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Progress of TASK code in RF modeling

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Outline

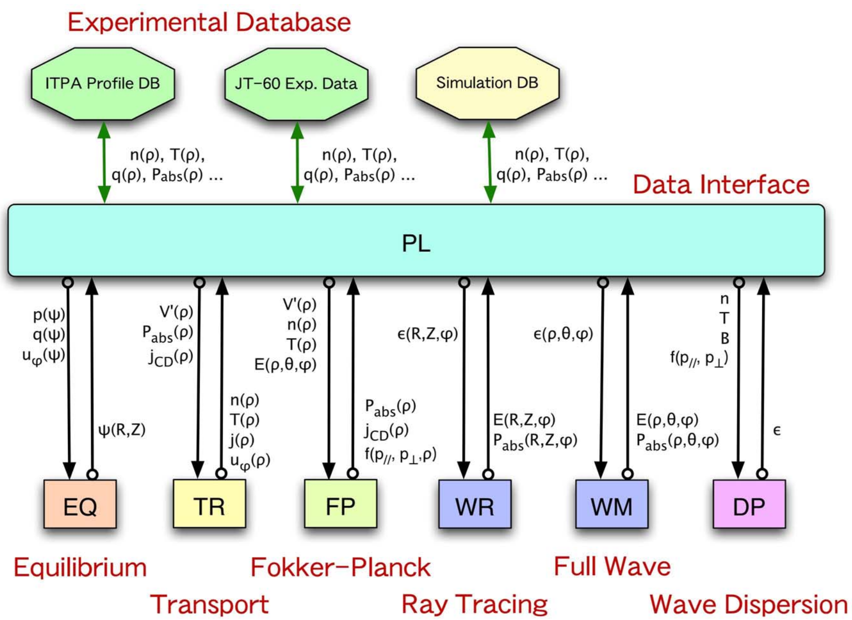
- Present Status of TASK Code
- Full wave analysis of ECH in small ST
- Integro-differential analysis including FLR effects
- Self-consistent analysis of wave heating and current drive
- Summary

TASK Code

- **Transport Analysing System for TokamaK**
- **Features**
  - **A Core of Integrated Modeling Code in BPSI**
    — Modular structure, UnifiedStandard data interface
  - **Various Heating and Current Drive Scheme**
    — EC, LH, IC, AW, (NB)
  - **High Portability**
    — Most of Library Routines Included
  - **Development using CVS** (Concurrent Version System)
    — Open Source (V0.93 http://bpsi.nucleng.kyoto-u.ac.jp/task/)
  - **Parallel Processing using MPI Library**
  - **Extension to Toroidal Helical Plasmas**

Structures of TASK

EQ	**2D Equilibrium**	Fixed boundary, Toroidal rotation
TR	**1D Transport**	Diffusive Transport, Transport models
WR	**3D Geometrical Optics**	EC, LH: Ray tracing, Beam tracing
WM	**3D Full Wave**	IC, AW: Antenna excitation, Eigen mode
FP	**3D Fokker-Planck**	Relativistic, Bounce-averaged
DP	**Wave Dispersion**	Local dielectric tensor, Arbitrary $f(\boldsymbol{v})$
PL	**Data Interface**	Data conversion, Profile database



Wave Dispersion Analysis : TASK/DP

- **Various Models of Dielectric Tensor** $\overleftrightarrow{\epsilon}(\omega, \boldsymbol{k}; \boldsymbol{r})$:
  - **Resistive MHD** model
  - **Collisional cold** plasma model
  - **Collisional warm** plasma model
  - **Kinetic plasma** model (**Maxwellian**, non-relativistic)
  - **Kinetic plasma** model (**Arbitrary** $f(\boldsymbol{v})$, relativistic)
  - **Gyro-kinetic plasma** model (Maxwellian)
- **Numerical Integration in momentum space**: **Arbitrary** $f(\boldsymbol{v})$
  - Relativistic Maxwellian
  - Output of TASK/FP: Fokker-Planck code

