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Tokamak Transport Simulation with Gyrokinetic Transport Model Including the Effect of Zonal Flow

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Plasma Turbulence
Itoh Project 2004-2008

Motivations

- **Prediction and Control of Highly Autonomous Burning Plasmas**
  - Development of a reliable turbulent transport model
    - Validation by systematic comparison with experimental data
    - Validation by quantitative comparison with direct numerical simulation
  - Integrated tokamak modeling code including various phenomena
- **Comparison of transport models by transport simulation**
  - Experimental data from ITPA profile database: 55 shots
  - Heat transport simulation by TASK/TR code
  - Turbulent transport models: CDBM, CDBM05, GLF23, Weiland

Contents

- Transport simulation with a simple fluid transport model including the effect of zonal flow
- Development of gyrokinetic transport model including the effect of zonal flow
- Future development of the gyrokinetic transport model

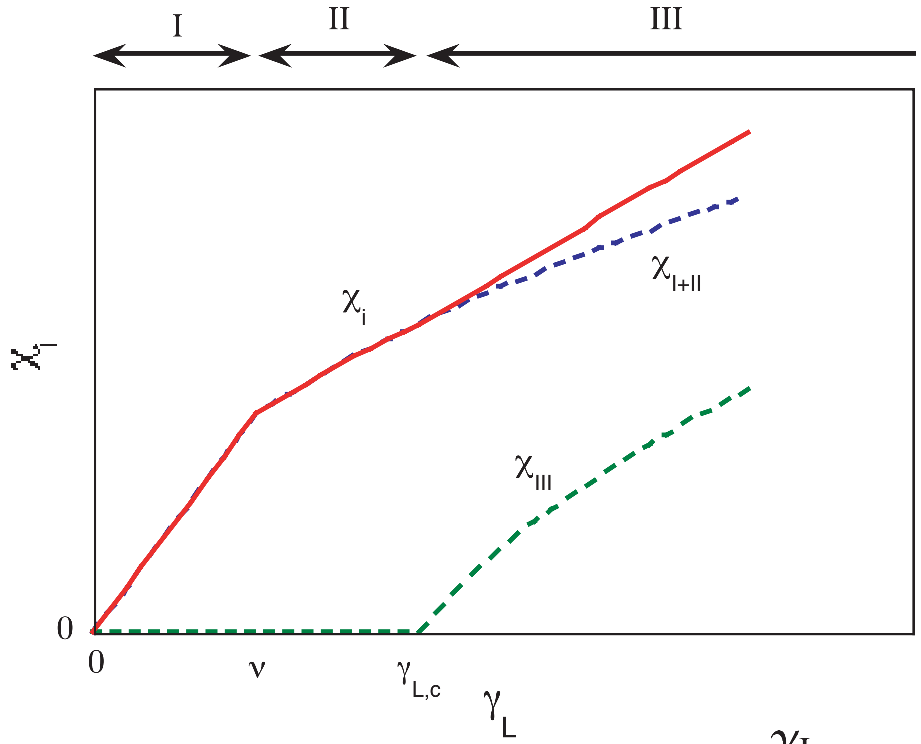
Turbulent Transport Model including Zonal Flow (1)

- **Ref.: K. Itoh et al., Phys. Plasmas 12 (2005) 062303**
- **Fitting Formula**: [$\Theta(\gamma_{\mathrm{L}} - \gamma_{\mathrm{L,c}})$: Heaviside function]

$$\chi_{\mathrm{i}} = \sqrt{\chi_{\mathrm{I+II}}^2 + \chi_{\mathrm{III}}^2 \Theta(\gamma_{\mathrm{L}} - \gamma_{\mathrm{L,c}})}$$

