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Integrated Full Wave Analysis of RF Heating and Current Drive in Toroidal Plasmas

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Outline

- Present Status of TASK Code
- Full wave analysis of ECH in small ST
- Self-consistent analysis of wave heating and current drive
- Integral formulation of full wave analysis
- Summary

Motivation

- **Full wave analysis is required in various situations in tokamaks and helical systems**
  - **Ion cyclotron wave** : wavelength $\sim$ inhomogeneous scale length
  - **Lower hybrid wave** : effect of multiple reflections
  - **Electron cyclotron wave** : tunneling through a cutoff layer
- **Further extension is required for integrated analysis**
  - **Consistent analysis including the modification of $f_0(v)$**
  - **Application to high-frequency waves**
  - **Precise analysis including the FLR effects**
- **Integrated analysis using the TASK code**

TASK Code

- **Transport Analysing System for TokamaK**
- **Features**
  - **A Core of Integrated Modeling Code in BPSI**
    — Modular structure, Unified Standard data interface
  - **Various Heating and Current Drive Scheme**
    — EC, LH, IC, AW, (NB)
  - **High Portability**
    — Most of Library Routines Included
  - **Development using CVS** (Concurrent Version System)
    — Open Source (V0.93 http://bpsi.nucleng.kyoto-u.ac.jp/task/)
  - **Parallel Processing using MPI Library**
  - **Extension to Toroidal Helical Plasmas**

Modules of TASK

EQ	**2D Equilibrium**	Fixed/Free boundary, Toroidal rotation
TR	**1D Transport**	Diffusive transport, Transport models
WR	**3D Geometr. Optics**	EC, LH: Ray tracing, Beam tracing
WM	**3D Full Wave**	IC, AW: Antenna excitation, Eigen mode
FP	**3D Fokker-Planck**	Relativistic, Bounce-averaged
DP	**Wave Dispersion**	Local dielectric tensor, Arbitrary $f(\boldsymbol{v})$
PL	**Data Interface**	Data conversion, Profile database
LIB	**Libraries**	