Full wave analysis: TASK/WM

- **magnetic surface coordinate**: (ψ, θ, φ)
- Boundary-value problem of **Maxwell's equation**

$$\nabla \times \nabla \times \boldsymbol{E} = \frac{\omega^2}{c^2} \overleftrightarrow{\epsilon} \cdot \boldsymbol{E} + \mathrm{i}\,\omega\mu_0 \boldsymbol{j}_{\mathrm{ext}}$$

- Kinetic **dielectric tensor**: $\overleftrightarrow{\epsilon}$
  - **Wave-particle resonance**: $Z[(\omega - n\omega_{\mathrm{c}})/k_{||}v_{\mathrm{th}}]$
  - **Fast ion: Drift-kinetic**

$$\left[\frac{\partial}{\partial t} + v_{||}\nabla_{||} + (\boldsymbol{v}_{\mathrm{d}} + \boldsymbol{v}_{\mathrm{E}}) \cdot \nabla + \frac{e_\alpha}{m_\alpha}(v_{||}E_{||} + \boldsymbol{v}_{\mathrm{d}} \cdot \boldsymbol{E})\frac{\partial}{\partial \varepsilon}\right] f_\alpha = 0$$

- Poloidal and toroidal **mode expansion**
  - **Accurate estimation of $k_{||}$**
- Eigenmode analysis: **Complex eigen frequency** which maximize wave amplitude for fixed excitation proportional to electron density

Fokker-Planck Analysis : TASK/FP

- **Fokker-Planck equation**
  for **velocity distribution function** $f(p_{||}, p_\perp, \psi, t)$

$$\frac{\partial f}{\partial t} = E(f) + C(f) + Q(f) + L(f)$$

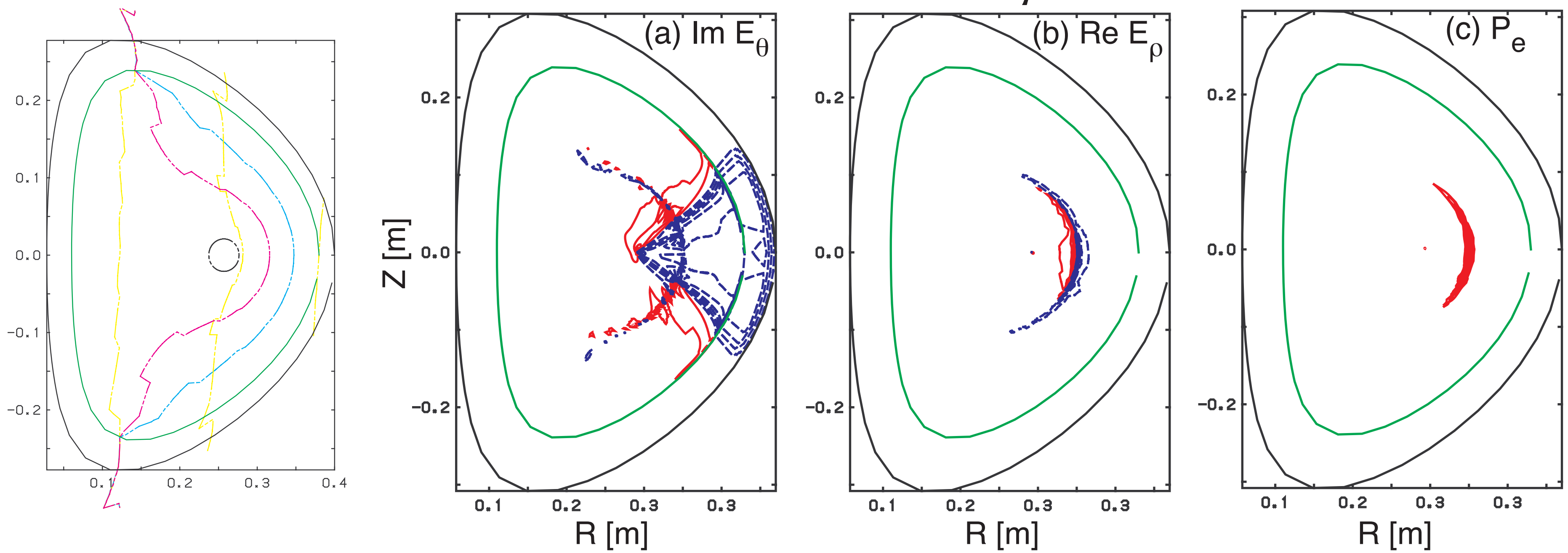
  - $E(f)$: Acceleration term due to DC electric field
  - $C(f)$: Coulomb collision term
  - $Q(f)$: Quasi-linear term due to wave-particle resonance
  - $L(f)$: Spatial diffusion term
- **Bounce-averaged**: Trapped particle effect, zero banana width
- **Relativistic**: momentum $\boldsymbol{p}$, weakly relativistic collision term
- **Nonlinear collision**: momentum or energy conservation
- **Three-dimensional**: spatial diffusion (neoclassical, turbulent)

