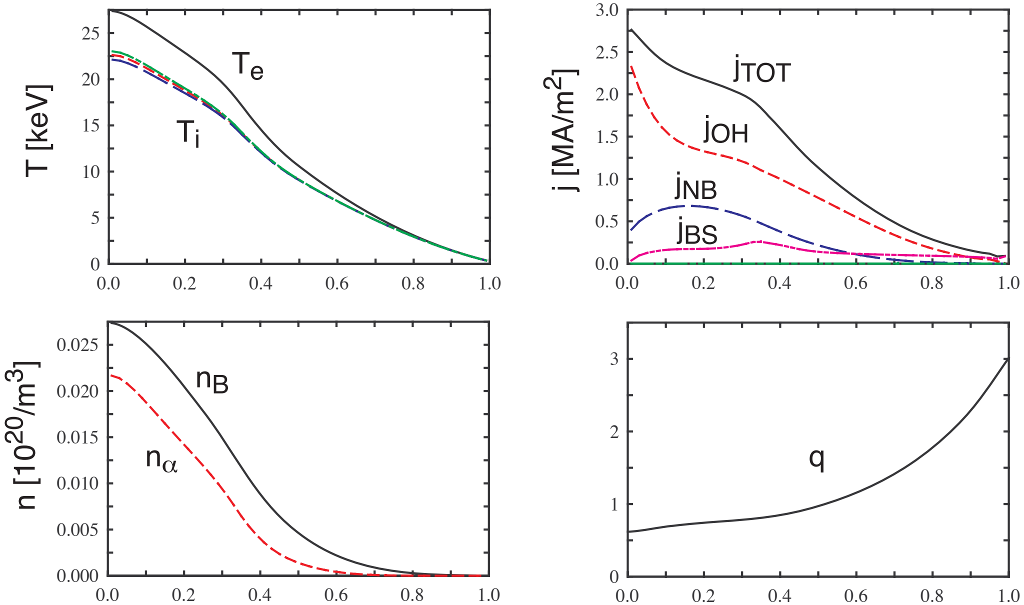


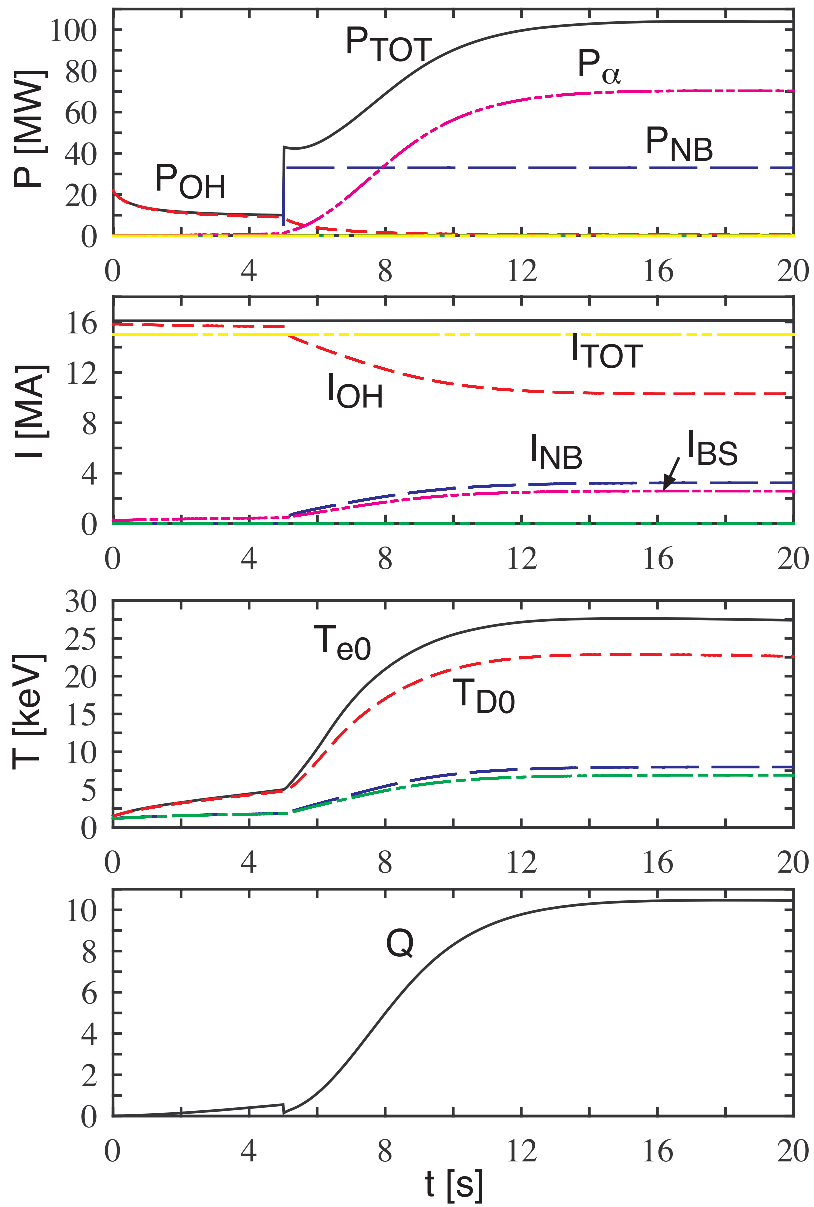
Integrated Analysis of AE in ITER Plasma

- **Combined Analysis**
  - **Equilibrium**: TASK/EQ
  - **Transport**: TASK/TR
    — Turbulent transport model: CDBM
    — Neoclassical transport model: NCLASS (**Houlberg**)
    — Heating and current profile: given profile
  - **Full wave analysis**: TASK/WM
- **Stability analysis**
  - Standard H-mode operation: $I_\mathrm{p} = 15\,\mathrm{MA}$, $Q \sim 10$
  - Hybrid operation: $I_\mathrm{p} = 12\,\mathrm{MA}$, flat q profile above 1
  - Steady-state operation: $I_\mathrm{p} = 9\,\mathrm{MA}$, reversed shear

