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Advanced Full Wave Analyses in Tokamak Plasmas

A. Fukuyama
Department of Nuclear Engineering, Kyoto University

Outline

- Present Status of TASK Code
- Full wave analysis of ECH in small ST
- Self-consistent analysis of wave heating and current drive
- Eigenmode analysis
- Summary
- Future Plan

TASK Code

- **T**ransport **A**nalysing **S**ystem for Tokama**K**
- **Features**
  - **A Core of Integrated Modeling Code in BPSI**
    - Modular structure, Unified Standard data interface
  - **Various Heating and Current Drive Scheme**
    - EC, LH, IC, AW, (NB)
  - **High Portability**
    - Most of Library Routines Included
  - **Development using CVS** (Concurrent Version System)
    - Open Source (V0.93 http://bpsi.nucleng.kyoto-u.ac.jp/task/)
  - **Parallel Processing using MPI Library**
  - **Extension to Toroidal Helical Plasmas**

Modules of TASK

EQ	**2D Equilibrium**	Fixed/Free boundary, Toroidal rotation
TR	**1D Transport**	Diffusive transport, Transport models
WR	**3D Geometr. Optics**	EC, LH: Ray tracing, Beam tracing
WM	**3D Full Wave**	IC, AW: Antenna excitation, Eigen mode
FP	**3D Fokker-Planck**	Relativistic, Bounce-averaged
DP	**Wave Dispersion**	Local dielectric tensor, Arbitrary $f(\boldsymbol{v})$
PL	**Data Interface**	Data conversion, Profile database
LIB	**Libraries**	

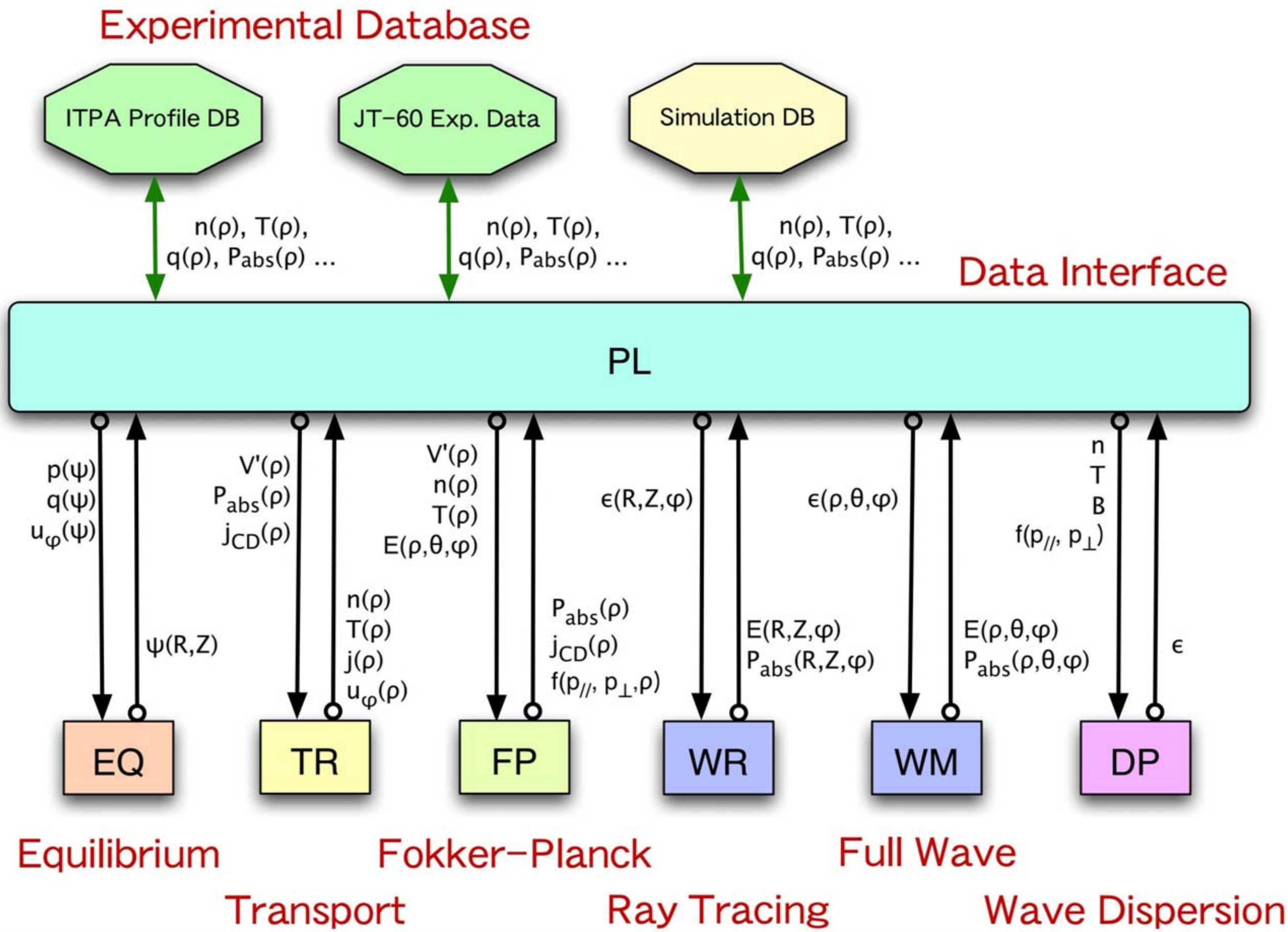
under development

TX	Transport analysis including plasma rotation and E_r
WA	Global linear stability analysis