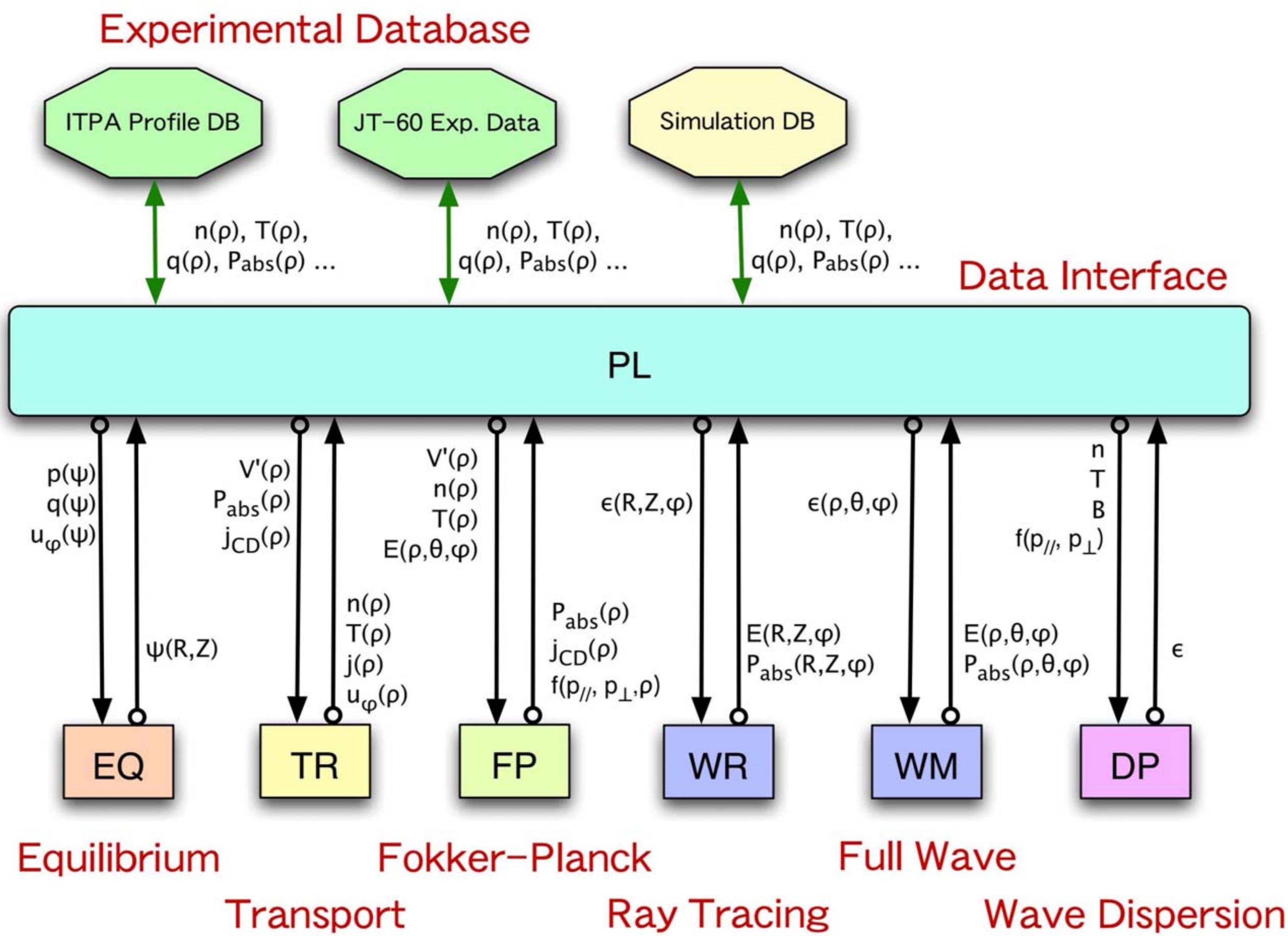
under development

TX	Transport analysis including plasma rotation and E_r
WA	Global linear stability analysis

in collaboration

TOPICS-EQU	Free-boundary equilibrium: Azumi (JAEA)

Modular Structure of TASK



Wave Dispersion Analysis : TASK/DP

- **Various Models of Dielectric Tensor** $\overleftrightarrow{\epsilon}(\omega, \boldsymbol{k}; \boldsymbol{r})$:
  - **Resistive MHD** model
  - **Collisional cold** plasma model
  - **Collisional warm** plasma model
  - **Kinetic plasma** model (**Maxwellian**, non-relativistic)
  - **Kinetic plasma** model (**Arbitrary** $f(\boldsymbol{v})$, relativistic)
  - **Gyro-kinetic plasma** model (Maxwellian)
- **Numerical Integration in momentum space**: **Arbitrary** $f(\boldsymbol{v})$
  - Relativistic Maxwellian
  - Output of TASK/FP: Fokker-Planck code

Full wave analysis: TASK/WM

- **magnetic surface coordinate**: (ψ, θ, φ)
- Boundary-value problem of **Maxwell's equation**

$$\nabla \times \nabla \times \boldsymbol{E} = \frac{\omega^2}{c^2} \overleftrightarrow{\epsilon} \cdot \boldsymbol{E} + \mathrm{i}\,\omega\mu_0 \boldsymbol{j}_{\text{ext}}$$

- Kinetic **dielectric tensor**: $\overleftrightarrow{\epsilon}$
  - **Wave-particle resonance**: $Z[(\omega - n\omega_{\text{c}})/k_{||}v_{\text{th}}]$
  - **Fast ion: Drift-kinetic**

$$\left[\frac{\partial}{\partial t} + v_{||}\nabla_{||} + (\boldsymbol{v}_{\text{d}} + \boldsymbol{v}_{\text{E}}) \cdot \nabla + \frac{e_\alpha}{m_\alpha}(v_{||}E_{||} + \boldsymbol{v}_{\text{d}} \cdot \boldsymbol{E})\frac{\partial}{\partial \varepsilon}\right] f_\alpha = 0$$