Standard H-mode Operation

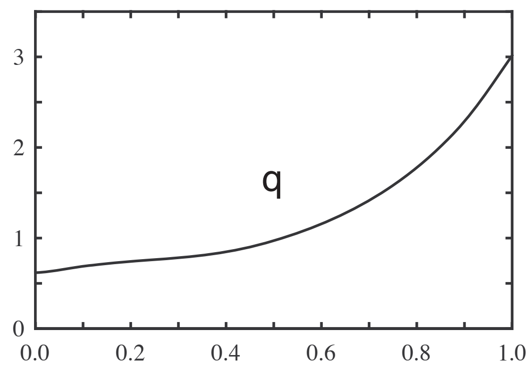
- $I_p = 15\,\mathrm{MA}$
- $P_{NB} = 33\,\mathrm{MW}$
- $\beta_N = 1.3$



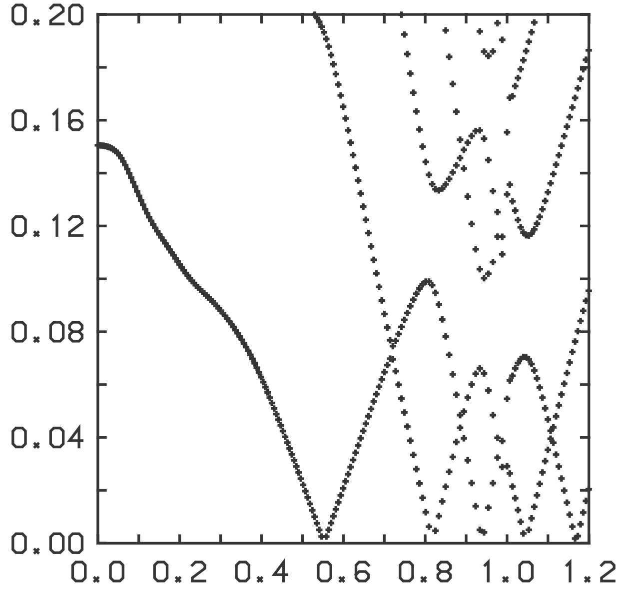


AE in Standard H-mode Operation

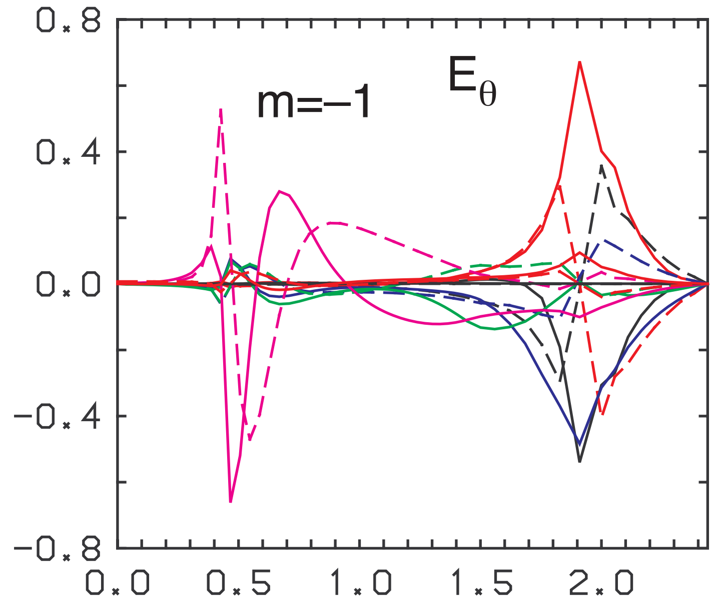
q profile



Alfvén Continuum



Mode structure ($n = 1$)