Region I + II	Region III
$\gamma_{\mathrm{L}} < \gamma_{\mathrm{L,c}}$	$\gamma_{\mathrm{L}} > \gamma_{\mathrm{L,c}}$
$\chi_{\mathrm{I+II}} = \frac{\sqrt{\nu}}{\sqrt{\gamma_{\mathrm{L}}} + \sqrt{\nu}} \frac{\gamma_{\mathrm{L}}}{k_r^2}$ where $\nu = \frac{1}{1-\mu} \frac{k_\perp^4}{k_\theta^2 q_r^2} \nu_{\mathrm{damp}},$ $\mu = (1 + 2q^2) \frac{\mu_\parallel}{D_{rr}}$	$\chi_{\mathrm{III}} = A \left(\sqrt{1 + \frac{2}{A} \frac{\gamma_{\mathrm{L}} - \gamma_{\mathrm{L,c}}}{\gamma_{\mathrm{L}}}} - 1 \right) \frac{\gamma_{\mathrm{L}}}{k_r^2}$ where $A = \frac{\mu H \rho_{\mathrm{s}}^2 k_\perp^4}{4(1-\mu) q_r^2}, \qquad \gamma_{\mathrm{L,c}} = \frac{(1-\mu)^2}{\sqrt{2}\mu H} \frac{k_r^2}{k_\perp^2} \omega_*$ $H = \frac{2}{1 + k_\perp^2 \rho_{\mathrm{s}}^2} + \frac{6 q_r^2}{\Delta\omega^2 \rho_{\mathrm{s}}^2} \frac{\partial v_{\mathrm{g}}^2}{\partial k_r^2}$

Turbulent Transport Model including Zonal Flow (2)



Region I: Negligible zonal flow $\chi_{\mathrm{I}} \sim \dfrac{\gamma_{\mathrm{L}}}{k_r^2}$ for $\gamma_{\mathrm{L}} \ll \nu$

Region II: Collisional damping of zonal flow $\chi_{\mathrm{II}} \sim \sqrt{\dfrac{\nu}{\gamma_{\mathrm{L}}}} \dfrac{\gamma_{\mathrm{L}}}{k_r^2}$ for $\gamma_{\mathrm{L}} \gg \nu$

Region III: Nonlinear damping of zonal flow $\chi_{\mathrm{III}} \sim \dfrac{\gamma_{\mathrm{L}} - \gamma_{\mathrm{L,c}}}{k_r^2}$ for $\gamma_{\mathrm{L}} \gtrsim \gamma_{\mathrm{L,c}}$

$\sqrt{2A} \sim O(k_\perp \rho_{\mathrm{s}})$ $\chi_{\mathrm{III}} \sim \sqrt{2A}\, \dfrac{\gamma_{\mathrm{L}}}{k_r^2}$ for $\gamma_{\mathrm{L}} \gg \gamma_{\mathrm{L,c}}$

Transport Simulation with an ITG Model including Zonal Flow

- **Choice of linear growth rate γ_L**
- **Choice of dominant wave number k_θ, k_r, q_r**
- **Simple model ： ITG model proposed by R. Waltz (1984)**
  - Linear growth rate:

