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Modeling of Plasma Rotation and Density Modification

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Contents

Motivation

- **Transport Simulation including Core and SOL Plasmas**
  - **Role of Separatrix**
    - — Closed magnetic surface $\Longleftrightarrow$ Open magnetic field line
    - — Difference of dominant transport process
- **Tansient Behavior of Plasma Rotation**
  - **Radial Electric Field**: Radial force balance, (Poisson equation)
  - **Poloidal rotation**: Equation of motion
  - **Toroidal rotation**: Equation of motion
  - Equation of motion rather than transport matrix
- **Analysis including Atomic Processes**

1D Transport code: TASK/TX

- **Dynamic Transport Equation**: *Fukuyama et al. PPCF (1994)*
  - **Two fluid equation for electrons and ions**
    - — Flux surface averaged
    - — Coupled with Maxwell equation
    - — Neutral diffusion equation
  - **Neoclassical transport**
  - **Turbulent transport**
    - — Current diffusive ballooning mode
    - — Ambipolar diffusion through poloidal momentum transfer
    - — Thermal diffusivity, Perpendicular viscosity
  - **Maxwell's equation, Poisson's equation**
  - **Slowdown equation for beam component**
  - **Diffusion equation for neutral particles**

Model Equation (1)

- **Fluid equations** (electrons and ions)

$$\frac{\partial n_s}{\partial t} = -\frac{1}{r}\frac{\partial}{\partial r}(r n_s u_{sr}) + S_s$$

$$\frac{\partial}{\partial t}(m_s n_s u_{sr}) = -\frac{1}{r}\frac{\partial}{\partial r}(r m_s n_s u_{sr}^2) + \frac{1}{r} m_s n_s u_{s\theta}^2 + e_s n_s (E_r + u_{s\theta} B_\phi - u_{s\phi} B_\theta) - \frac{\partial}{\partial r} n_s T_s$$

$$\frac{\partial}{\partial t}(m_s n_s u_{s\theta}) = -\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 m_s n_s u_{sr} u_{s\theta}) + e_s n_s (E_\theta - u_{sr} B_\phi) + \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^3 n_s m_s \mu_s \frac{\partial}{\partial r}\frac{u_{s\theta}}{r}\right)$$

$$+F_{s\theta}^{\mathrm{NC}} + F_{s\theta}^{\mathrm{C}} + F_{s\theta}^{\mathrm{W}} + F_{s\theta}^{\mathrm{X}} + F_{s\theta}^{\mathrm{L}}$$

$$\frac{\partial}{\partial t}\left(m_s n_s u_{s\phi}\right) = -\frac{1}{r}\frac{\partial}{\partial r}(r m_s n_s u_{sr} u_{s\phi}) + e_s n_s (E_\phi + u_{sr} B_\theta) + \frac{1}{r}\frac{\partial}{\partial r}\left(r n_s m_s \mu_s \frac{\partial}{\partial r} u_{s\phi}\right)$$

$$+F_{s\phi}^{\mathrm{C}} + F_{s\phi}^{\mathrm{W}} + F_{s\phi}^{\mathrm{X}} + F_{s\phi}^{\mathrm{L}}$$

$$\frac{\partial}{\partial t}\frac{3}{2} n_s T_s = -\frac{1}{r}\frac{\partial}{\partial r} r\left(\frac{5}{2} u_{sr} n_s T_s - n_s \chi_s \frac{\partial}{\partial r} T_{\mathrm{e}}\right) + e_s n_s (E_\theta u_{s\theta} + E_\phi u_{s\phi})$$

$$+P_s^{\mathrm{C}} + P_s^{\mathrm{L}} + P_s^{\mathrm{H}}$$

Model Equation (2)

- **Diffusion equation for (fast and slow) neutral particles**

$$\frac{\partial n_0}{\partial t} = -\frac{1}{r}\frac{\partial}{\partial r}\left(-rD_0\frac{\partial n_0}{\partial r}\right) + S_0$$