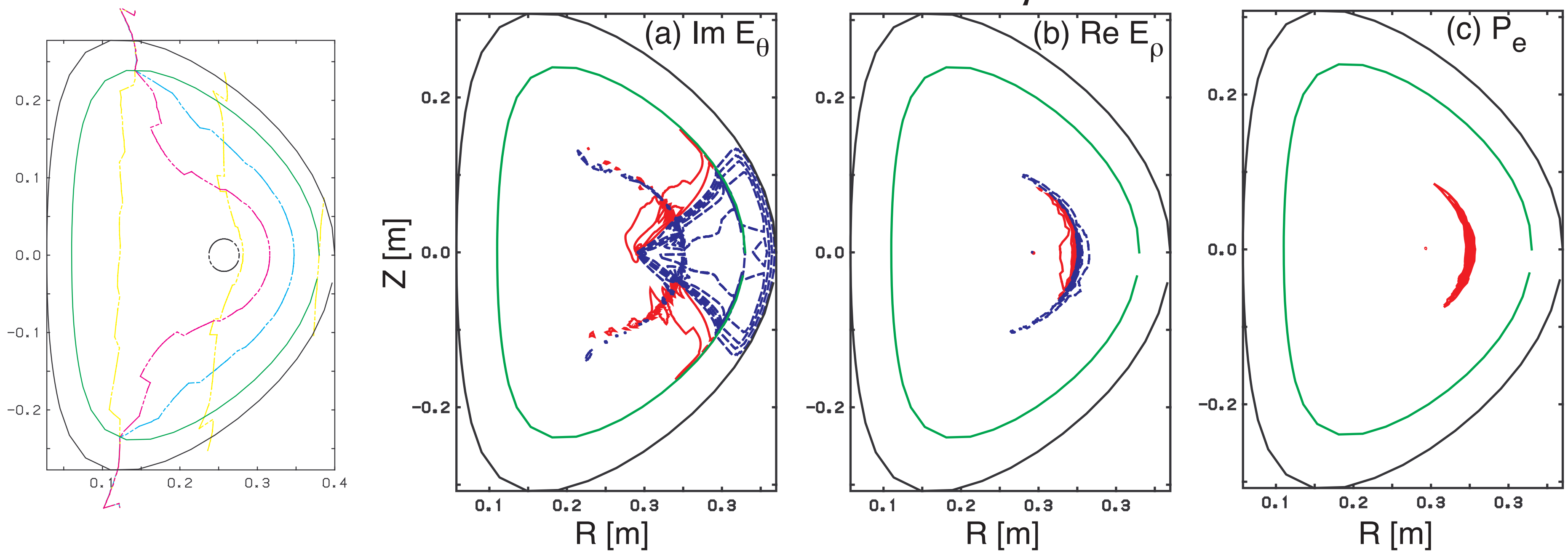
- Poloidal and toroidal **mode expansion**
  - **Accurate estimation of $k_{||}$**
- Eigenmode analysis: **Complex eigen frequency** which maximize wave amplitude for fixed excitation proportional to electron density

Full Wave Analysis of ECH in a Small-Size ST

- **Small-size spherical tokamak: LATE** (Kyoto University)
  - **T. Maekawa et al., IAEA-CN-116/EX/P4-27 (Vilamoura, Portugal, 2004)**
  - $R = 0.22\,\mathrm{m}$, $a = 0.16\,\mathrm{m}$, $B_0 = 0.0552\,\mathrm{T}$, $I_p = 6.25\,\mathrm{kA}$, $\kappa = 1.5$
  - $f = 2.8\,\mathrm{GHz}$, Toroidal mode number $n = 12$, Extraordinary mode

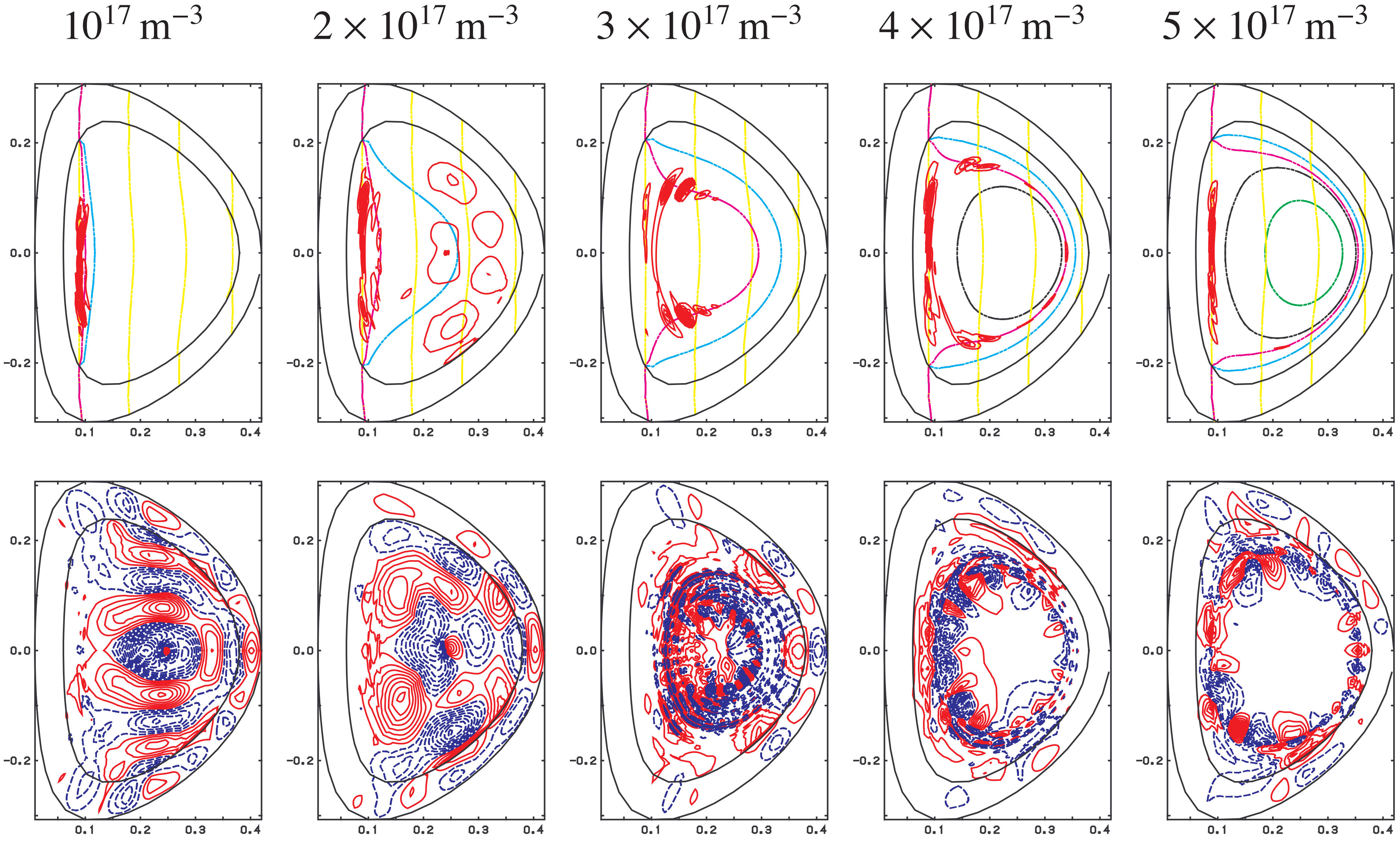
Penetration through cutoff layer	$\Longrightarrow$	Propagation along the UHR layer	$\Longrightarrow$	Collisional damping near the UHR layer