Full Wave Analysis of ECH in a Small-Size ST

- **Small-size spherical tokamak: LATE** (Kyoto University)
  - **T. Maekawa et al., IAEA-CN-116/EX/P4-27 (Vilamoura, Portuga, 2004)**
  - $R = 0.22\,\mathrm{m}$, $a = 0.16\,\mathrm{m}$, $B_0 = 0.0552\,\mathrm{T}$, $I_p = 6.25\,\mathrm{kA}$, $\kappa = 1.5$
  - $f = 2.8\,\mathrm{GHz}$, Toroidal mode number $n = 12$, Extraordinary mode

Penetration through cutoff layer ⟹ Propagation along the UHR layer ⟹ Collisional damping near the UHR layer

UHR,cutoff, ImE_θ ReE_ρ P_{abs}



Integro-Differential Full Wave Analysis

- **Purpose of Integro-Differential Analysis**
  - **FLR effects**
    - — **Absorption by energetic particles** (α particles, beam ions)
    - — **Absorption at higher cyclotron harmonics** (IBW, EBW)
  - **Inhomogeneous k along k**
    - — Landau damping in inhomogeneous plasmas
    - — Cyclotron resonance along the field line

- **Various analyses of FLR effects**
  - **Differential analysis**: $k_\perp \rho < 1$, complicated for higher harmonics
  - **Spectral analysis**: Necessary to solve dense matrix equation
  - **Integro-differential analysis**: Localized within 3 gyroradius

1D Full Wave Analysis including FLR Effects

- **Formulation of Integro-Differential equation:**
  - Spatial inhomogeneity ρ/L: neglected; isotropic temperature:

    — **O. Sauter, J. Vaclavik, NF 32 (1992) 1455**
  - Spatial inhomogeneity ρ/L: any order; non-isotropic temperature:

    — **Y. Uetani, A. Fukuyama (unpublished, 1998)**

- **Maxwell's equation**

$$\nabla \times \nabla \times \boldsymbol{E}(x) - \frac{\omega^2}{c^2} \int_{-\infty}^{\infty} \mathrm{d}x' \overleftrightarrow{\epsilon}(x, x') \cdot \boldsymbol{E}(x') = \mathrm{i}\,\omega\mu_0 \boldsymbol{J}_{\mathrm{ext}}(x)$$

- **Dielectric tensor**

$$\overleftrightarrow{\epsilon}(x, x') = \delta(x - x') - \sum_s \int_{-\infty}^{\infty} \mathrm{d}\bar{x} \, \frac{\omega_{ps}^2}{\omega^2} \left(\frac{\Omega_s}{v_{T\perp s}} \right)^2 \sum_{n=-\infty}^{\infty} \overleftrightarrow{H}_{ns}(x, x', \bar{x})$$