in collaboration

TOPICS-EQU	Free-boundary equilibrium: Azumi (JAEA)

Modular Structure of TASK



Wave Dispersion Analysis : TASK/DP

- **Various Models of Dielectric Tensor** $\overleftrightarrow{\epsilon}(\omega, \boldsymbol{k}; \boldsymbol{r})$:
  - **Resistive MHD** model
  - **Collisional cold** plasma model
  - **Collisional warm** plasma model
  - **Kinetic plasma** model (**Maxwellian**, non-relativistic)
  - **Kinetic plasma** model (**Arbitrary** $f(\boldsymbol{v})$, relativistic)
  - **Gyro-kinetic plasma** model (Maxwellian)
- **Numerical Integration in momentum space**: **Arbitrary** $f(\boldsymbol{v})$
  - Relativistic Maxwellian
  - Output of TASK/FP: Fokker-Planck code

Relativistic Dielectric Tensor

- **Dielectric Tensor**: $\omega_p = \sqrt{n_s e_s^2/m_s\epsilon_0}$, $\omega_c = e_s B/m_s$

$$\epsilon_{ij} = \delta_{ij} + \frac{\omega_p^2}{\omega^2}\int \mathrm{d}\boldsymbol{p}\, p_\perp \sum_n \Pi_{in}^* \Pi_{jn} L_n f_0$$

$$+ \frac{\omega_p^2}{\omega^2}\delta_{3i}\delta_{3j}\int \mathrm{d}\boldsymbol{p}\frac{p_\|}{\gamma}\left[\frac{\partial f_0}{\partial p_\|} - \frac{p_\|}{p_\perp}\frac{\partial f_0}{\partial p_\perp}\right]$$

- **Factor Π_{in}**: $\xi \equiv k_\perp p_\perp / m\omega_c$

$$\Pi_{1n} \equiv \frac{n}{\xi}J_n(\xi) \qquad \Pi_{2n} \equiv \mathrm{i}\, J_n'(\xi) \qquad \Pi_{3n} \equiv \frac{p_\|}{p_\perp}J_n(\xi)$$

- **Operator L_n**:

$$L_n \equiv \frac{1}{\gamma - n\dfrac{\omega_c}{\omega} - \dfrac{k_\| p_\|}{m\omega}}\left[\left(1 - \frac{k_\| p_\|}{m\omega\gamma}\right)\frac{\partial}{\partial p_\perp} + \frac{k_\| p_\perp}{m\omega\gamma}\frac{\partial}{\partial p_\|}\right]$$

Full wave analysis: TASK/WM

- **magnetic surface coordinate**: (ψ, θ, φ)
- Boundary-value problem of **Maxwell's equation**

$$\nabla \times \nabla \times \boldsymbol{E} = \frac{\omega^2}{c^2} \overleftrightarrow{\epsilon} \cdot \boldsymbol{E} + \mathrm{i}\,\omega\mu_0 \boldsymbol{j}_{\mathrm{ext}}$$

- Kinetic **dielectric tensor**: $\overleftrightarrow{\epsilon}$
  - **Wave-particle resonance**: $Z[(\omega - n\omega_{\mathrm{c}})/k_{\|}v_{\mathrm{th}}]$
  - **Fast ion: Drift-kinetic**