UHR,cutoff, $\quad \mathrm{Im} E_\theta \quad \mathrm{Re} E_\rho \quad P_{\mathrm{abs}}$



Density Dependence of ECW Propagation

LATE: 5 GHz, 0.072 T, $N_{||} = 0.5$

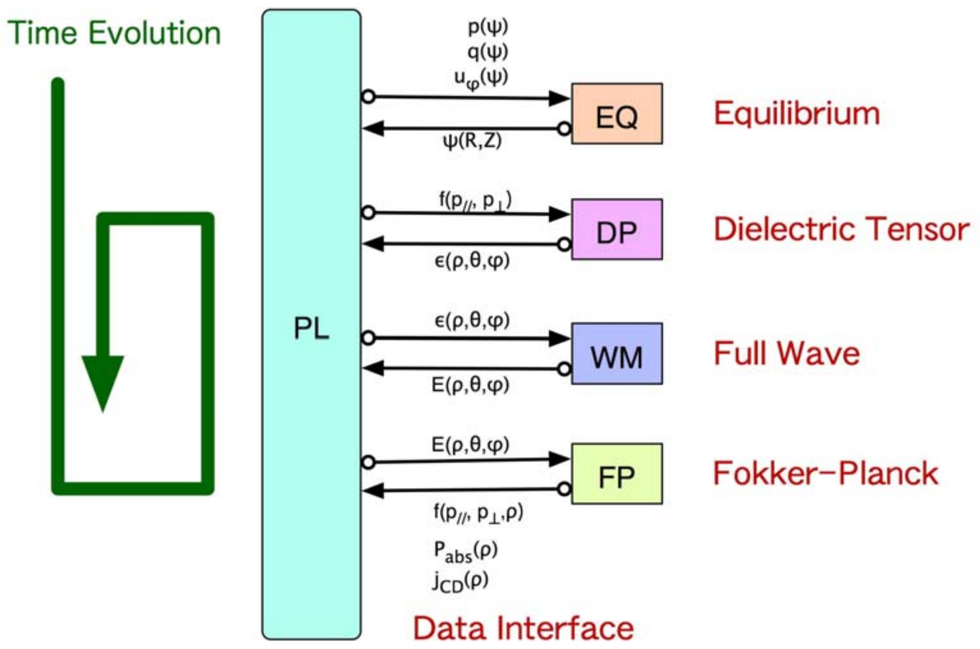


Fokker-Planck Analysis : TASK/FP

- **Fokker-Planck equation**
  for **velocity distribution function** $f(p_{||}, p_\perp, \psi, t)$
  $$\frac{\partial f}{\partial t} = E(f) + C(f) + Q(f) + L(f)$$
  - $E(f)$: Acceleration term due to DC electric field
  - $C(f)$: Coulomb collision term
  - $Q(f)$: Quasi-linear term due to wave-particle resonance
  - $L(f)$: Spatial diffusion term
- **Bounce-averaged**: Trapped particle effect, zero banana width
- **Relativistic**: momentum $\boldsymbol{p}$, weakly relativistic collision term
- **Nonlinear collision**: momentum or energy conservation
- **Three-dimensional**: spatial diffusion (neoclassical, turbulent)