- **Maxwell's equation**

$$\frac{1}{r}\frac{\partial}{\partial r}(rE_r) = \frac{1}{\epsilon_0}\sum_s e_s n_s$$

$$\frac{\partial B_\theta}{\partial t} = \frac{\partial E_\phi}{\partial r}, \qquad \frac{\partial B_\phi}{\partial t} = -\frac{1}{r}\frac{\partial}{\partial r}(rE_\phi)$$

$$\frac{1}{c^2}\frac{\partial E_\theta}{\partial t} = -\frac{\partial}{\partial r}B_\phi - \mu_0\sum_s e_s n_s u_{s\theta}, \qquad \frac{1}{c^2}\frac{\partial E_\phi}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}(rB_\theta) - \mu_0\sum_s e_s n_s u_{s\phi}$$

Neoclassical Transport Model

- **Neoclassical transport**
  - Viscosity force arises when plasma rotates in the poloidal direction.
  - Banana-Plateau regime

$$F_{s\theta}^{\mathrm{NC}} = -\sqrt{\pi} q^2 n_s m_s \frac{v_{\mathrm{T}s}}{qR} \frac{\nu_s^*}{1+\nu_s^*} u_{s\theta}$$

$$\nu_{\mathrm{s}}^* \equiv \frac{\nu_s qR}{\epsilon^{3/2} v_{\mathrm{T}s}}$$

- **This poloidal viscosity force induces**
  - Neoclassical radial diffusion
  - Neoclassical resistivity
  - Bootstrap current
  - Ware pinch

Turbulent Transport Model

- **Turbulent Diffusion**
  - Poloidal momentum exchange between electron and ion through the turbulent electric field
  - Ambipolar flux (electron flux = ion flux)

$$F_{\mathrm{i}\theta}^{\mathrm{W}} = -\ F_{\mathrm{e}\theta}^{\mathrm{W}}$$

$$= -ZeB_\phi n_\mathrm{i} D_\mathrm{i}\left[-\frac{1}{n_\mathrm{i}}\frac{dn_\mathrm{i}}{dr} + \frac{Ze}{T_\mathrm{i}}E_r - \langle\frac{\omega}{m}\rangle\frac{ZeB_\phi}{T_\mathrm{i}} - \left(\frac{\mu_\mathrm{i}}{D_\mathrm{i}} - \frac{1}{2}\right)\frac{1}{T_\mathrm{i}}\frac{dT_\mathrm{i}}{dr}\right]$$

- **Perpendicular viscosity**
  - Non-ambipolar flux (electron flux ≠ ion flux): μ_s = constant $\times\, D$
- **Diffusion coefficient** (proportional to $|E|^2$)
  - Current-diffusive ballooning mode turbulence model

Model of Scrape-Off Layer Plasma

- **Particle, momentum and heat losses along the field line**
  - Decay time

$$\nu_L = \begin{cases} 0 & (0 < r < a) \\ \dfrac{C_s}{2\pi r R\{1 + \log[1 + 0.05/(r - a)]\}} & (a < r < b) \end{cases}$$

  - **Electron source term**

$$S_e = n_0 \langle \sigma_{ion} v \rangle n_e - \nu_L (n_e - n_{e,div})$$

- **Recycling from divertor**
  - Recycling rate: $\gamma_0 = 0.8$
  - Neutral source

$$S_0 = \frac{\gamma_0}{Z_i} \nu_L (n_e - n_{e,div}) - \frac{1}{Z_i} n_0 \langle \sigma_{ion} v \rangle n_e + \frac{P_b}{E_b}$$

- **Gas puff from wall**

Steady State Flux (1)

- **Electron flux**
  - Inertia term in the equation of motion = 0
- **Radial flux**

$$u_{er} = -\frac{1}{1+\alpha}\frac{\bar{\nu}_e+\nu_{ei}}{n_e m_e \Omega_{e\phi}^2}\left[\frac{dP_e}{dr}+\frac{dP_i}{dr}\right] - \frac{\alpha}{1+\alpha}\frac{E_\phi}{B_\theta} + \frac{1}{1+\alpha}\frac{F_\theta^W}{n_e m_e \Omega_{e\phi}}$$

$$+\frac{1}{1+\alpha}\frac{\bar{\nu}_e}{\Omega_{e\phi}}u_{i\theta} + \frac{\alpha}{1+\alpha}\frac{B_\phi}{B_\theta}\frac{\nu_{eb}}{\Omega_{e\phi}}(u_{b\phi}-u_{i\phi})$$

where $\alpha \equiv \dfrac{\bar{\nu}_e+\nu_{ei}}{\nu_{ei}+\nu_{eb}}\dfrac{B_\theta^2}{B_\phi^2}, \quad \nu_{eb} = \dfrac{n_b m_b}{n_e m_e}\nu_{be}$

  - Poloidal neoclassical viscosity $\bar{\nu}_e$
  - Factor α represents parallel neoclassical viscosity
  - First three terms in RHS are neoclassical diffusion, Ware pinch and turbulent diffusion.