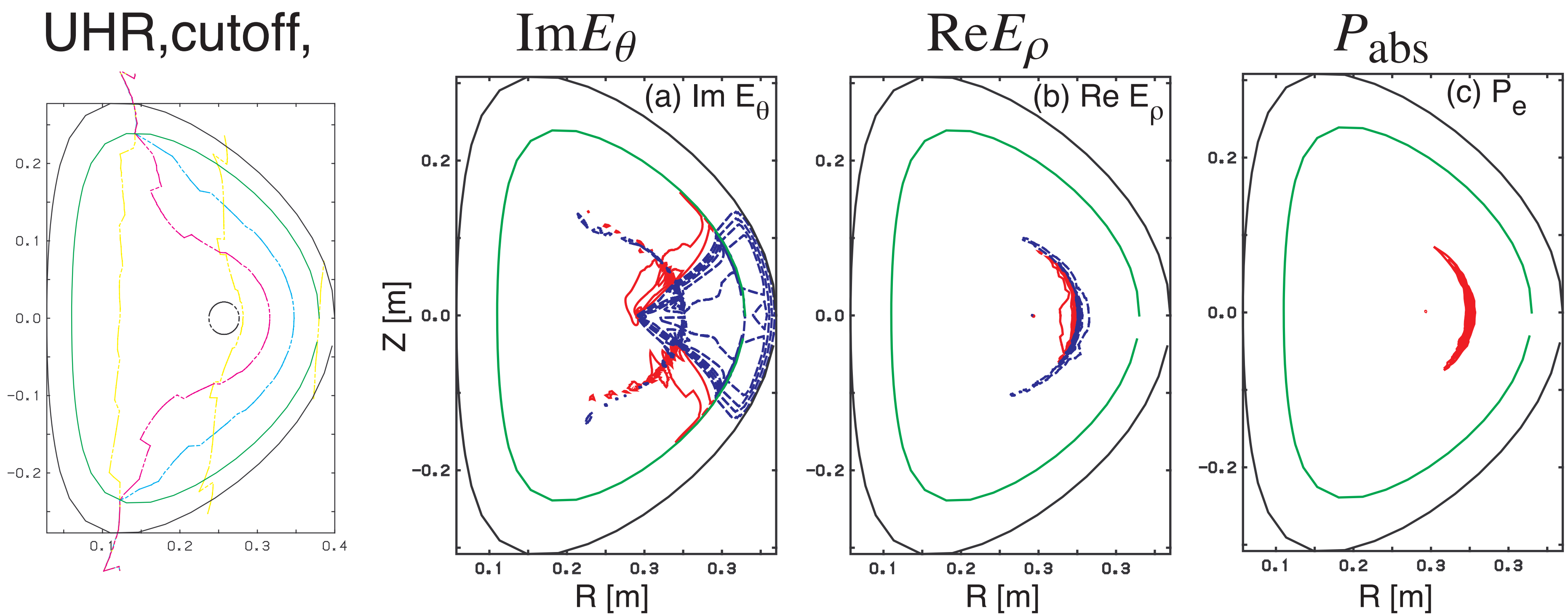
$$\left[\frac{\partial}{\partial t} + v_{\|}\nabla_{\|} + (\boldsymbol{v}_{\mathrm{d}} + \boldsymbol{v}_{\mathrm{E}}) \cdot \nabla + \frac{e_\alpha}{m_\alpha}(v_{\|}E_{\|} + \boldsymbol{v}_{\mathrm{d}} \cdot \boldsymbol{E})\frac{\partial}{\partial \varepsilon}\right] f_\alpha = 0$$

- Poloidal and toroidal **mode expansion**
  - **Accurate estimation of $k_{\|}$**
- Eigenmode analysis: **Complex eigen frequency** which maximize wave amplitude for fixed excitation proportional to electron density