Self-Consistent Wave Analysis with Modified $f(v)$

- **Modification of velocity distribution from Maxwellian**
  - Absorption of ICRF waves in the presence of energetic ions
  - Current drive efficiency of LHCD
  - NTM controllability of ECCD (absorption width)
- **Self-consistent wave analysis including modification of $f(v)$**

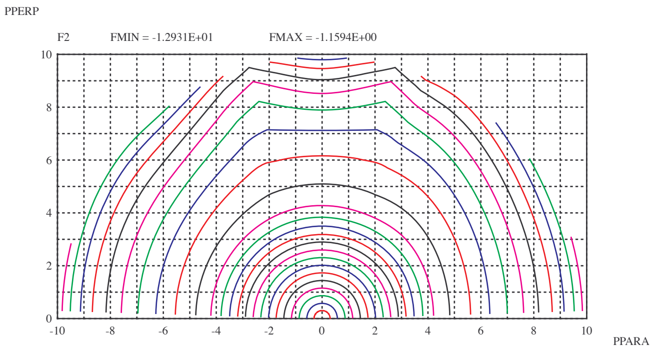


Development of Self-Consistent Wave Analysis

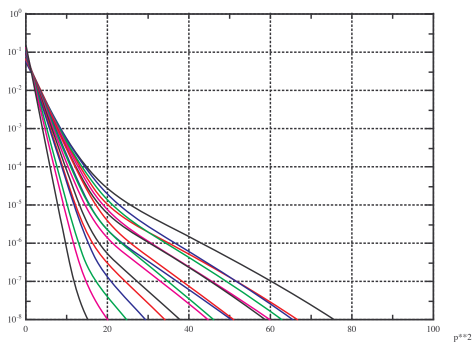
- **Code Development in TASK**
  - Ray tracing analysis with arbitrary $f(v)$: **Already done**
  - Full wave analysis with arbitrary $f(v)$: **Completed**
  - Fokker-Plank analysis of ray tracing results: **Already done**
  - Fokker-Plank analysis of full wave results: **Almost competed**
  - Self-consistent iterative analysis: **Preliminary**

- **Tail formation by ICRF minority heating**

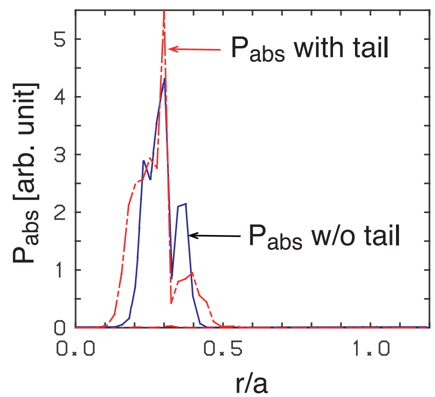
Momentum Distribution



Tail Formation



Power deposition



FLR Effects in Full Waves Analyses

- Several approaches to describe the FLR effects.
- **Differential operators**: $k_\perp \rho \rightarrow i\rho\partial/\partial r_\perp$
  - This approach cannot be applied to the case $k_\perp \rho \gtrsim 1$.
  - Extension to the third and higher harmonics is difficult.
- **Spectral method**: Fourier transform in inhomogeneous direction
  - This approach can be applied to the case $k_\perp \rho > 1$.
  - All the wave field spectra are coupled with each other.
  - Solving a dense matrix equation requires large computer resources.
- **Integral operators**: $\int \epsilon(x - x') \cdot E(x')\mathrm{d}x'$
  - This approach can be applied to the case $k_\perp \rho > 1$
  - Correlations are localized within several Larmor radii
  - Necessary to solve a large band matrix

Full Wave Analysis
Using an Integral Form of Dielectric Tensor

- **Maxwell's equation**:

$$\nabla \times \nabla \times \boldsymbol{E}(\boldsymbol{r}) + \frac{\omega^2}{c^2} \int \overleftrightarrow{\epsilon}(\boldsymbol{r}, \boldsymbol{r}') \cdot \boldsymbol{E}(\boldsymbol{r}')\mathrm{d}\boldsymbol{r} = \mu_0 \boldsymbol{J}_{ext}(\boldsymbol{r})$$