Steady State Flux (2)

- **Toroidal current**

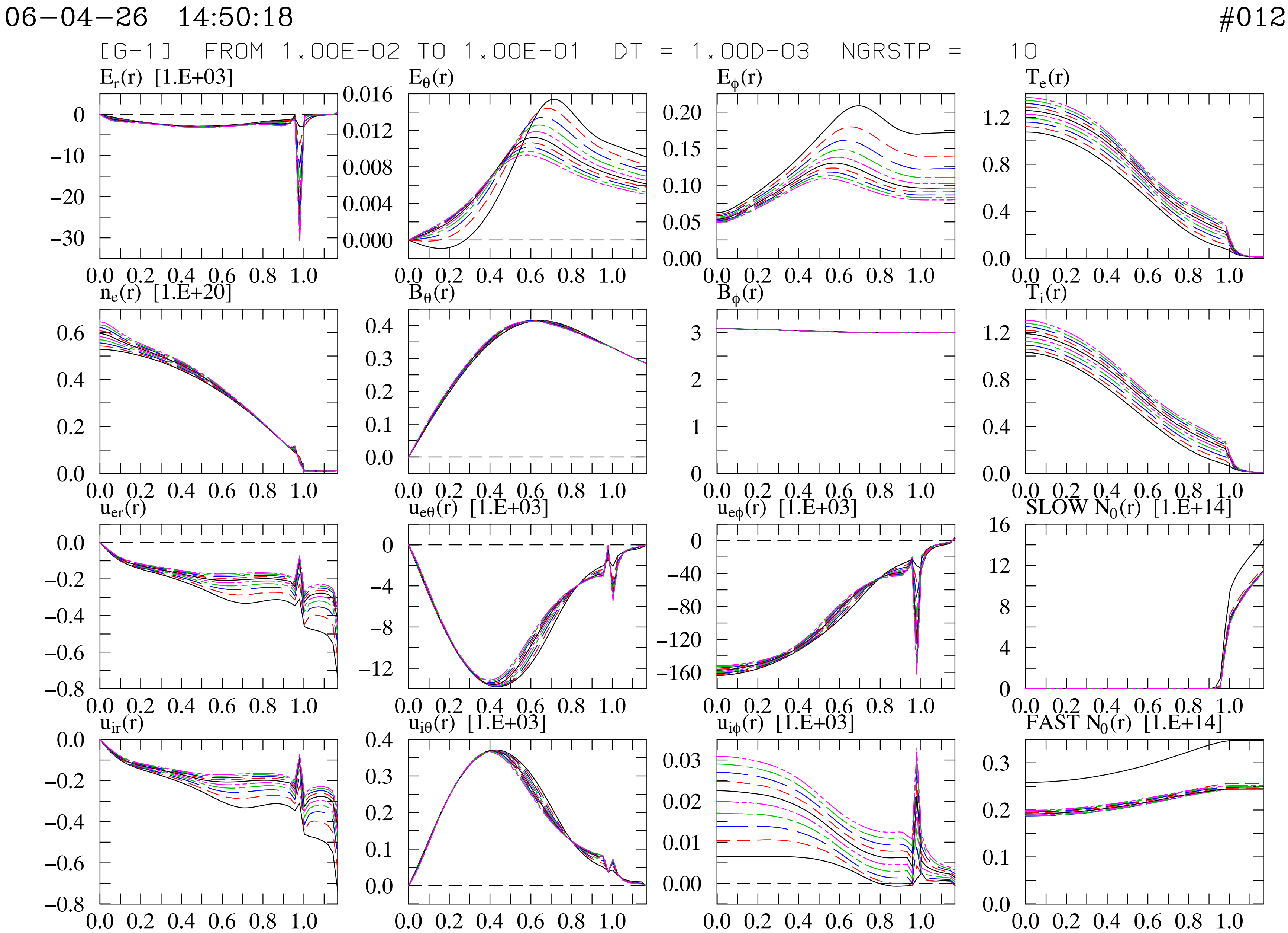
$$u_{\mathrm{e}\phi} = \frac{-1}{\nu_{\mathrm{ei}} + \nu_{\mathrm{eb}}} \left\{ \frac{1}{1+\alpha} \frac{e}{m_{\mathrm{e}}} E_\phi + \frac{1}{1+\alpha} \frac{B_\theta}{B_\phi} \frac{\bar{\nu}_{\mathrm{e}} + \nu_{\mathrm{ei}}}{n_{\mathrm{e}} m_{\mathrm{e}} \Omega_{\mathrm{e}\phi}} \left[\frac{\mathrm{d}P_{\mathrm{e}}}{\mathrm{d}r} + \frac{\mathrm{d}P_{\mathrm{i}}}{\mathrm{d}r} \right] \right.$$
$$+ \frac{1}{1+\alpha} \frac{B_\theta}{B_\phi} \frac{F_\theta^{\mathrm{W}}}{n_{\mathrm{e}} m_{\mathrm{e}}} + \frac{\bar{\nu}_{\mathrm{e}}}{1+\alpha} \frac{B_\theta}{B_\phi} u_{\mathrm{i}\theta}$$
$$\left. - \left(\nu_{\mathrm{eb}} - \frac{\alpha}{1+\alpha} \nu_{\mathrm{ei}} \right) u_{\mathrm{b}\phi} - \left(\nu_{\mathrm{ei}} + \frac{\alpha}{1+\alpha} \nu_{\mathrm{eb}} \right) u_{\mathrm{i}\phi} \right\}$$