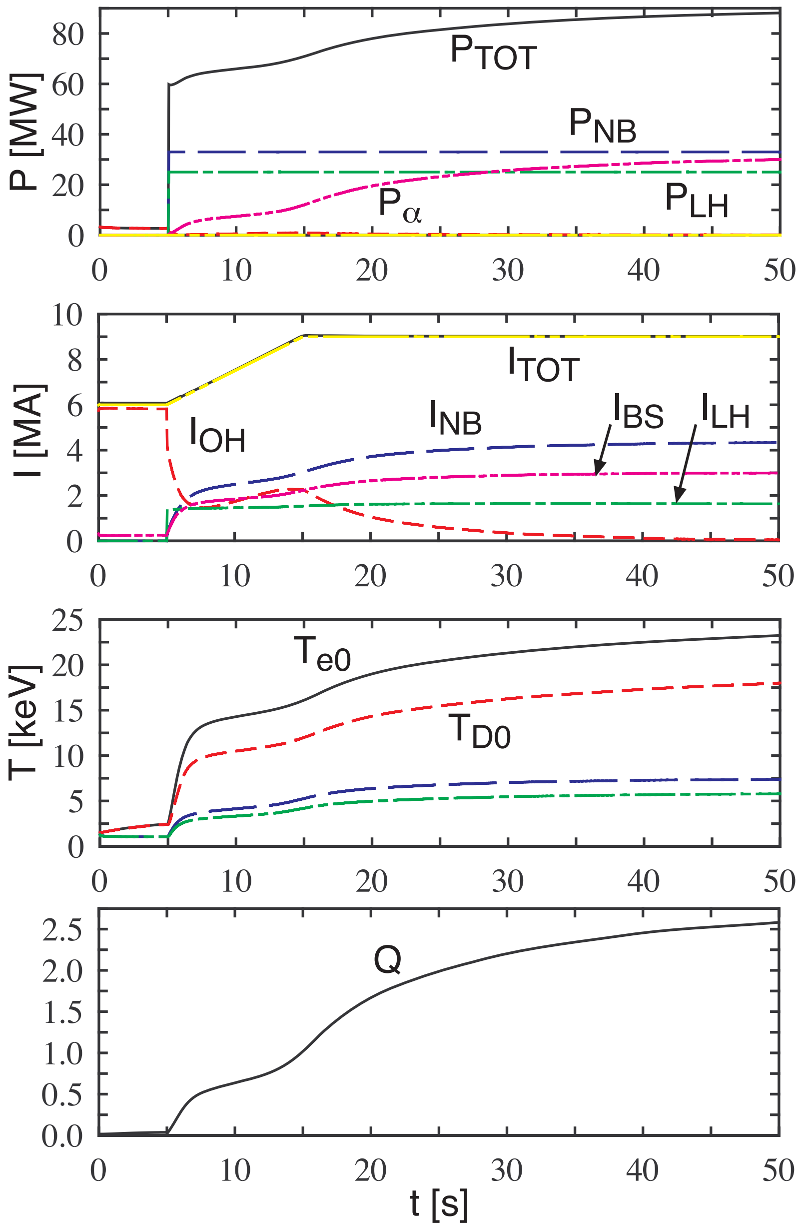


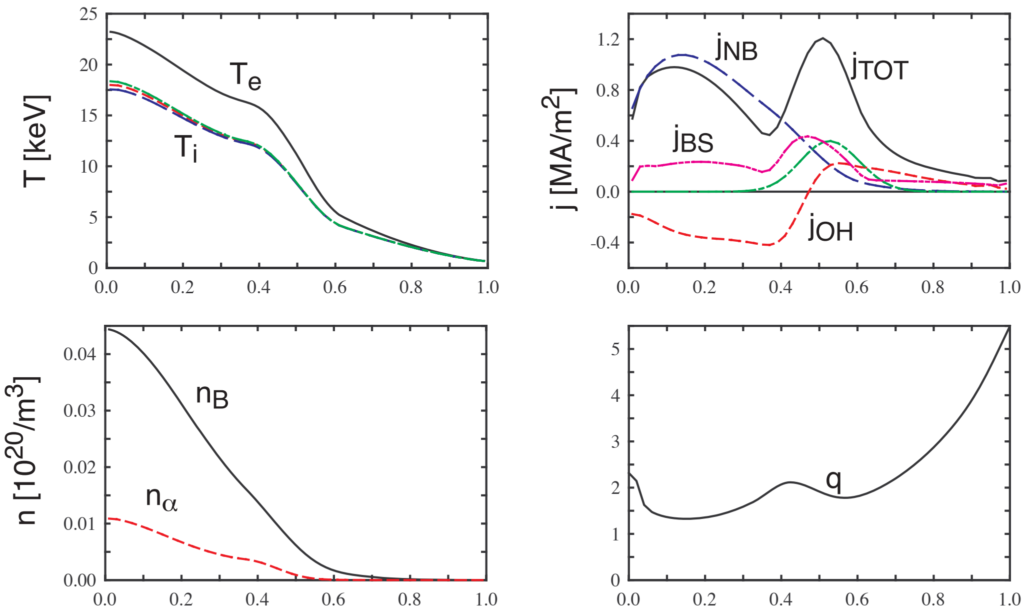
$$f_r = 95.95\,\text{kHz}$$
$$f_i = -1.95\,\text{kHz}$$

Stabilization due to $q = 1$

Steady-State Operation I

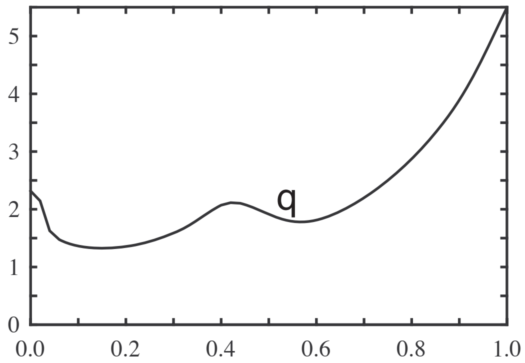
- $I_p = 6 \rightarrow 9\,\mathrm{MA}$
- $P_{NB} = 33\,\mathrm{MW}$
- $P_{LH} = 20\,\mathrm{MW}$
- $Q = 10.4$
- $\beta_N = 1.8$



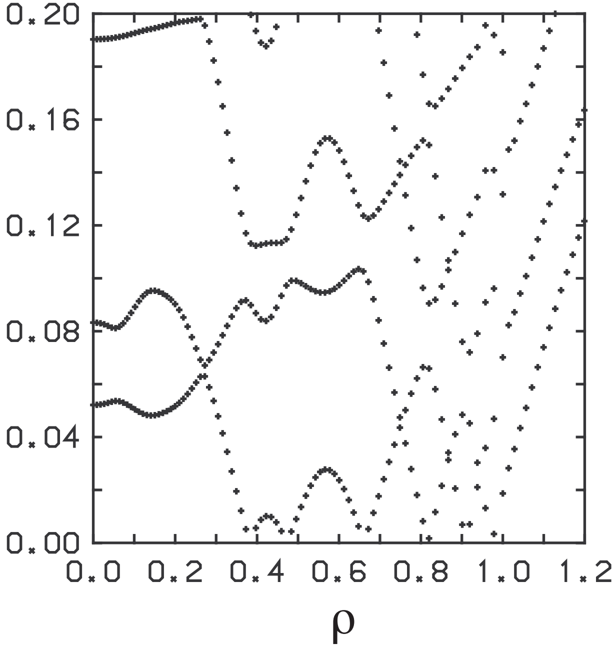


AE in Steady-State Operation I

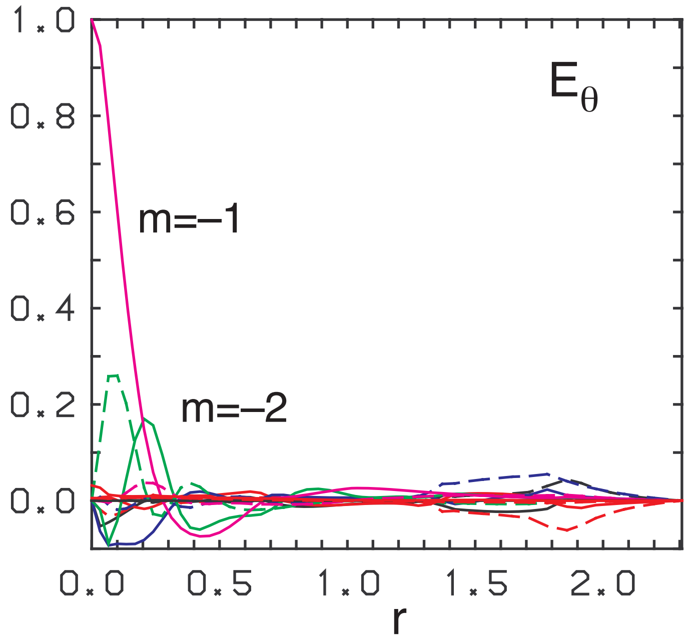
q profile



Alfvén Continuum



Mode structure ($n = 1$)