- **Matrix elements**

$$
\begin{aligned}
H_{xx} &= -nA_1F_n^{(2)} \\
H_{yx} &= \mathrm{i}A_1(X-Y)\left\{(X-Y)F_n^{(3)}-(X+Y)F_n^{(4)}\right\} \\
H_{zx} &= -\mathrm{i}A_2\left\{(X-Y)F_n^{(3)}-(X+Y)F_n^{(4)}\right\} \\
H_{xy} &= -\mathrm{i}A_1(X+Y)\left\{(X+Y)F_n^{(3)}-(X-Y)F_n^{(4)}\right\} \\
H_{yy} &= -A_1(X+Y)(X-Y)F_n^{(1)} \\
H_{zy} &= A_2(X+Y)F_n^{(1)} \\
H_{xz} &= \mathrm{i}A_2\left\{(X+Y)F_n^{(3)}-(X-Y)F_n^{(4)}\right\} \\
H_{yz} &= A_2(X-Y)F_n^{(1)} \\
H_{zz} &= \frac{\sqrt{2}v_{T\parallel}\eta}{v_{T\perp}}A_2F_n^{(1)}
\end{aligned}
$$

- **Coefficients**

$$
\begin{aligned}
A_1 &\equiv \frac{\omega}{\sqrt{2}k_\parallel v_{T\parallel}}Z(\eta)+\left(1-\frac{T_\perp}{T_\parallel}\right)\frac{Z'(\eta)}{2} \\
A_2 &\equiv \frac{\omega}{2k_\parallel v_{T\perp}}\left\{+\mathrm{n}\frac{\Omega}{\omega}\left(1-\frac{\mathrm{T}_\perp}{\mathrm{T}_\parallel}\right)\right\}Z'(\eta)
\end{aligned}
$$

- **Kernel function**

$$
\begin{aligned}
F_n^{(i)}(X,Y) \equiv& \frac{1}{2\pi^2}\int_0^\pi \mathrm{d}\theta \\
&\times\exp\left[-\frac{X^2}{1+\cos\theta}-\frac{Y^2}{1-\cos\theta}\right]f_n^{(i)}(\theta)
\end{aligned}
$$

with

$$
f_n^{(i)}(\theta) = \begin{cases}
\dfrac{\cos n\theta}{\sin\theta} & (i=1) \\
\sin n\theta & (i=2) \\
\dfrac{\sin n\theta}{\sin^2\theta} & (i=3) \\
\dfrac{\cos\theta\sin n\theta}{\sin^2\theta} & (i=4)
\end{cases}
$$

where

$$
\begin{aligned}
X &\equiv \frac{\Omega}{v_{T\perp}}\left(\bar{x}-\frac{x+x'}{2}\right), \\
Y &\equiv \frac{\Omega}{2v_{T\perp}}(x-x')
\end{aligned}
$$