Full Wave Analysis of ECH in a Small-Size ST

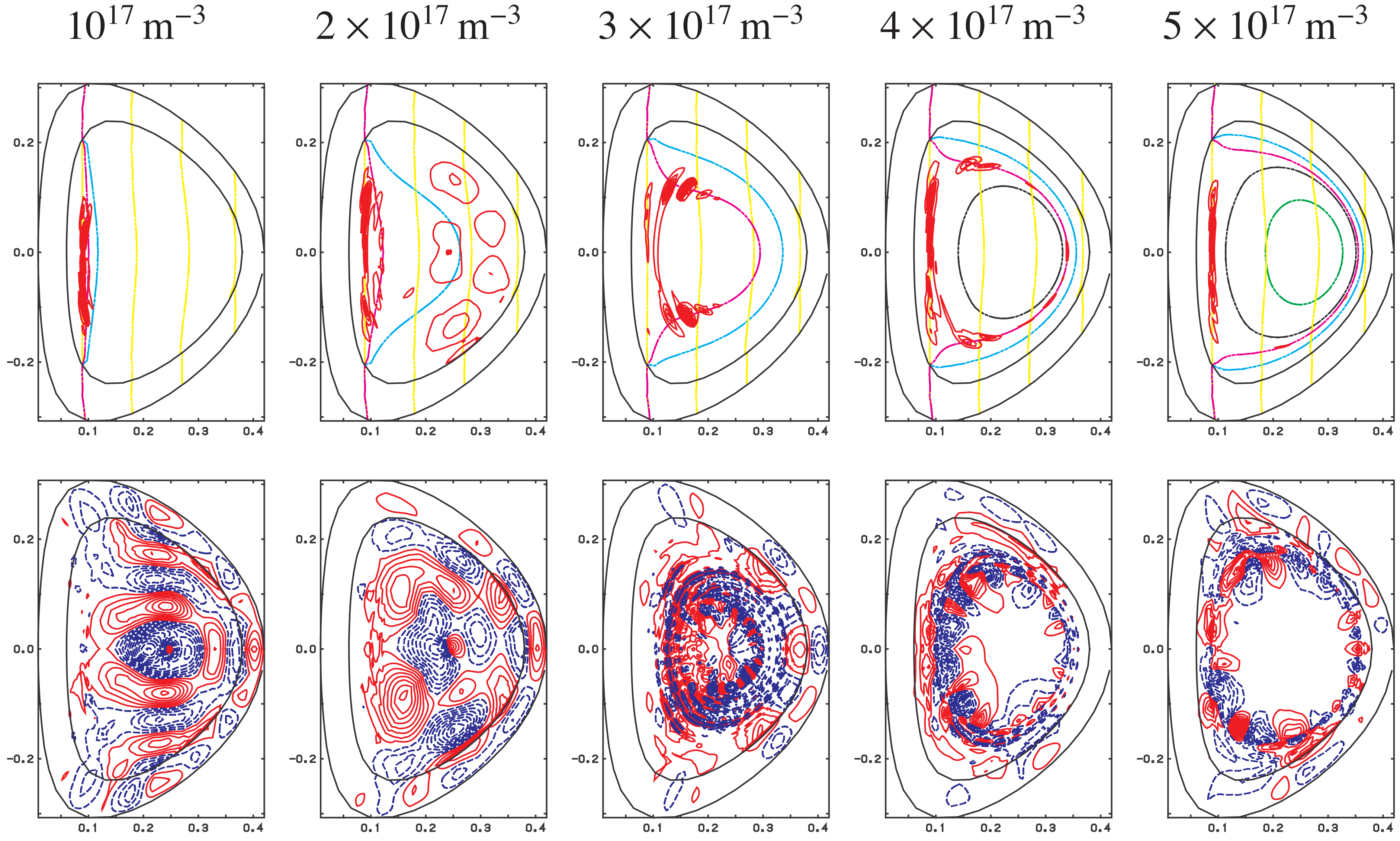
- **Small-size spherical tokamak: LATE** (Kyoto University)
  - **T. Maekawa et al., IAEA-CN-116/EX/P4-27 (Vilamoura, Portuga, 2004)**
  - $R = 0.22\,\mathrm{m}$, $a = 0.16\,\mathrm{m}$, $B_0 = 0.0552\,\mathrm{T}$, $I_p = 6.25\,\mathrm{kA}$, $\kappa = 1.5$
  - $f = 2.8\,\mathrm{GHz}$, Toroidal mode number $n = 12$, Extraordinary mode

Penetration through cutoff layer	$\Longrightarrow$	Propagation along the UHR layer	$\Longrightarrow$	Collisional damping near the UHR layer



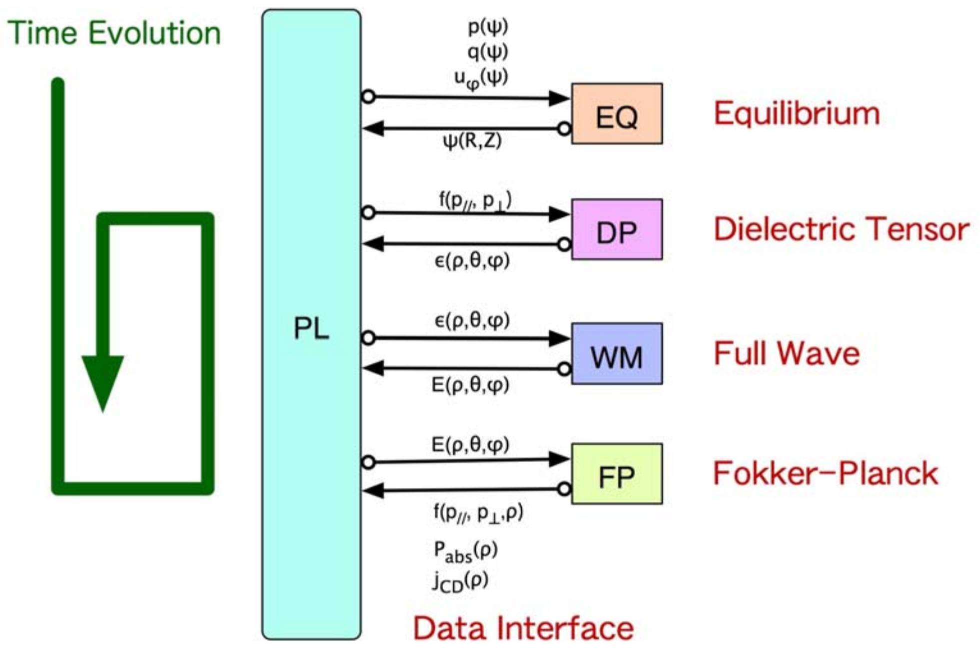
Density Dependence of ECW Propagation

LATE: $5\,\mathrm{GHz}, 0.072\,\mathrm{T}, N_{\|} = 0.5$



Self-Consistent Wave Analysis with Modified $f(v)$

- **Modification of velocity distribution from Maxwellian**
  - Absorption of ICRF waves in the presence of energetic ions
  - Current drive efficiency of LHCD
  - NTM controllability of ECCD (absorption width)
- **Self-consistent wave analysis including modification of $f(v)$**