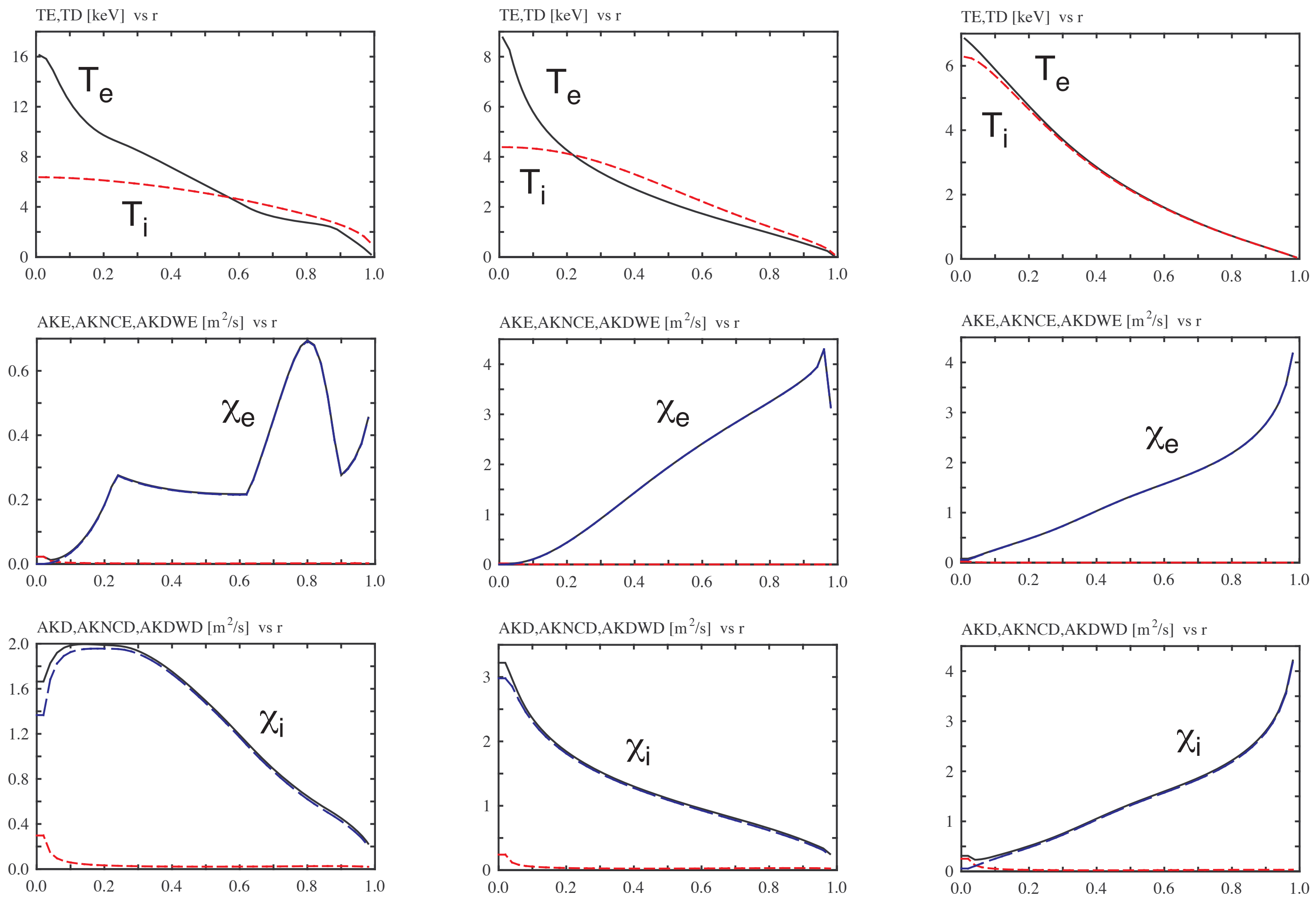
$$\gamma_L = 2.5\, H(\xi) \sqrt{\frac{2\eta_i T_i L_n}{T_e R}}\, \omega_*$$

$$H(\xi) = \frac{1}{1 + \exp[-6(\eta_i - \eta_c)]}$$

  - Electron: Trapped electron and resistive circulating electron mode
  - Assumption: $k_\perp \rho_s = 0.3$, $k_\theta \sim k_r \sim q_r$, $\mu = 1/2$
  - Coefficients required for reproducing experimental data
    - — Original Waltz model: 0.05
    - — Zonal flow model: 1

Result of Transport Simulation

ITG (Waltz) | ITG with Zonal Flow | CDBM



Gyrokinetic Transport Model including Zonal flow

1. **Ballooning model** : **TASK/EGK**

Gyrokinetic linear eigen mode analysis using Ballooning approximation

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Fitting formula of zonal effect