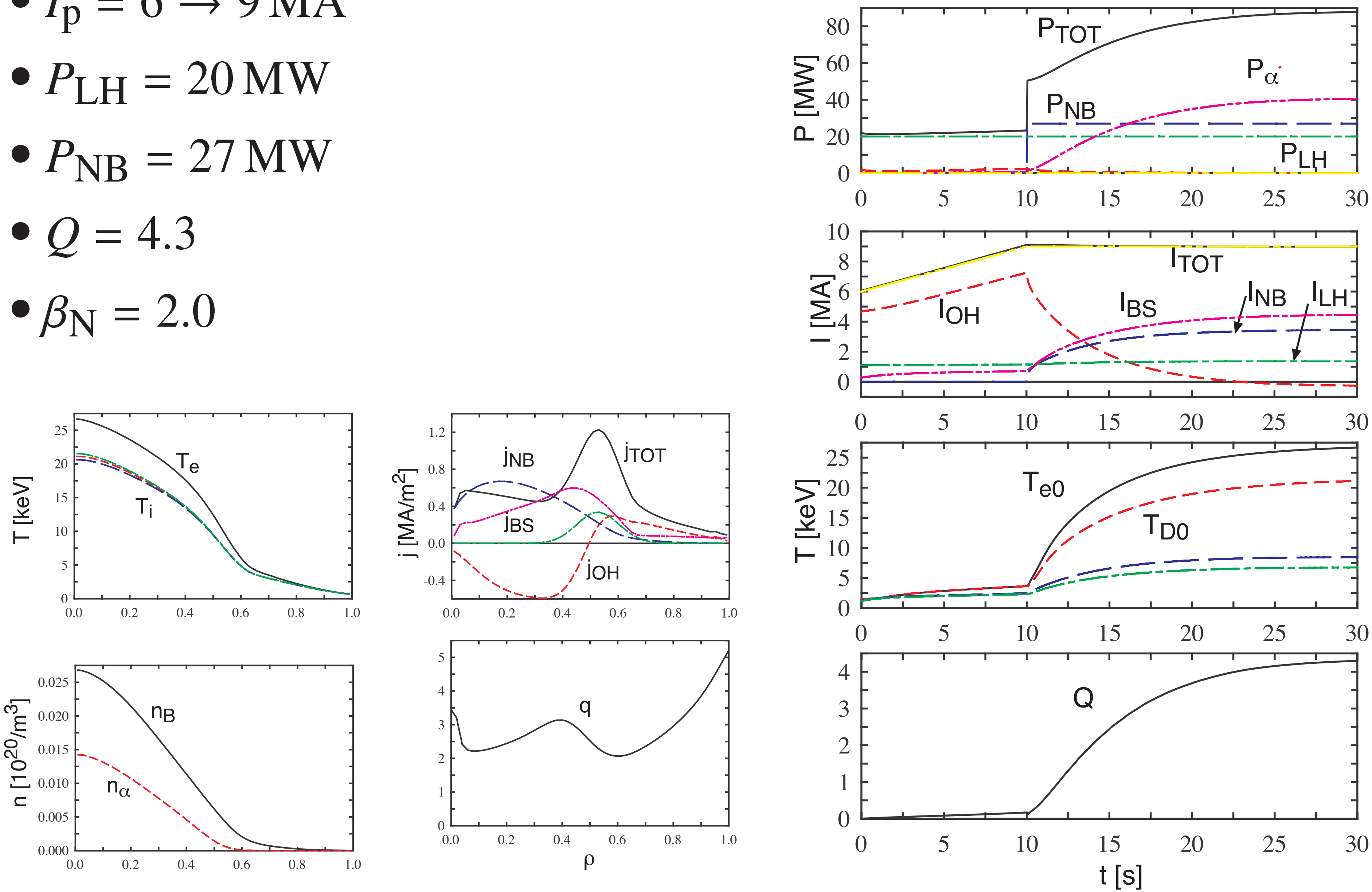


$$f_r = 109.10\,\text{kHz}$$
$$f_i = 0.77\,\text{kHz}$$

Unstable core localized mode

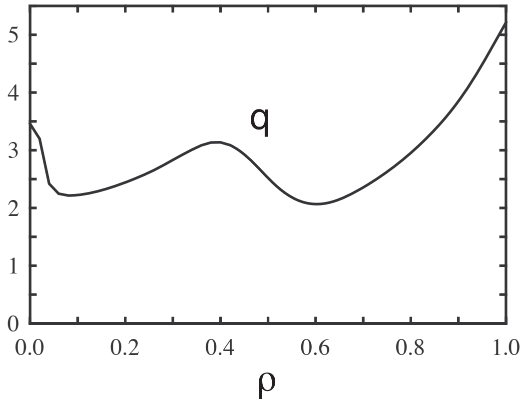
Steady-State Operation II

- $I_p = 6 \rightarrow 9\,\text{MA}$
- $P_{LH} = 20\,\text{MW}$
- $P_{NB} = 27\,\text{MW}$
- $Q = 4.3$
- $\beta_N = 2.0$