Fokker-Planck Analysis : TASK/FP

- **Fokker-Planck equation**
  for **velocity distribution function** $f(p_{||}, p_\perp, \psi, t)$

$$\frac{\partial f}{\partial t} = E(f) + C(f) + Q(f) + L(f)$$

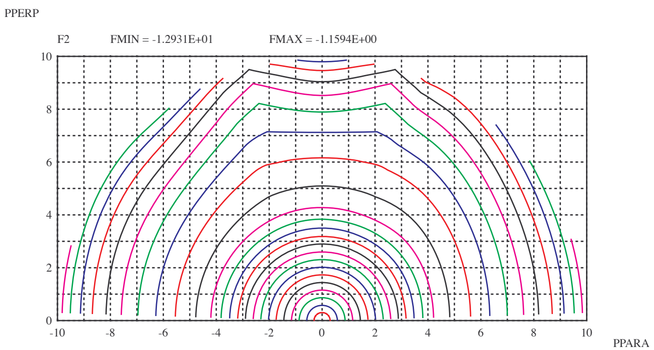
  - $E(f)$: Acceleration term due to DC electric field
  - $C(f)$: Coulomb collision term
  - $Q(f)$: Quasi-linear term due to wave-particle resonance
  - $L(f)$: Spatial diffusion term
- **Bounce-averaged**: Trapped particle effect, zero banana width
- **Relativistic**: momentum $\boldsymbol{p}$, weakly relativistic collision term
- **Nonlinear collision**: momentum or energy conservation
- **Three-dimensional**: spatial diffusion (neoclassical, turbulent)

Development of Self-Consistent Wave Analysis

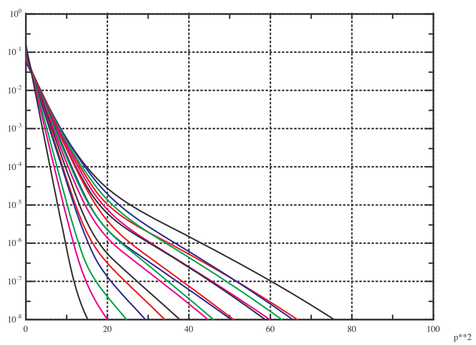
- **Code Development in TASK**
  - Ray tracing analysis with arbitrary $f(v)$: **Already done**
  - Full wave analysis with arbitrary $f(v)$: **Completed**
  - Fokker-Plank analysis of ray tracing results: **Already done**
  - Fokker-Plank analysis of full wave results: **Almost competed**
  - Self-consistent iterative analysis: **Preliminary**