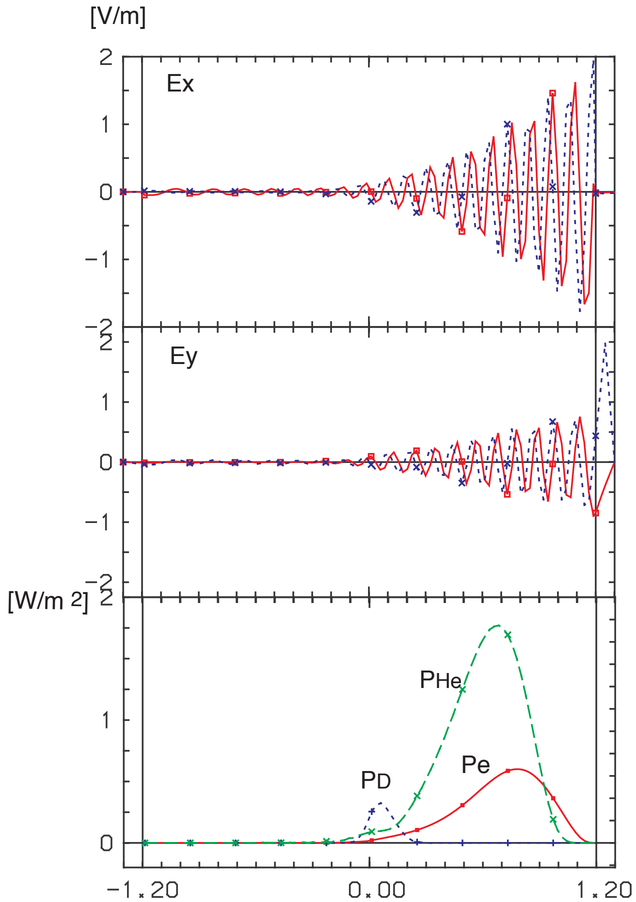
and

$$
\eta \equiv \frac{\omega-n\Omega}{\sqrt{2}k_\parallel v_{T\parallel}}
$$

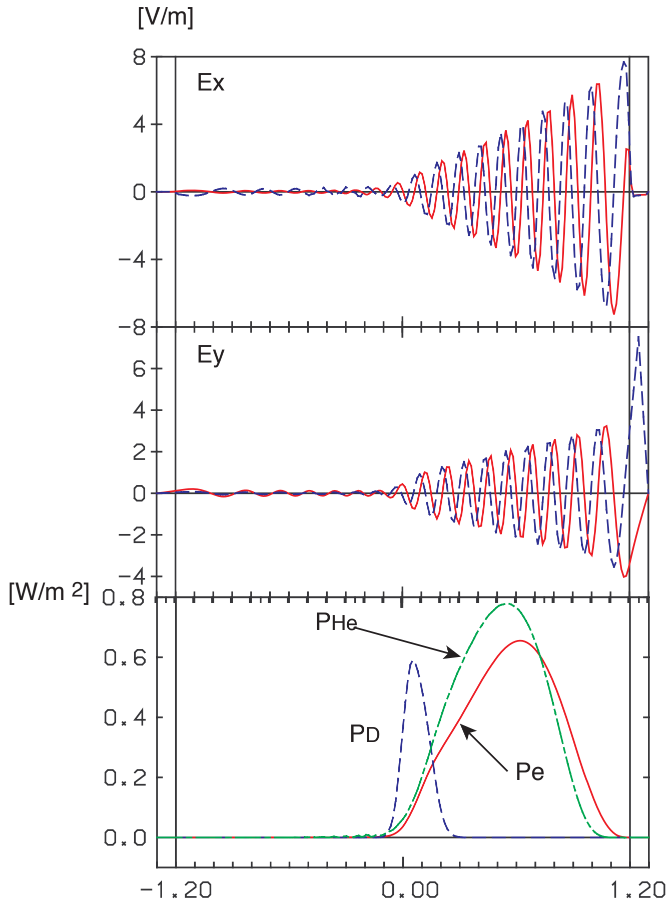
1D Full Wave Analysis including FLR Effects

- **Absorption of ICRF wave by α particles:**

Differential analysis up to $k_\perp^2 \rho_\mathrm{H}^2$



Integro-Differential Analysis



Overestimate α absorption

2D Formulation in Tokamaks (1)

- **Induced current**

$$\begin{aligned} \boldsymbol{J}_{\text{ind}}(\boldsymbol{r},t) &= \int \mathrm{d}\boldsymbol{v} q \boldsymbol{v} f(\boldsymbol{r},\boldsymbol{v},t) \\ &= -\frac{q}{m}\int \mathrm{d}\boldsymbol{v} q \boldsymbol{v} \int_0^\infty \mathrm{d}\tau \left[\boldsymbol{E}(\boldsymbol{r}',t-\tau) + \boldsymbol{v}' \times \boldsymbol{B}(\boldsymbol{r}',t-\tau)\right] \cdot \frac{\partial f_0(\boldsymbol{r}',\boldsymbol{v}')}{\partial \boldsymbol{v}'} \end{aligned}$$