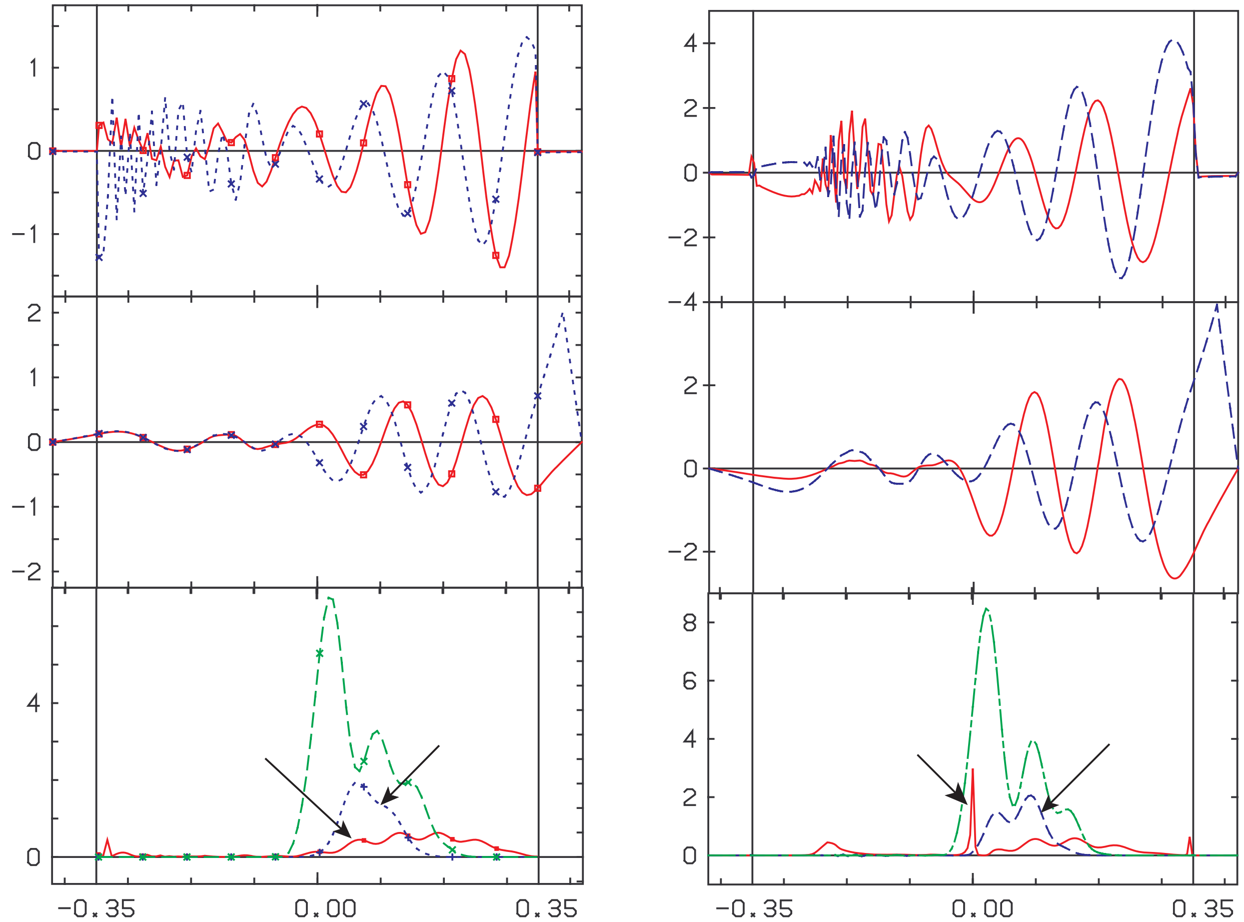
- **Integral form of dielectric tensor**: $\overleftrightarrow{\epsilon}(\boldsymbol{r}, \boldsymbol{r}')$
  - Integration along the unperturbed cyclotron orbit
- **1D analysis in tokamaks** (in the direction of major radius)
  - To confirm the applicability of an integral form of dielectric tensor
  - **Another formulation in the lowest order of ρ/L**

    — Sauter O, Vaclavik J, Nucl. Fusion **32** (1992) 1455.
- **2D analysis in tokamaks**
  - In more realistic configurations

One-Dimensional Analysis (1)

ICRF minoring heating without energetic particles ($n_H/n_D = 0.1$)