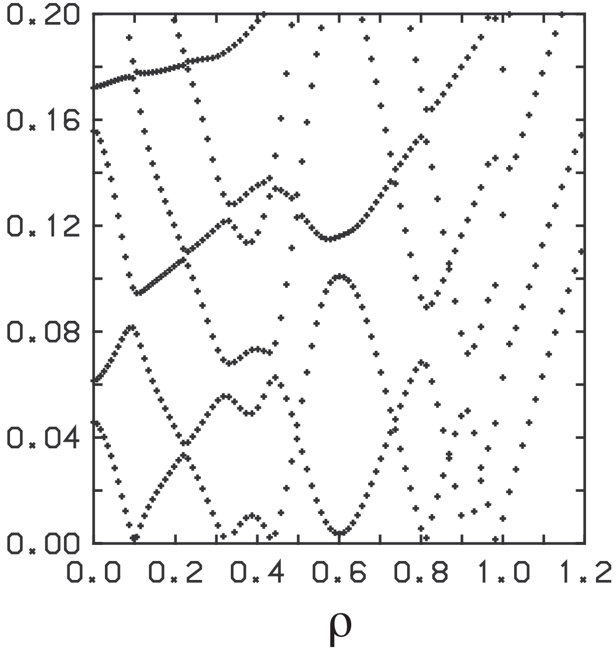


AE in Steady-State Operation II

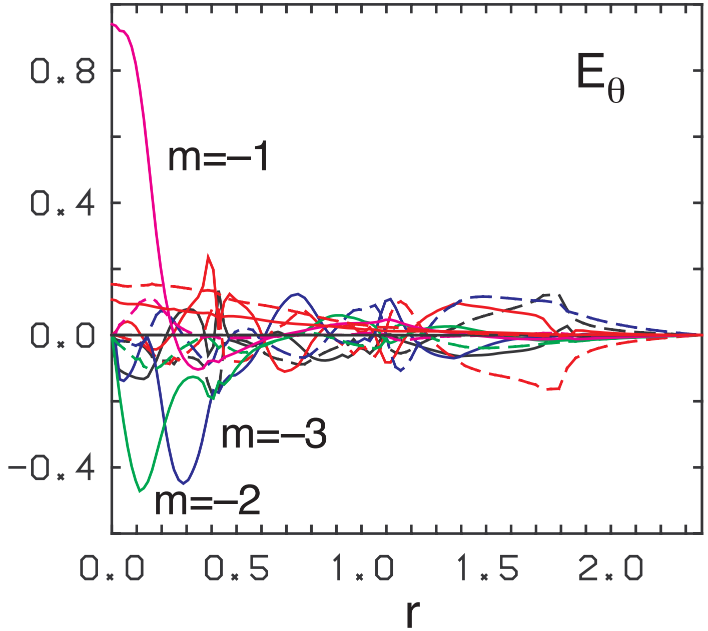
q profile



Alfvén Continuum



Mode structure ($n = 1$)



$$f_r = 102.40\,\text{kHz}$$

$$f_i = 0.085\,\text{kHz}$$

Unstable core localized mode

Self-Consistent Full Wave Analysis

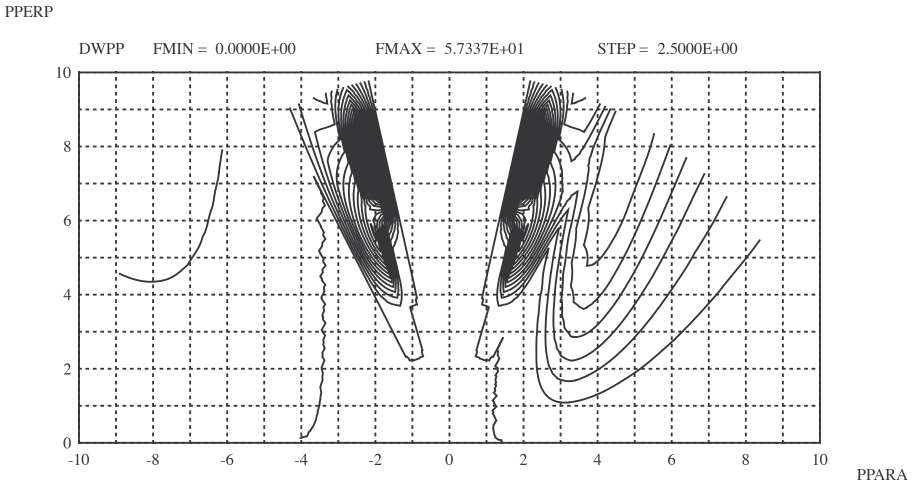
- **Deviation of velocity distribution from Maxwellian affects**
  - Power absorption of ICRF waves in the presence of energetic ions
  - Stability of modes driven by velocity anisotropy
  - Growth rate of Alfvén eigenmode

- **Systematic analyses including the modification of velocity distribution by TASK code is under way.**
  - **Full wave analysis with arbitrary velocity distribution**
  - **Bounce averaged Fokker-Plank analysis**

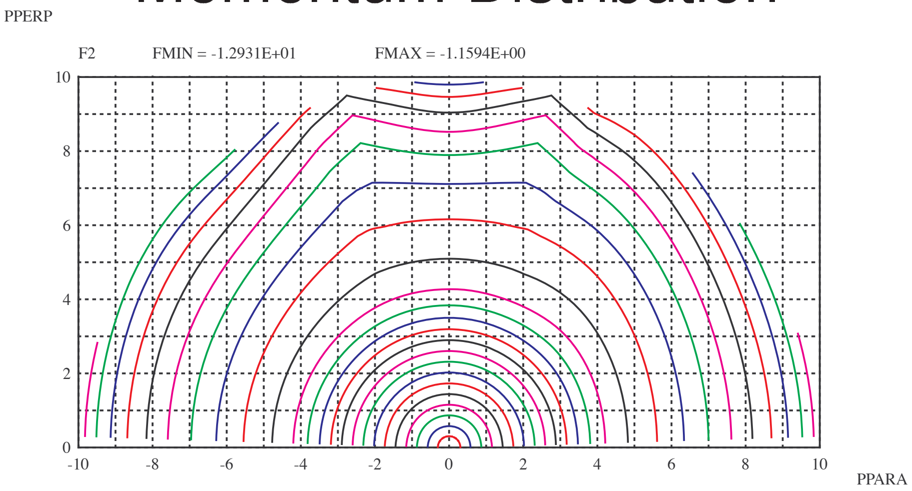
- **Upgrade of Fokker-Planck module is not completed yet.**