  - The first two terms in RHS are neoclassical resistivity and boot-strap current

- **Similar expression for poloidal rotation**

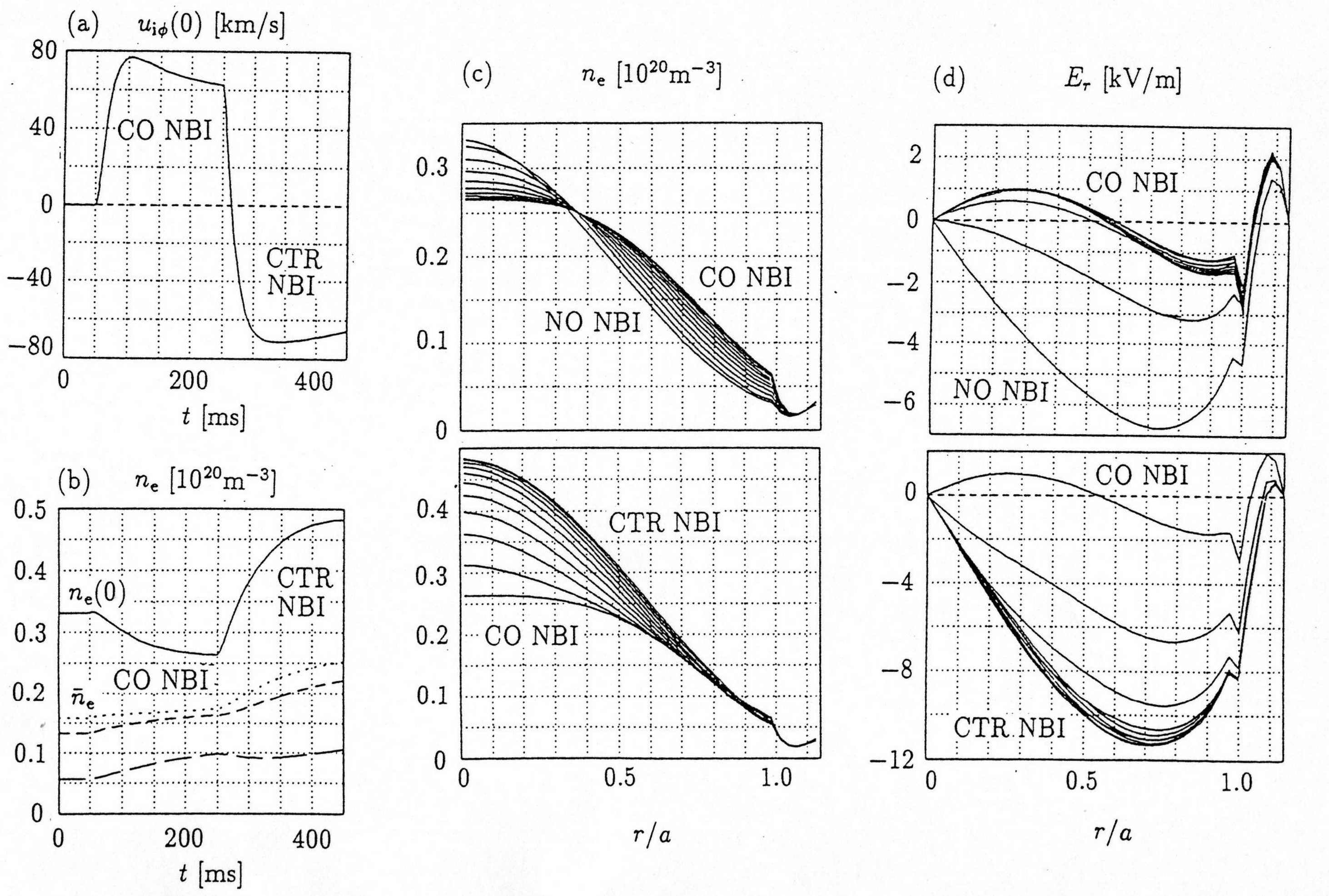
Model equations include dominant neoclassical transport.