2. **Two-dimensional model** : **TASK/WM**

Two-dimensional linear eigen mode analysis using gyro-kinetic dielectric

Fitting formula of zonal effect

3. **Poloidal Flow model** : **TASK/TX**

Gyrokinetic linear eigen mode analysis using Ballooning approximation

＋

Fitting formula of zonal effect +

Dynamic tranport simulation

Ballooning type model (1)

- **Ref.: G. Rewoldt et al., Phys. Fluids 25 (1982) 480.**
- **Particle phase-space distribution**: $h_s = h_{s\sigma}(E, \mu, \theta)$

$$f_s = F_{\mathrm{M}s} - \frac{e_s \phi}{T_s} F_{\mathrm{M}s} + h_s$$

- **Ballooning representation**:

$$\Phi(\psi, \theta, \zeta, t) = \exp[-\mathrm{i}\,\omega t + \mathrm{i}\,n\zeta - \mathrm{i}\,nq(\psi)\theta] \times \sum_{\ell=-\infty}^{\infty} \phi(\theta - 2\ell\pi, \psi) \exp[\mathrm{i}\,nq(\psi) 2\ell\pi]$$

- **Gyrokinetic equation**: $J_n = J_n(k_\perp \rho_s)$

$$\left[\omega + \mathrm{i}\,\nu_{\mathrm{C}s} - \omega_{\mathrm{d}s} + \mathrm{i}\,v_\parallel \frac{B^\theta}{B} \frac{\mathrm{d}}{\mathrm{d}\theta}\right] h_s = (\omega - \omega_{\mathrm{T}s}) \frac{e_s}{T_s} F_{\mathrm{M}s} \left(J_0 \phi - v_\parallel J_0 A_\parallel - \mathrm{i}\,v_\perp J_1 A_\perp\right)$$

Ballooning type model (1)

- **Poisson equation**

$$-\nabla^2\phi = \frac{1}{\epsilon_0}\sum_s e_s\delta n_s = \frac{1}{\epsilon_0}\sum_s e_s\int \mathrm{d}v^3\left(-\frac{e_s\phi}{T_s}F_{\mathrm{M}s} + J_0\,h_s\right)$$

- **Ampere's law**

$$-\nabla^2 A_{\|} = \mu_0\sum_s \delta j_{s\|} = \mu_0\sum_s e_s\int \mathrm{d}v^3\,v_{\|}\,J_0\,h_s$$

$$-\nabla^2 A_{\perp} = \mu_0\sum_s \delta_{s\perp} = \mu_0\sum_s e_s\int \mathrm{d}v^3\,(-\mathrm{i}\,v_{\perp})\,J_1\,h_s$$

- Eigenmode equation

$\Longrightarrow$ Real frequency ω_{R}, Linear growth rate γ_{L}

$\Longrightarrow$ Fitting formula of ZOnal flow

Two-dimensional model

- **Gyrokinetic dielectric tensor**:
  - **Particle distribution function**：$h_s = h_{s\sigma}(E, \mu, \theta)$

$$f_s = F_{\mathrm{M}s} - \frac{e_s \phi}{T_s} F_{\mathrm{M}s} + h_s$$

  - **Gyrokinetic equation**: $J_n = J_n(k_\perp \rho_s)$

$$\left[\omega + \mathrm{i}\,\nu_{\mathrm{C}s} - \omega_{\mathrm{d}s} - k_\parallel v_\parallel\right] h_s = (\omega - \omega_{\mathrm{T}s}) \frac{e_s}{T_s} F_{\mathrm{M}s} \left(J_0\,\phi - v_\parallel J_0\,A_\parallel - \mathrm{i}\,v_\perp\,J_1 A_\perp\right)$$

  - **Induced current**:

$$\boldsymbol{j}_s = e_s \int \mathrm{d}v^3\,\boldsymbol{v}\,\left(-\frac{e_s \phi}{T_s} F_{\mathrm{M}s} + h_s\right)$$