Fast Ion Tail Formation by ICRF

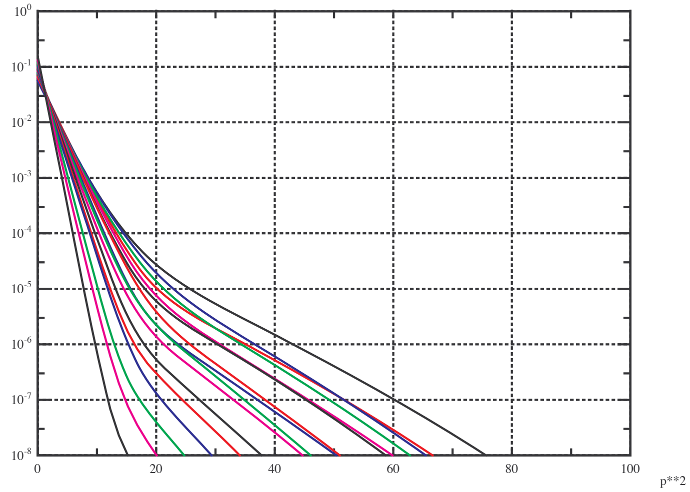
Quasi-linear Diffusion



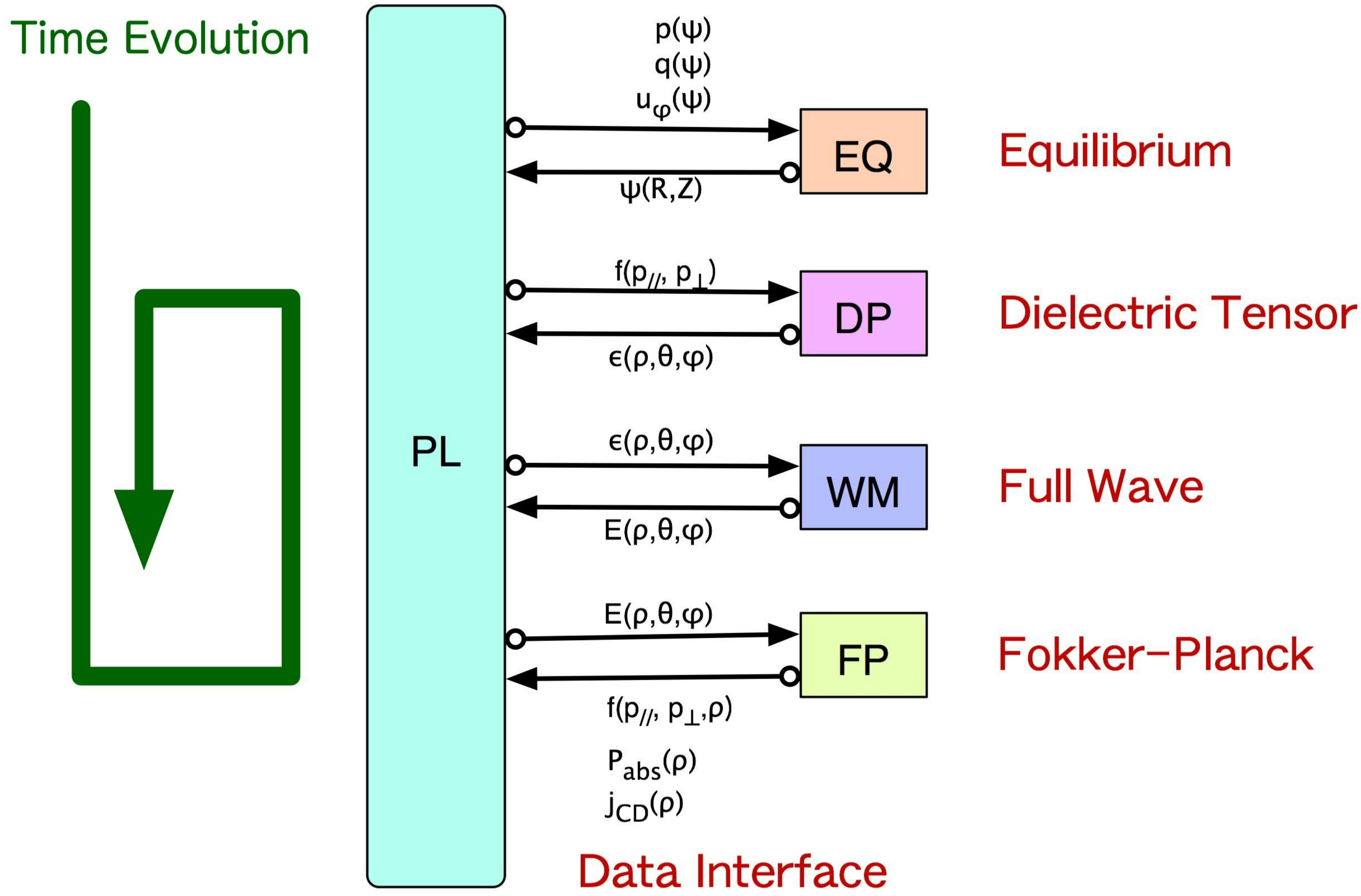
Momentum Distribution



Tail Formation



Procedure of Self-consistent Full Wave Analysis



Integro-Differential Full Wave Analysis

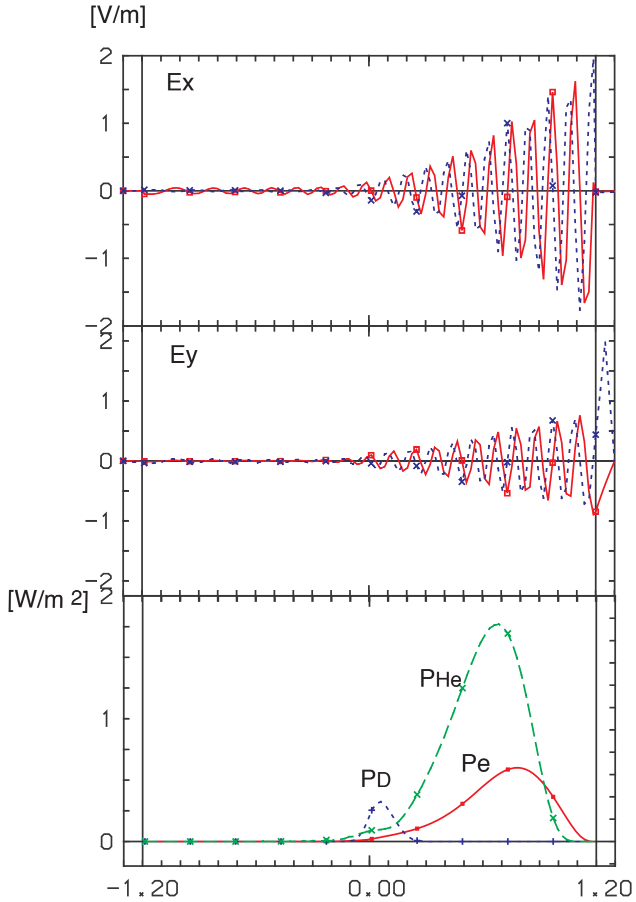
- **Purpose**
  - **FLR effects**
    - **Existence of energetic particles**
    - **Cyclotron higher harmonics**
  - **Inhomogeneous k along k**
    - Landau damping in inhomogeneous plasmas
    - Cyclotron resonance along the field line

- **Previous analyses: FLR effects**
  - Estimation of $k_\perp$ from fast wave dispersion relation
  - Differential operator up to the second order (**TORIC, old WM, ...**)
  - Spectral expansion in three dimensions (**AORSA**)
  - Integral operator approach (1D)