Typical Profiles



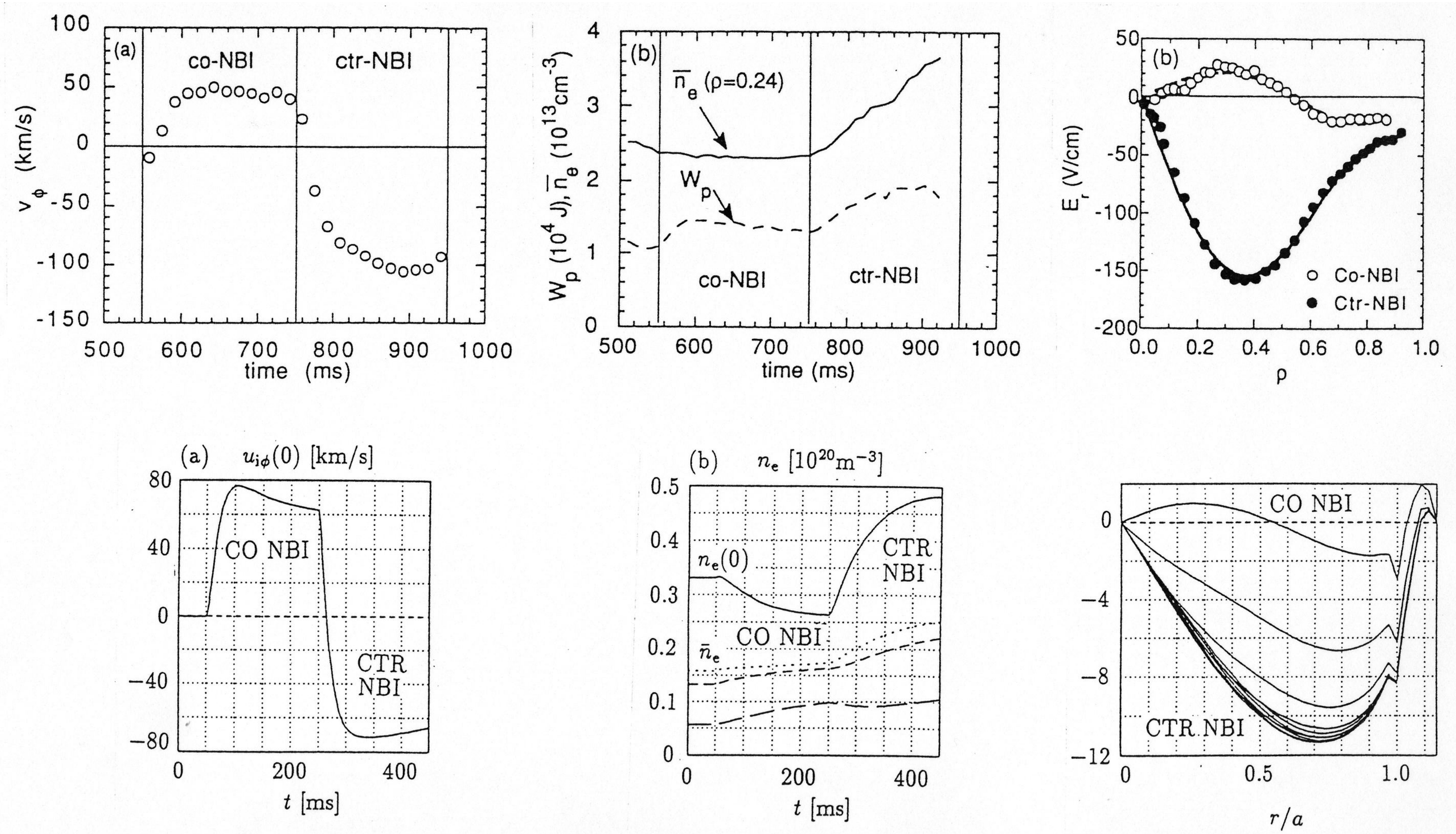
Simulation of plasma rotation and radial electric field