Differential form | Integral form



$$R_0 = 1.31\text{m}$$

$$a = 0.35\text{m}$$

$$B_0 = 1.4\text{T}$$

$$T_{e0} = 1.5\text{keV}$$

$$T_{D0} = 1.5\text{keV}$$

$$T_{H0} = 1.5\text{keV}$$

$$n_{s0} = 10^{20}\text{m}^{-3}$$

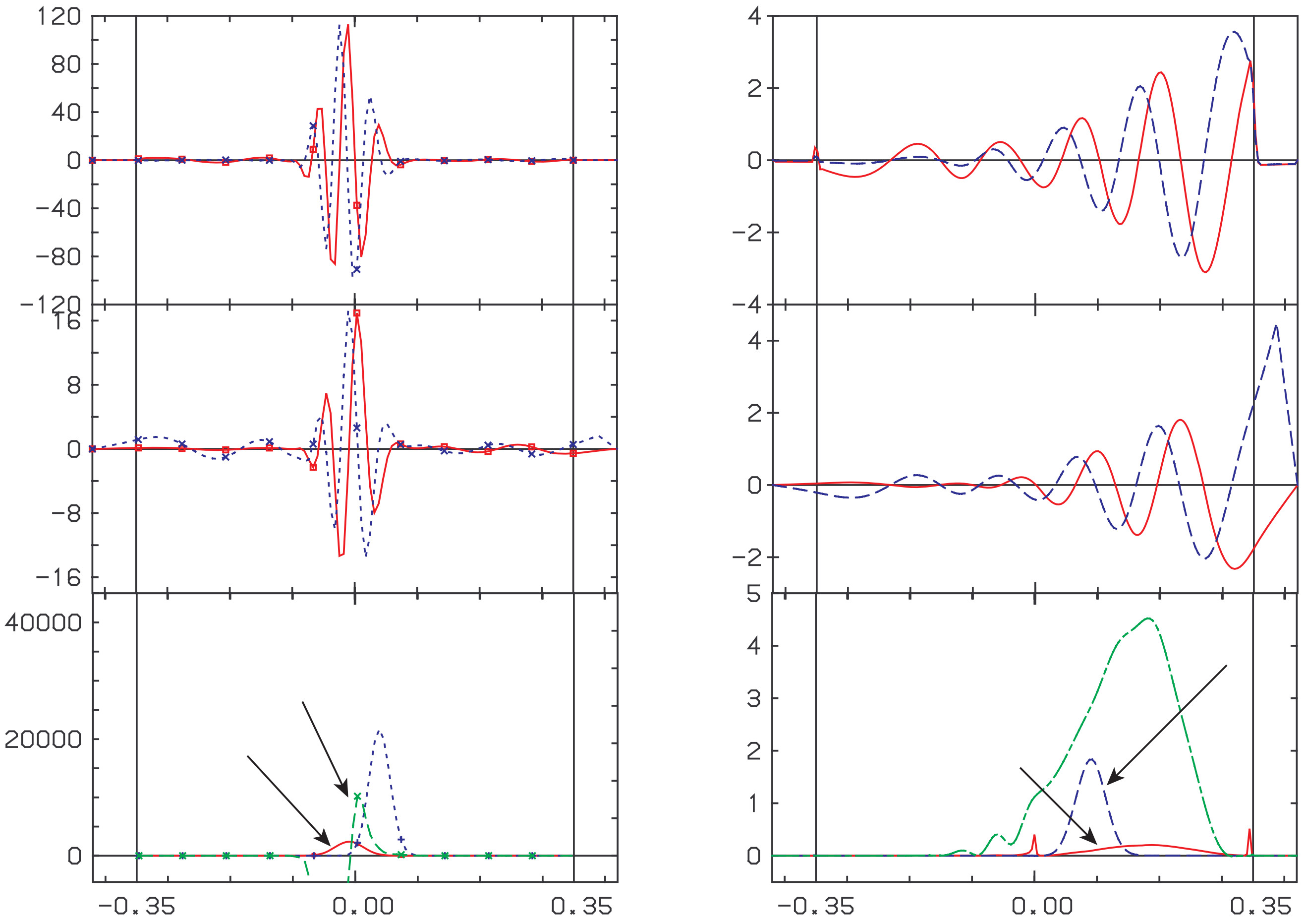
$$\omega/2\pi = 20\text{MHz}$$

Differential approach is applicable

One-Dimensional Analysis (2)

ICRF minoring heating with energetic particles $(n_{\mathrm{H}}/n_{\mathrm{D}} = 0.1)$