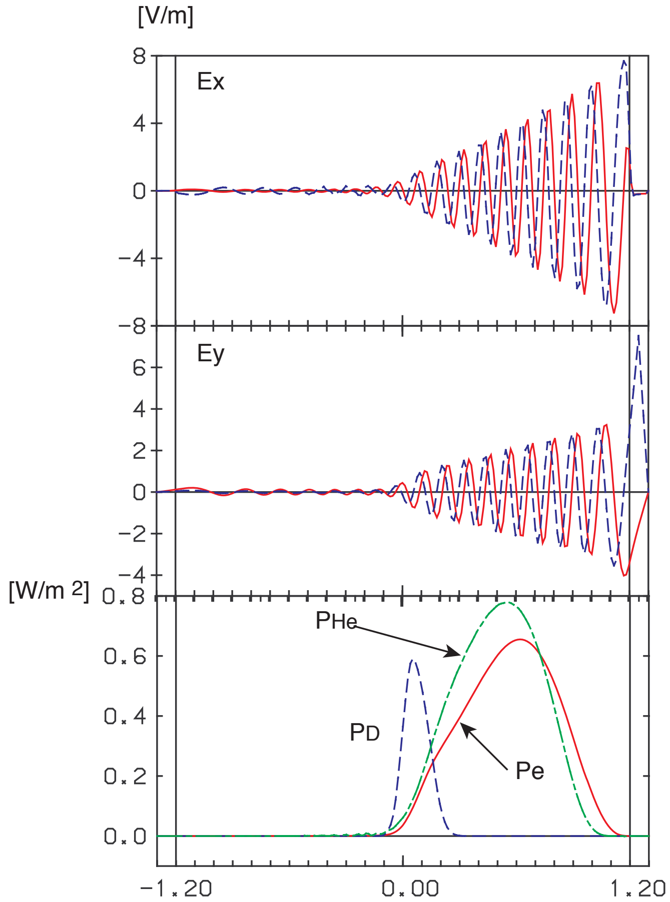
1D Full Wave Analysis including FLR Effects

- **Absorption of ICRF wave by α particles:**
  O. Sauter, J. Vaclavik (1992), Y. Uetani, A. Fukuyama (1998)

Differential analysis up to $k_\perp^2 \rho_H^2$



Integro-Differential Analysis



Overestimate α absorption

2D Formulation in Tokamaks (1)

- **Induced current**

$$\boldsymbol{J}_{\rm ind}(\boldsymbol{r},t) = \int {\rm d}\boldsymbol{v} q\boldsymbol{v} f(\boldsymbol{r},\boldsymbol{v},t)$$

$$= -\frac{q}{m}\int {\rm d}\boldsymbol{v} q\boldsymbol{v} \int_0^\infty {\rm d}\tau \left[\boldsymbol{E}(\boldsymbol{r}',t-\tau) + \boldsymbol{v}'\times\boldsymbol{B}(\boldsymbol{r}',t-\tau)\right]\cdot\frac{\partial f_0(\boldsymbol{r}',\boldsymbol{v}')}{\partial\boldsymbol{v}'}$$

- **Wave electric field**

$$\boldsymbol{E}(\boldsymbol{r},t) = \sum_{MN}\boldsymbol{E}_{MN}(\boldsymbol{r})\exp\{iM\theta + iN\phi - i\omega t\}$$

- **Equilibrium distribution function**

$$f_0(r_0,\boldsymbol{v}) = n_0(r_0)\left(\frac{m}{2\pi T_\perp(r_0)}\right)^{\frac{3}{2}}\left(\frac{T_\perp(r_0)}{T_\parallel(r_0)})\right)\exp\left\{-\frac{mv_\perp^2}{2T_\perp(r_0)} - \frac{mv_\parallel^2}{2T_\parallel(r_0)}\right\}$$

2D Formulation in Tokamaks (2)

- **Induced current**

$$
\begin{aligned}
\boldsymbol{J}_{\rm ind}(\boldsymbol{r}, t_0) \;=\; & -\int {\rm d}v_{\parallel} \int {\rm d}v_{\perp} \int {\rm d}\psi \int_0^{\infty} {\rm d}\tau \\
& \cdot\; n_0 q^2 \left(\frac{m}{2\pi T_{\perp}}\right)^{\frac{3}{2}} \left(\frac{T_{\perp}}{T_{\parallel}}\right)^{\frac{1}{2}} \exp\left\{-\frac{m v_{\perp}^2}{2T_{\perp}} - \frac{m v_{\parallel}^2}{2T_{\parallel}}\right\} \overleftrightarrow{G}_0 \\
& \cdot \sum_{M,N} \begin{pmatrix} E_{r' MN} \\ E_{\theta' MN} \\ E_{\phi' MN} \end{pmatrix} \exp i\left[M\theta + N\phi - \omega t_0 - k_{\parallel} v_{\parallel}\tau + \omega\tau\right] \\
& \times \sum_{n_1, n_2 = -\infty}^{\infty} J_{n_1}\left(\frac{k_{\perp} v_{\perp}}{\Omega}\right) J_{n_2}\left(\frac{k_{\perp} v_{\perp}}{\Omega}\right) \exp i\left\{n_1(\Omega\tau - \theta_0 + \psi) - n_2(-\theta_0 + \psi)\right\}
\end{aligned}
$$

$$
\overleftrightarrow{G}_0 = \begin{pmatrix} v_{\perp}^2 \sin(-\theta_0 + \psi) \\ -v_{\perp}^2 \cos(-\theta_0 + \psi) \\ v_{\parallel} v_{\perp} \end{pmatrix} \cdot \begin{pmatrix} -\dfrac{v_{\perp}}{T_{\perp}} \sin(\Omega\tau - \theta_0 + \psi) + v_{\perp} v_{\parallel} \left(\dfrac{1}{T_{\perp}} - \dfrac{1}{T_{\parallel}}\right) \dfrac{N}{R\omega} \sin(\Omega\tau - \theta_0 + \psi) \\ \dfrac{v_{\perp}}{T_{\perp}} \cos(\Omega\tau - \theta_0 + \psi) - v_{\perp} v_{\parallel} \left(\dfrac{1}{T_{\perp}} - \dfrac{1}{T_{\parallel}}\right) \dfrac{N}{R\omega} \cos(\Omega\tau - \theta_0 + \psi) \\ -\dfrac{v_{\parallel}}{T_{\parallel}} + v_{\perp} v_{\parallel} \left(\dfrac{1}{T_{\perp}} - \dfrac{1}{T_{\parallel}}\right) \left\{\dfrac{M}{r_0\omega} \cos(\Omega\tau - \theta_0 + \psi) + \dfrac{i}{\omega} \sin(\Omega\tau - \theta_0 + \psi) \dfrac{\partial}{\partial r'}\right\} \end{pmatrix}
$$

2D Formulation in Tokamaks (3)