- **JFT-2M parameter**: NBI co-injection $\longrightarrow$ counter-injection
- Toroidal rotation $\Longrightarrow$ Negative E_r $\Longrightarrow$ Density peaking
- **TASK/TX**: Particle Diffusivity: $0.3\,\mathrm{m^2/s}$, Ion viscosity: $10\,\mathrm{m^2/s}$



Comparison with JFT-2M Experiment

- **JFT-2M Experiment**: **Ida et al.: Phys. Rev. Lett. 68 (1992) 182**
- Good agreement with experimental observation

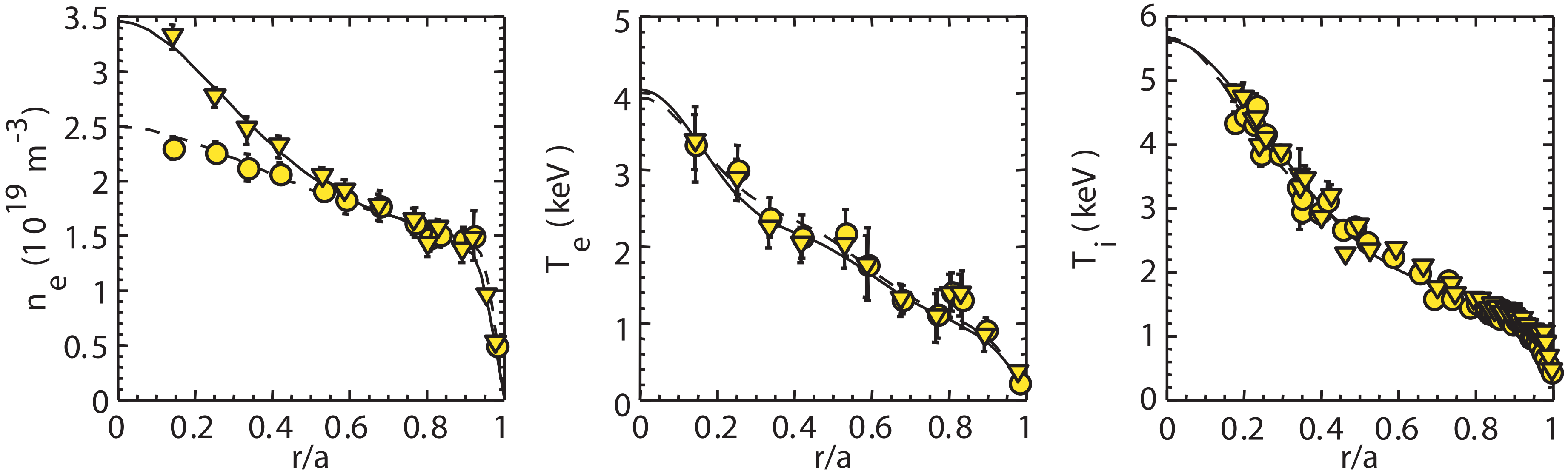


Density Peaking Due to Momentum Input

- Density peaking was observed in NBI counter injection on JT60-U.
  Ref. Takenaga et al. (ITPA, 2005)