- **Maxwell's equation**: two-dimensional boundary value problem

$$\nabla \times \nabla \times \boldsymbol{E} = \frac{\omega^2}{c^2} \overleftrightarrow{\epsilon} \cdot \boldsymbol{E} + \mathrm{i}\,\omega\mu_0 \boldsymbol{j}_{\mathrm{ext}}$$

- Eigenmode equation $\Longrightarrow$ Linear growth reate γ_{L} $\Longrightarrow$ Fitting formula of the effect of Zonal Flow

Poloidal Flow Model

- **Time Evolutioin of poloidal flow driven by zonal flow**
- **Dynamic transport equatioin (Flux-Averaged fluid equation)**

$$\frac{\partial n_s}{\partial t} = -\frac{1}{r}\frac{\partial}{\partial r}(r n_s u_{sr}) + S_s$$

$$\frac{\partial}{\partial t}(m_s n_s u_{sr}) = -\frac{1}{r}\frac{\partial}{\partial r}(r m_s n_s u_{sr}^2) + \frac{1}{r} m_s n_s u_{s\theta}^2 + e_s n_s (E_r + u_{s\theta} B_\phi - u_{s\phi} B_\theta) - \frac{\partial}{\partial r} n_s T_s$$

$$\frac{\partial}{\partial t}(m_s n_s u_{s\theta}) = -\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 m_s n_s u_{sr} u_{s\theta}) + e_s n_s (E_\theta - u_{sr} B_\phi) + \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^3 n_s m_s \mu_s \frac{\partial}{\partial r}\frac{u_{s\theta}}{r}\right)$$

$$+F_{s\theta}^{\mathrm{NC}} + F_{s\theta}^{\mathrm{C}} + F_{s\theta}^{\mathrm{W}} + F_{s\theta}^{\mathrm{X}} + F_{s\theta}^{\mathrm{L}} + F_{s\theta}^{\mathrm{ZF}}$$

$$\frac{\partial}{\partial t}\left(m_s n_s u_{s\phi}\right) = -\frac{1}{r}\frac{\partial}{\partial r}(r m_s n_s u_{sr} u_{s\phi}) + e_s n_s (E_\phi + u_{sr} B_\theta) + \frac{1}{r}\frac{\partial}{\partial r}\left(r n_s m_s \mu_s \frac{\partial}{\partial r} u_{s\phi}\right)$$

$$+F_{s\phi}^{\mathrm{C}} + F_{s\phi}^{\mathrm{W}} + F_{s\phi}^{\mathrm{X}} + F_{s\phi}^{\mathrm{L}} + F_{s\phi}^{\mathrm{ZF}}$$

$$\frac{\partial}{\partial t}\frac{3}{2} n_s T_s = -\frac{1}{r}\frac{\partial}{\partial r} r \left(\frac{5}{2} u_{sr} n_s T_s - n_s \chi_s \frac{\partial}{\partial r} T_{\mathrm{e}}\right) + e_s n_s (E_\theta u_{s\theta} + E_\phi u_{s\phi})$$

$$+P_s^{\mathrm{C}} + P_s^{\mathrm{L}} + P_s^{\mathrm{H}}$$

Summary

- Using a linear growth rate of the fluid ITG mode and a fitting formula of the ion heat transport including the effect of zonal flow derived by K. Itoh et al., we have carried out heat transport simulations. The reduction of ion heat transport due to the zonal flow contributes to reproduce experimental observations.

- In order to estimate the linear growth rate of gyrokinetic microscopic instabilities, we are developing a ballooning-type eigenmode solver, TASK/EGK. The linear growth rate will be used to estimate the ion heat transport coefficiet fby the fitting formula and to carry out transport simulation.

- Two-dimensional gyrokinetic analysis of micro instabilities and tranport simulations including the effect of poloidal flow explicitly are being studied as a part of long term researches.