- **Tail formation by ICRF minority heating**

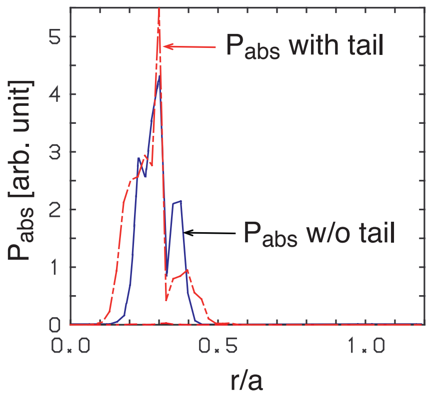
Momentum Distribution



Tail Formation



Power deposition

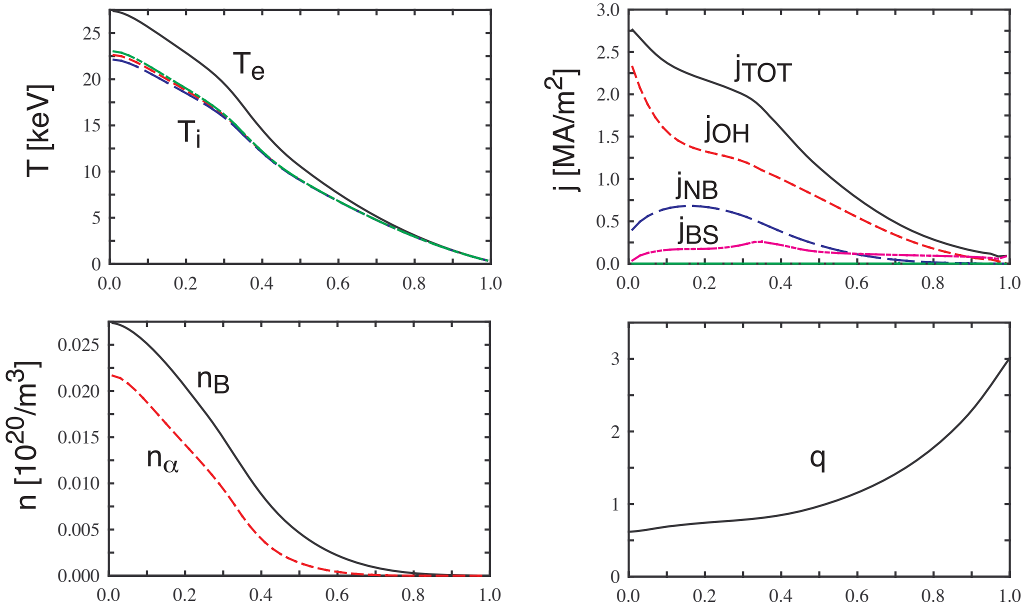


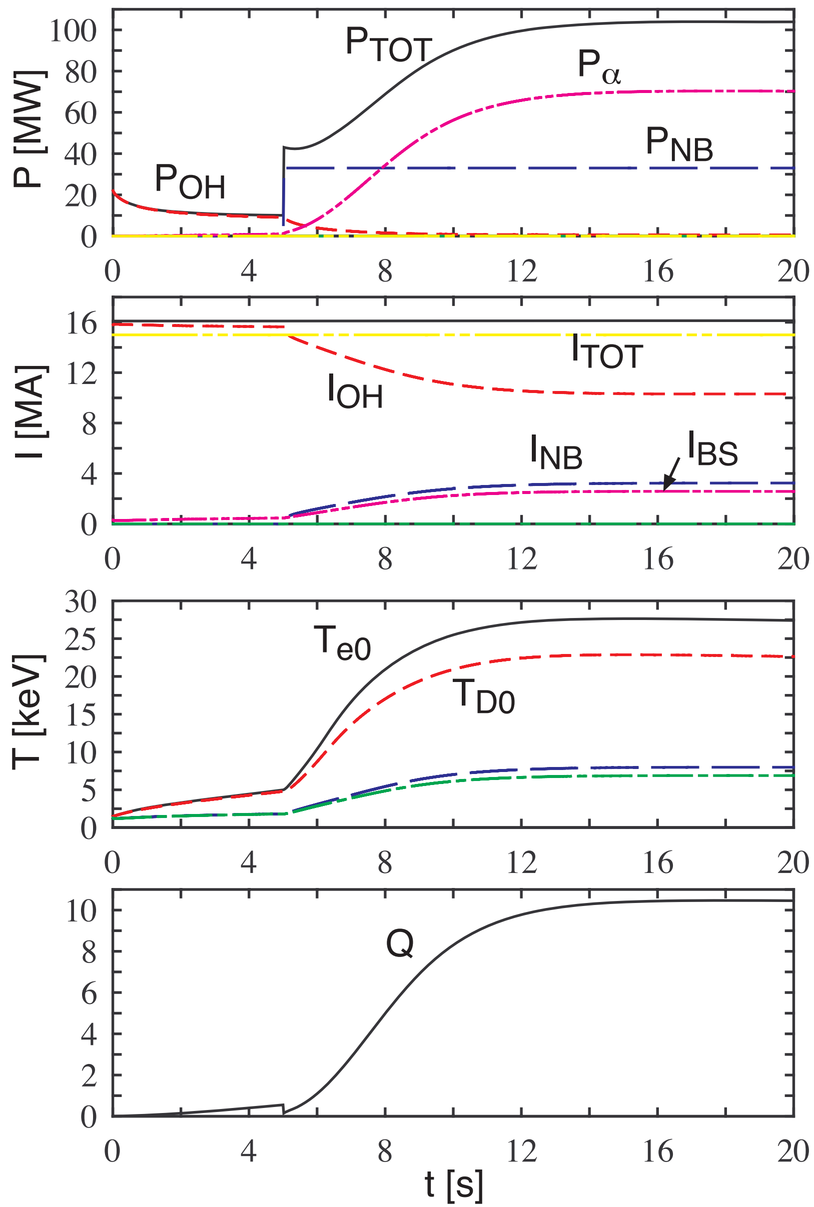
Integrated Analysis of AE in ITER Plasma

- **Combined Analysis**
  - **Equilibrium**: TASK/EQ
  - **Transport**: TASK/TR
    - Turbulent transport model: CDBM
    - Neoclassical transport model: NCLASS (**Houlberg**)
    - Heating and current profile: given profile
  - **Full wave analysis**: TASK/WM
- **Stability analysis**
  - Standard H-mode operation: $I_{\rm p} = 15\,{\rm MA}$, $Q \sim 10$
  - Hybrid operation: $I_{\rm p} = 12\,{\rm MA}$, flat q profile above 1
  - Steady-state operation: $I_{\rm p} = 9\,{\rm MA}$, reversed shear