- **Wave electric field**

$$\boldsymbol{E}(\boldsymbol{r},t) = \sum_{MN} \boldsymbol{E}_{MN}(\boldsymbol{r}) \exp\{iM\theta + iN\phi - i\omega t\}$$

- **Equilibrium distribution function**

$$f_0(r_0,\boldsymbol{v}) = n_0(r_0)\left(\frac{m}{2\pi T_\perp(r_0)}\right)^{\frac{3}{2}} \left(\frac{T_\perp(r_0)}{T_\parallel(r_0)}\right) \exp\left\{-\frac{mv_\perp^2}{2T_\perp(r_0)} - \frac{mv_\parallel^2}{2T_\parallel(r_0)}\right\}$$

2D Formulation in Tokamaks (2)

- **Induced current**

$$
\begin{aligned}
\boldsymbol{J}_{\rm ind}(\boldsymbol{r},t_0) \;=\; & \int \mathrm{d}v_\parallel \int \mathrm{d}v_\perp \int \mathrm{d}\psi \int_0^\infty \mathrm{d}\tau(-q^2)\exp i\omega\tau \\
& \cdot\; n_0\left(\frac{m}{2\pi T_\perp}\right)^{\frac{3}{2}}\left(\frac{T_\perp}{T_\parallel}\right)^{\frac{1}{2}}\exp\left\{-\frac{mv_\perp^2}{2T_\perp}-\frac{mv_\parallel^2}{2T_\parallel}\right\}\overleftrightarrow{G}_0 \\
& \cdot\; \sum_{M,N}\begin{pmatrix} E_{r'MN} \\ E_{\theta' MN} \\ E_{\phi' MN}\end{pmatrix}\exp i\left[M\theta+N\phi-\omega t_0-k_\parallel v_\parallel\tau+\omega\tau\right] \\
& \times\; \sum_{n_1,n_2=-\infty}^{\infty} J_{n_1}\left(\frac{k_\perp v_\perp}{\Omega}\right)J_{n_2}\left(\frac{k_\perp v_\perp}{\Omega}\right)\exp i\left\{n_1(\Omega\tau-\theta_0+\psi)-n_2(-\theta_0+\psi)\right\}
\end{aligned}
$$

$$
\overleftrightarrow{G}_0 = \begin{pmatrix} v_\perp^2\sin(-\theta_0+\psi) \\ -v_\perp^2\cos(-\theta_0+\psi) \\ v_\parallel v_\perp \end{pmatrix}\cdot\begin{pmatrix}
-\dfrac{v_\perp}{T_\perp}\sin(\Omega\tau-\theta_0+\psi)+v_\perp v_\parallel\left(\dfrac{1}{T_\perp}-\dfrac{1}{T_\parallel}\right)\dfrac{N}{R\omega}\sin(\Omega\tau-\theta_0+\psi) \\
\dfrac{v_\perp}{T_\perp}\cos(\Omega\tau-\theta_0+\psi)-v_\perp v_\parallel\left(\dfrac{1}{T_\perp}-\dfrac{1}{T_\parallel}\right)\dfrac{N}{R\omega}\cos(\Omega\tau-\theta_0+\psi) \\
-\dfrac{v_\parallel}{T_\parallel}+v_\perp v_\parallel\left(\dfrac{1}{T_\perp}-\dfrac{1}{T_\parallel}\right)\left\{\dfrac{M}{r_0\omega}\cos(\Omega\tau-\theta_0+\psi)+\dfrac{i}{\omega}\sin(\Omega\tau-\theta_0+\psi)\dfrac{\partial}{\partial r'}\right\}
\end{pmatrix}
$$

2D Formulation in Tokamaks (3)