co-injection : $n_e(r/a=0.2)/\langle n_e\rangle=1.41$
$P_{IN}=8.4MW$, $P_{ABS}=6.3MW$, $P_{LOSS}^{FAST}/P_{IN}=0.26$, $H_H^{98(y,2)}=0.86$

ctr-injection : $n_e(r/a=0.2)/\langle n_e\rangle=1.74$
$P_{IN}=10.2MW$, $P_{ABS}=6.5MW$, $P_{LOSS}^{FAST}/P_{IN}=0.38$, $H_H^{98(y,2)}=0.81$

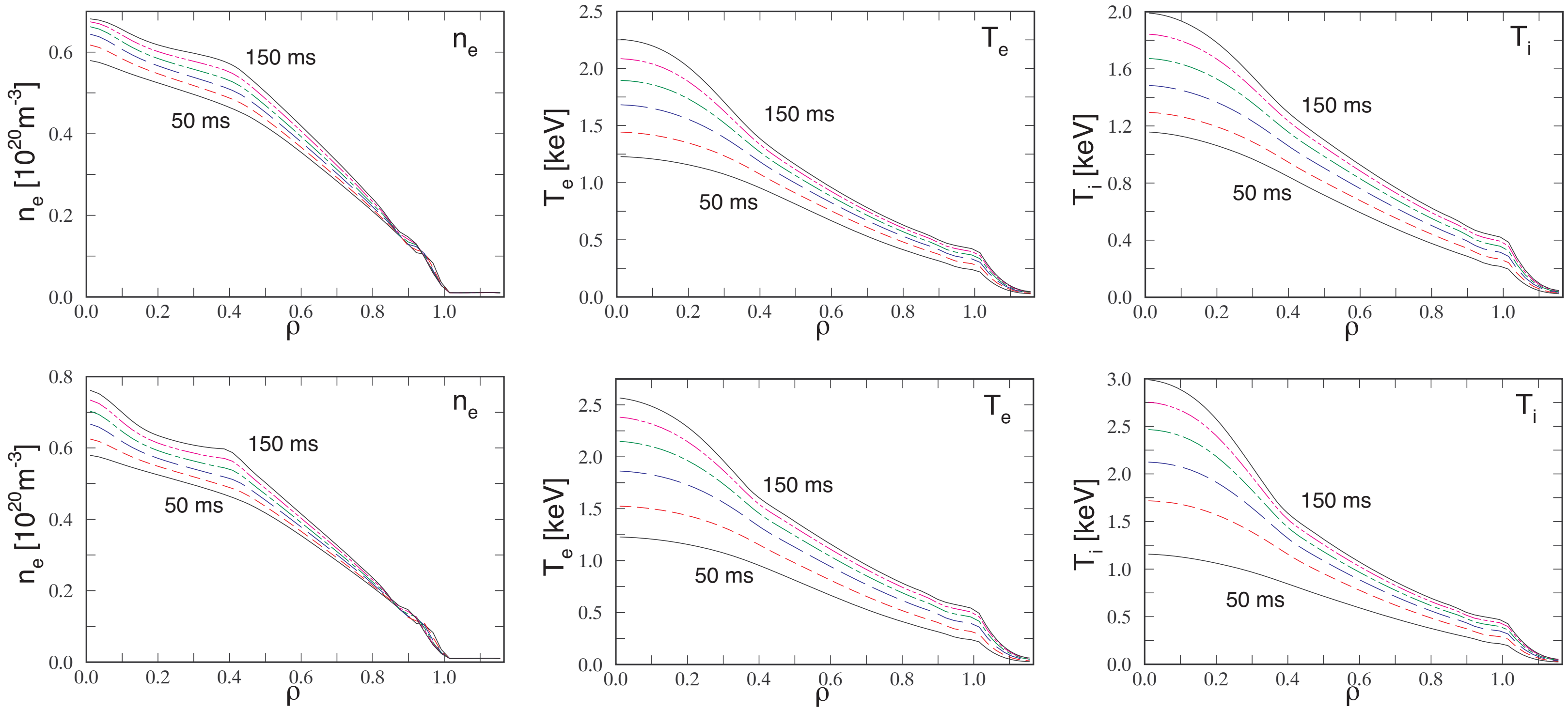


Density Peaking Simulation (1)