Standard H-mode Operation

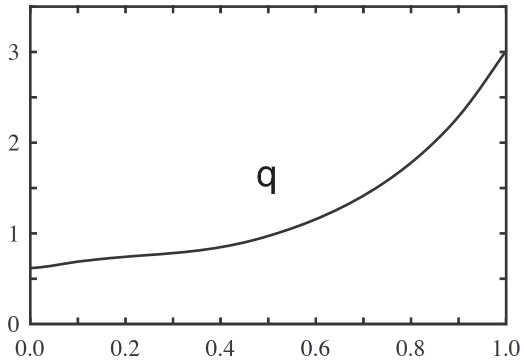
- $I_p = 15\,\mathrm{MA}$
- $P_{NB} = 33\,\mathrm{MW}$
- $\beta_N = 1.3$



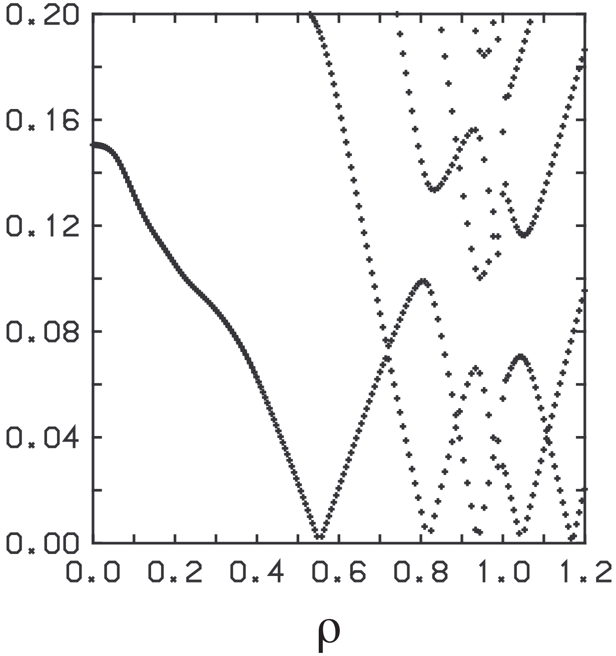


AE in Standard H-mode Operation

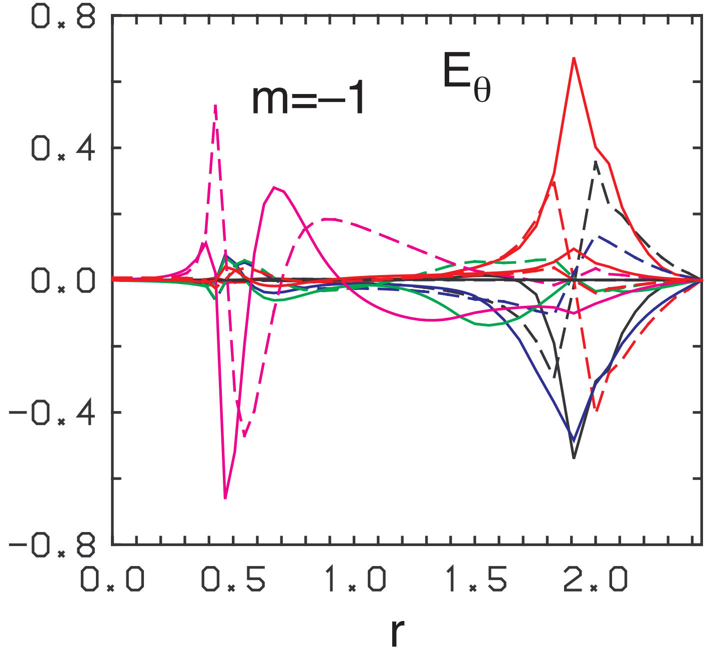
q profile



Alfvén Continuum



Mode structure ($n = 1$)



$$f_r = 95.95\,\mathrm{kHz}$$
$$f_i = -1.95\,\mathrm{kHz}$$

Stabilization due to $q = 1$

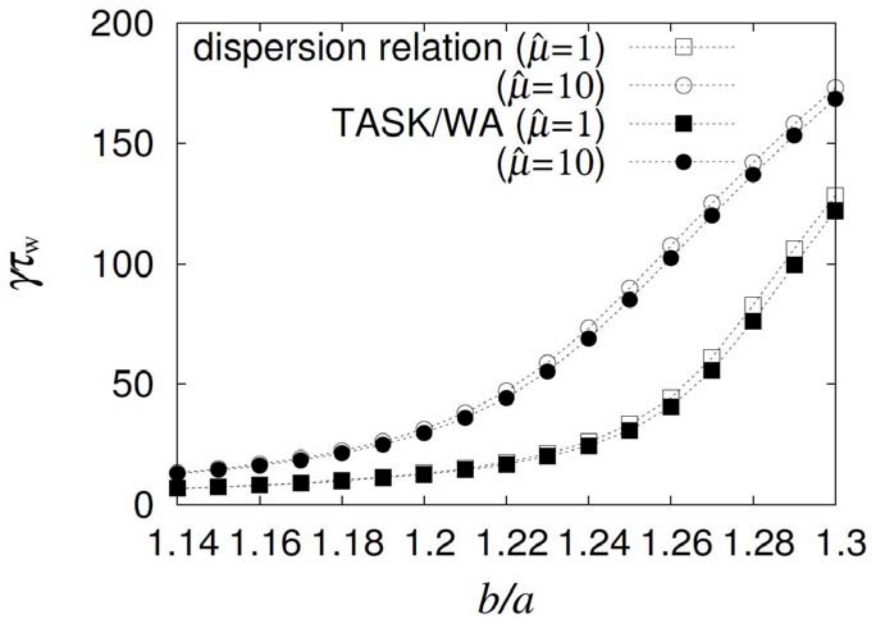
Full Wave Analysis of RWM (TASK/WA)