- **Variable transformation**
  - Velocity $(v_\perp, \psi) \to$ Position (r', r_0)

$$\begin{cases} r = r_0 + \dfrac{v_\perp}{\Omega}\cos(-\theta_0 + \psi) \\ r' = r_0 + \dfrac{v_\perp}{\Omega}\cos(\Omega\tau - \theta_0 + \psi) \end{cases}$$

  - Jacobian: $J = \dfrac{\partial(v_\perp, \psi)}{\partial(r', r_0)} = -\dfrac{\Omega^2}{v_\perp \sin \Omega\tau}$
  - Then

$$\int_0^\infty \mathrm{d}v_\perp \int_0^{2\pi} \mathrm{d}\psi = \int_0^R \mathrm{d}r' \int_0^R \mathrm{d}r_0 \, |J| = \int_0^R \mathrm{d}r' \int_0^R \mathrm{d}r_0 \, \frac{\Omega^2}{v_\perp |\sin \Omega\tau|}$$

- **Fourier expansion with respect to $\Omega\tau$**
- **Time integral with respect to τ**
- **Velocity integral with respect to $v_\parallel$**

2D Formulation in Tokamaks (4)

- **Finally we obtain:**

$$\boldsymbol{J}_{\text{ind}}(\boldsymbol{r},t) = \sum_{MN} \int_0^R \mathrm{d}r' \overleftrightarrow{\sigma}_{MN}(r,r')\boldsymbol{E}_{MN}(r') \exp i\{M\theta + N\phi - \omega t\}$$

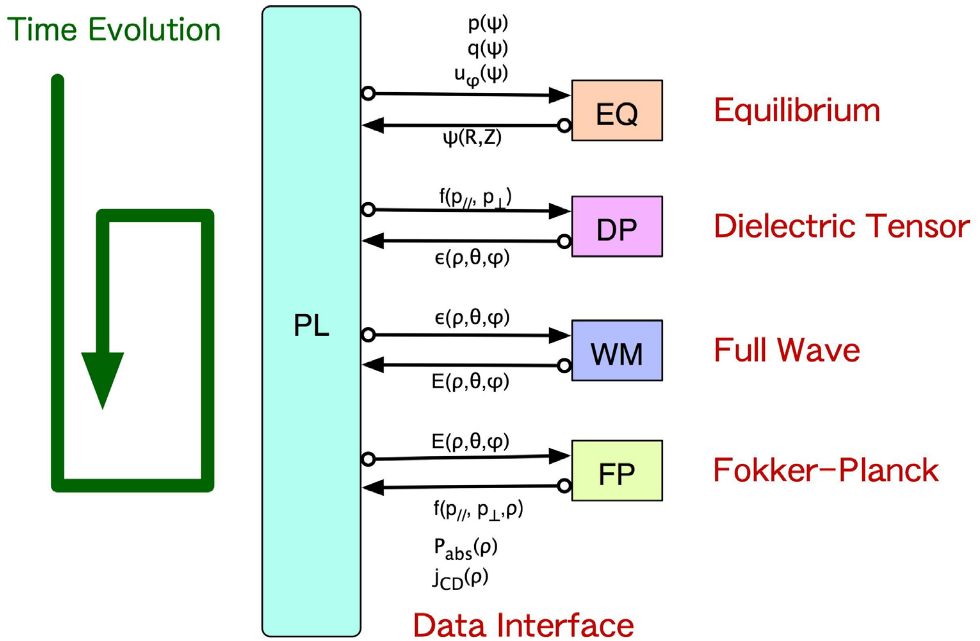
$$\begin{aligned}
\overleftrightarrow{\sigma}_{MN}(r,r') &= -\sum_{n_1 n_2}\sum_{\ell=-\infty}^{\infty}\int_0^R \mathrm{d}r_0\, q^2\Omega^2 n_0 \left(\frac{1}{2\pi}\right)^{\frac{5}{2}} \frac{i}{v_{T\perp}^2} \\
&\quad \times \int_0^{2\pi} \mathrm{d}\lambda \frac{1}{|\sin\lambda|} \exp\left\{-\frac{R_v^2\Omega^2}{2v_{T\perp}^2}\right\} \overleftrightarrow{G}_2 J_{n_1}(k_\perp R_v) J_{n_2}(k_\perp R_v) \\
&\quad \times \exp(-in_1\lambda)\left\{\frac{r - r_0 + iR_m}{R_v}\right\}^{n_1-n_2} \mathrm{e}^{i\ell\lambda}
\end{aligned}$$