- Transport model: CDBM (particle diffusivity, thermal diffusivity)
- **NBI** 6.5 MW was injected 50 ms after the simulation started.
- **Simulation results**
  - $n(0)$ **for co injection is 12% higher than that for counter injection**
  - **Temperature is higher for counter injection ↔ experiments**

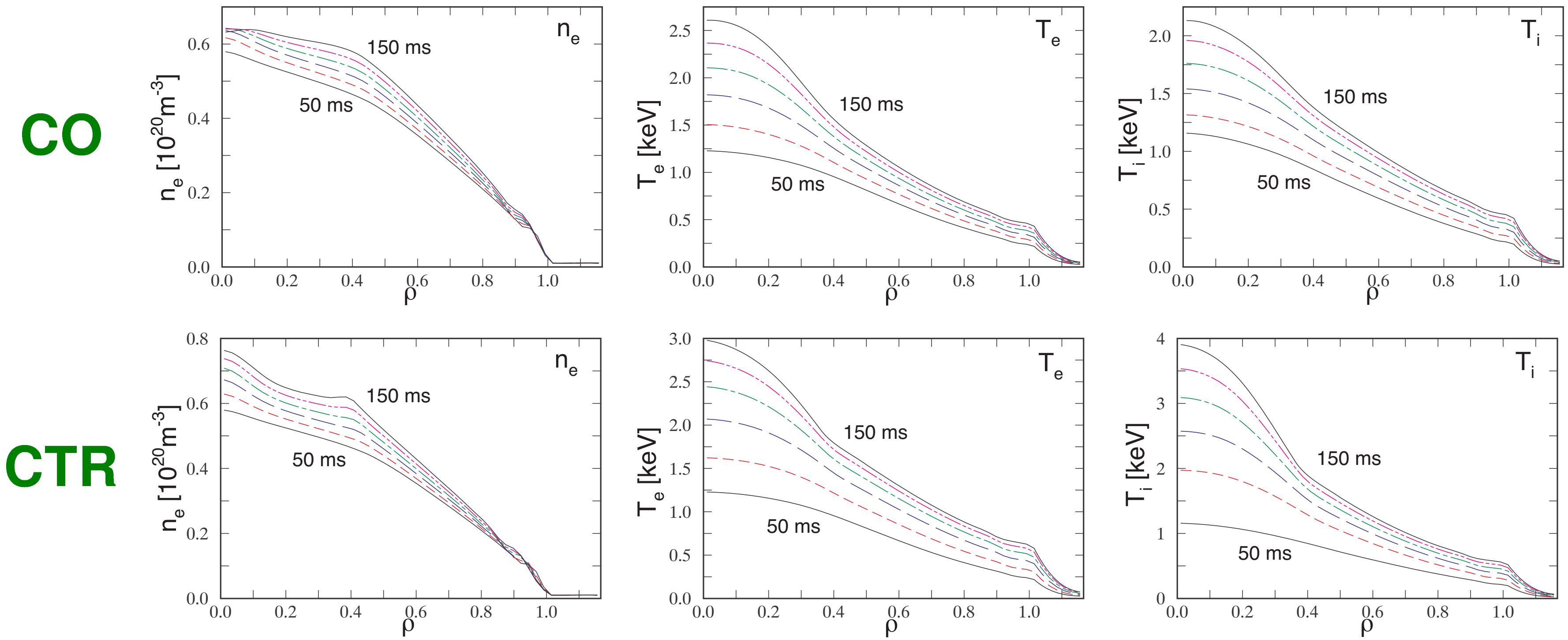
CO

CTR



Density Peaking Simulation (2)

- **NBI $10\,\mathrm{MW}$ heating**
  - **Co-injection: Density flattening**
  - **Counter-injection: ITB formation**



Summary

- In order to describe the plasma rotation consistently, we have formulated a set of dynamical transport equations and implemented it a transport code TASK/TX.
- The numerical stability of TASK/TX was improved by using cubic-spline FEM version.
- Density modification due to toroidal momentum input was successfully reproduced.
- Further improvement in modeling is required for realistic and robust simulation.