- **Full wave analysis**: solving Maxwell's equation

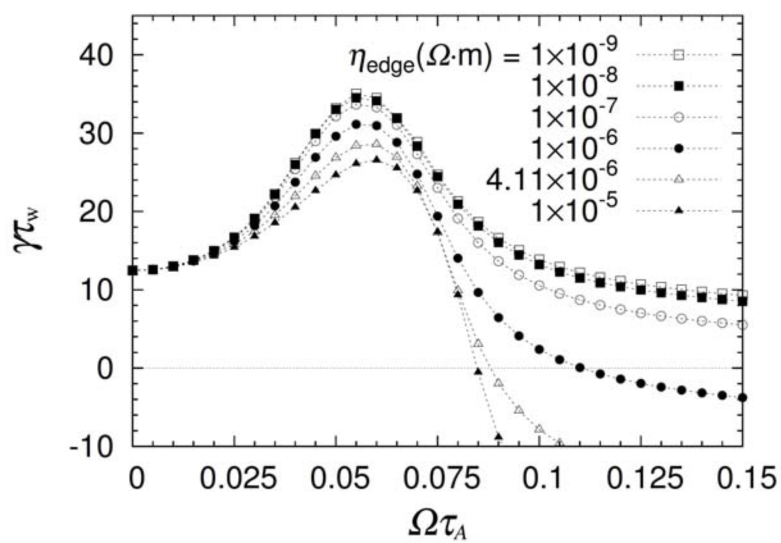
$$\nabla \times \nabla \times \boldsymbol{E} = \frac{\omega^2}{c^2} \overleftrightarrow{\epsilon} \cdot \boldsymbol{E} + \mathrm{i}\,\omega\mu_0 \boldsymbol{j}_{\mathrm{ext}}$$

- **Resistive MHD dielectric tensor including diamagnetic flow**
- **Ferromagnetic Resistive wall**

b/a dependence



Rotation dependence



Summary (1)

- **Several improvement of the TASK code for full wave analysis of wave heating and current drive is under way.**

- **Full wave analysis of EC wave propagation in a small-size ST**
  - Tunneling through the cutoff layer and absorption on the upper hybrid layer were described.
  - The description of electron Bernstein waves requires to include FLR effects in TASK/WM.

- **Formulation of 2D integro-differential full wave analysis including FLR effects**: **Next talk**

Summary (2)

- **Self-consistent analysis including modification of velocity distribution**
  - Full wave analysis with arbitrary velocity distribution was completed.
  - Fokker-Planck analysis uses wave fields calculated by the full wave module.
  - Coupling of the full-wave and Fokker-Planck modules is almost completed.

Future Plan of TASK Code

	Present Status	In 2 years	In 5 years
Equlibrium	Fixed/Free Boundary	Equilibrium Evolution	Start Up Analysis
Core Transport	1D Diffusive TR	Kinetic TR	2D Fluid TR
	1D Dynamic TR		
SOL Transport		2D Fluid TR	Plasma-Wall Interaction
Neutral Tranport	1D Diffusive TR	Orbit Following	
Energetic Ions	Kinetic Evolution	Orbit Following	
Wave Beam	Ray/Beam Tracing	Beam Propagation	
Full Wave	Kinetic ϵ	Gyro Integral ϵ	Orbit Integral ϵ
Stabilities	Sawtooth Osc.	Tearing Mode	Systematic Stability Analysis
	ELM Model	Resistive Wall Mode	
Turbulent Transport	CDBM Model	Linear GK + ZF	Nonlinear ZK + ZF
		Diagnostic Module	
		Control Module	

Future Plan of Integrated Full Wave Analysis

- **DP**: dielectric tensor
  - **2D integral operator for Maxwewllean**: under way
  - **2D integral operator for arbitrary $f(v)$**: planned
  - **Gyrokinetic arbitrary $f(v)$**: planned
- **WM**: full wave analysis
  - **Update to FEM version**: under way
  - **Waveguide excitation**: under way
- **FP**: Fokker-Planck analysis
  - **Integral quasi-linear operator**: formulation
  - **Radial diffusion**: Re-installation