- **Variable transformation**
  - Velocity $(v_\perp, \psi) \rightarrow$ Position (r', r_0)

$$\begin{cases} r = r_0 + \dfrac{v_\perp}{\Omega}\cos(-\theta_0 + \psi) \\ r' = r_0 + \dfrac{v_\perp}{\Omega}\cos(\Omega\tau - \theta_0 + \psi) \end{cases}$$

  - Jacobian: $J = \dfrac{\partial(v_\perp, \psi)}{\partial(r', r_0)} = -\dfrac{\Omega^2}{v_\perp \sin \Omega\tau}$
  - Then

$$\int_0^\infty \mathrm{d}v_\perp \int_0^{2\pi} \mathrm{d}\psi = \int_0^R \mathrm{d}r' \int_0^R \mathrm{d}r_0 \, |J| = \int_0^R \mathrm{d}r' \int_0^R \mathrm{d}r_0 \frac{\Omega^2}{v_\perp |\sin \Omega\tau|}$$

- **Fourier expansion with respect to $\Omega\tau$**
- **Time integral with respect to τ**
- **Velocity integral with respect to $v_\parallel$**

2D Formulation in Tokamaks (4)

- **Finally we obtain:**

$$\boldsymbol{J}_{\mathrm{ind}}(\boldsymbol{r}, t) = \sum_{MN} \int_0^R \mathrm{d}r' \overleftrightarrow{\sigma}_{MN}(r, r') \boldsymbol{E}_{MN}(r') \exp i \{M\theta + N\phi - \omega t\}$$

$$\begin{aligned}
\overleftrightarrow{\sigma}_{MN}(r, r') &= -\sum_{n_1 n_2} \sum_{\ell=-\infty}^{\infty} \int_0^R \mathrm{d}r_0 \; q^2 \Omega^2 n_0 \left(\frac{1}{2\pi}\right)^{\frac{5}{2}} \frac{i}{v_{T_\perp}^2} \\
&\times \int_0^{2\pi} \mathrm{d}\lambda \frac{1}{|\sin\lambda|} \exp\left\{-\frac{R_v^2 \Omega^2}{2 v_{T_\perp}^2}\right\} \overleftrightarrow{G}_2 J_{n_1}(k_\perp R_v) J_{n_2}(k_\perp R_v) \\
&\times \exp(-i n_1 \lambda) \left\{\frac{r - r_0 + i R_m}{R_v}\right\}^{n_1 - n_2} \mathrm{e}^{i\ell\lambda}
\end{aligned}$$

$$\begin{aligned}
G_{rr} &= A_1 R_v^2 \\
G_{r\theta} &= -(r' - r_0) A_1 R_m \\
G_{r\phi} &= R_m \left[A_2 + A_3 \left\{ \frac{M}{r'}(r' - r_0) + i R_p \frac{\partial}{\partial r'} \right\} \right] \\
G_{\theta r} &= -(r - r_0) A_1 R_p
\end{aligned}$$

$$G_{\theta\theta} = A_1(r - r_0)(r' - r_0)$$

$$G_{\theta\phi} = -(r - r_0)\left[A_2 + A_3\left\{\frac{M}{r'}(r' - r_0) + iR_p\frac{\partial}{\partial r'}\right\}\right]$$

$$G_{\phi r} = \left\{A_2 + \frac{1}{k_\parallel}\frac{\omega - \ell\Omega}{\Omega}\frac{N}{R}A_3\right\}R_p$$

$$G_{\phi\theta} = -\left\{A_2 + \frac{1}{k_\parallel}\frac{\omega - \ell\Omega}{\Omega}\frac{N}{R}A_3\right\}(r' - r_0)$$

$$G_{\phi\phi} = \frac{1}{k_\parallel}\frac{\omega - \ell\Omega}{\Omega}\left[A_2 + A_3\left\{\frac{M}{r'}(r' - r_0) + iR_p\frac{\partial}{\partial r'}\right\}\right]$$

where A_1, A_2, A_3, R_v. R_p. R_m are defined by

$$A_1 \equiv \frac{\Omega^2}{T_\perp}\frac{\sqrt{\pi}}{k_\parallel v_{T_\parallel}}Z(\eta_\ell) + \sqrt{\frac{\pi}{2}}\frac{\Omega^2}{k_\parallel}Z'(\eta_\ell)\left(\frac{1}{T_\perp} - \frac{1}{T_\parallel}\right)\frac{N}{R\omega}$$

$$A_2 \equiv -\sqrt{\frac{\pi}{2}}\frac{\Omega}{k_\parallel T_\parallel}Z'(\eta_\ell)$$

$$A_3 \equiv \sqrt{\frac{\pi}{2}}\frac{\Omega^2}{k_\parallel \omega}Z'(\eta_\ell)\left(\frac{1}{T_\perp} - \frac{1}{T_\parallel}\right)$$

$$R_v^2 = \left(\frac{r + r'}{2} - r_0\right)^2\frac{1}{\cos^2\frac{1}{2}\lambda} + \left(\frac{r - r'}{2}\right)^2\frac{1}{\sin^2\frac{1}{2}\lambda}$$

$$R_p = \frac{r - r'}{2}\frac{1}{\tan\frac{1}{2}\lambda} + \left(\frac{r + r'}{2} - r_0\right)\tan\frac{1}{2}\lambda$$

$$R_m = \frac{r - r'}{2}\frac{1}{\tan\frac{1}{2}\lambda} - \left(\frac{r + r'}{2} - r_0\right)\tan\frac{1}{2}\lambda$$

and

$$\eta_\ell \equiv \frac{\omega - \ell\Omega}{k_\parallel v_{\mathrm{T}\parallel}}, \quad k_\parallel \equiv \frac{M}{r_0}\frac{B_\theta}{B} + \frac{N}{R}\frac{B_\phi}{B}, \quad k_\perp \equiv \frac{M}{r_0}\frac{B_\phi}{B} - \frac{N}{R}\frac{B_\theta}{B}$$

Code development using FEM is under way

Summary

- **One of the targets of the integrated code, TASK, is to consistently analyze the behavior of AE (Alfvén Eigenmodes) driven by energetic particles.**

- **Coupled with 1-1/2D transport module, TASK/WM was used to study AEs in ITER operation scenarios.**

- **Self-consistent analysis including modification of $f(v)$**
  - Full wave module with arbitrary velocity distribution: OK
  - Fokker-Planck module with full wave electric fields: OK
  - Coupling of the two modules will be completed soon.

- **2D integro-differential full wave analysis including FLR**
  - Formulation was extended to 2D configuration.
  - Implementation is under way.