Code development using FEM is under way

Self-Consistent Wave Analysis with Modified $f(v)$

- **Modification of velocity distribution from Maxwellian**
  - Absorption of ICRF waves in the presence of energetic ions
  - Current drive efficiency of LHCD
  - NTM controllability of ECCD (absorption width)

- **Self-consistent wave analysis including modification of $f(v)$**

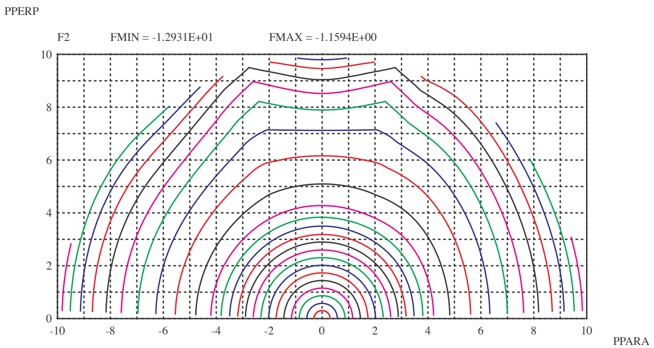


Development of Self-Consistent Wave Analysis

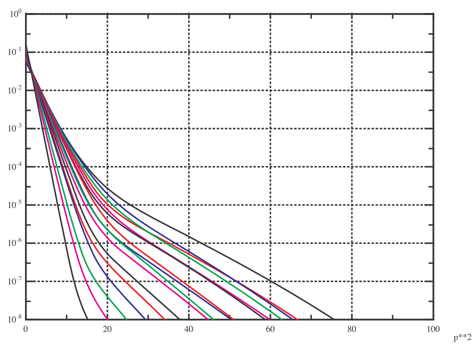
- **Code Development in TASK**
  - Ray tracing analysis with arbitrary $f(v)$: **Already done**
  - Full wave analysis with arbitrary $f(v)$: **Completed**
  - Fokker-Plank analysis of ray tracing results: **Already done**
  - Fokker-Plank analysis of full wave results: **Almost competed**
  - Self-consistent iterative analysis: **Preliminary**

- **Tail formation by ICRF minority heating**

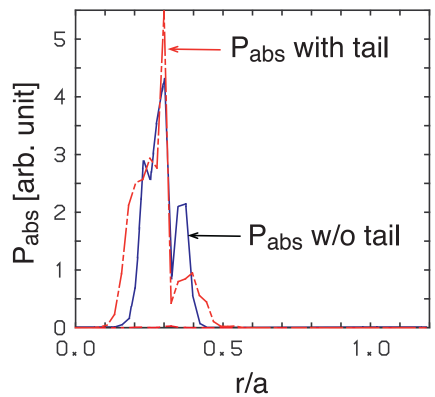
Momentum Distribution



Tail Formation



Power deposition



Summary

- **Several improvement of the TASK code for full wave analysis of wave heating and current drive is under way.**
- **Full wave analysis of EC wave propagation in a small-size ST**
- **Formulation of 2D integro-differential full wave analysis including FLR effects**
  - Formulation was extended to 2D configuration.
  - Implementation is under way.
  - Power absorption in ITER will be examined by next meeting.
- **Self-consistent analysis including modification of velocity distribution**
  - Full wave analysis with arbitrary velocity distribution was completed.
  - Fokker-Planck analysis uses wave fields calculated by the full wave module.
  - Coupling of the full-wave and Fokker-Planck modules is almost completed.