Differential form Integral form



$R_0 = 1.31\mathrm{m}$

$a = 0.35\mathrm{m}$

$B_0 = 1.4\mathrm{T}$

$T_{e0} = 1\mathrm{keV}$

$T_{\mathrm{D}0} = 1\mathrm{keV}$

$T_{\mathrm{H}0} = 100\mathrm{keV}$

$n_{s0} = 10^{20}\mathrm{m}^{-3}$

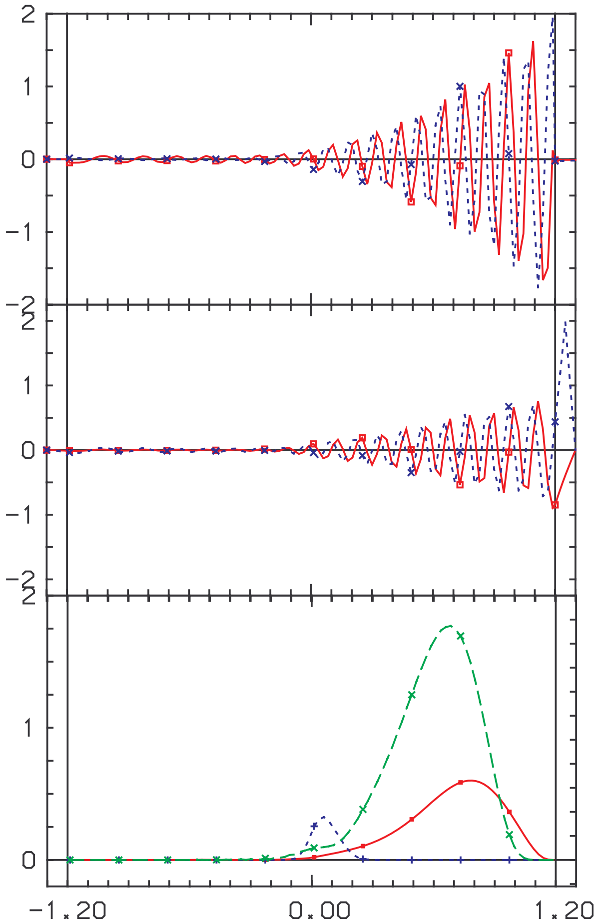
$\omega/2\pi = 20\mathrm{MHz}$

Differential approach cannot be applied since $k_\perp \rho_i > 1$.

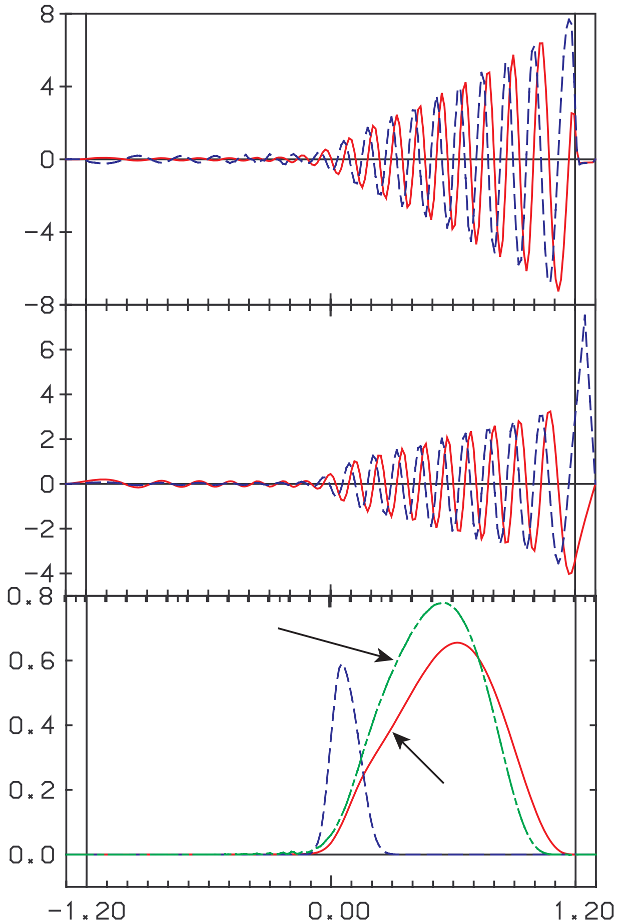
One-Dimensional Analysis (3)

ICRF minoring heating with α-particles ($n_D : n_{He} = 0.96 : 0.02$)

Differential form



Integral form



$R_0 = 3.0\mathrm{m}$

$a = 1.2\mathrm{m}$

$B_0 = 3\mathrm{T}$

$T_{e0} = 10\mathrm{keV}$

$T_{D0} = 10\mathrm{keV}$

$T_{\alpha 0} = 3.5\mathrm{MeV}$

$n_{s0} = 10^{20}\mathrm{m}^{-3}$

$\omega/2\pi = 45\mathrm{MHz}$

Absorption by α may be overestimated by differential approach.

2-D Formulation

- **Coordinates**
  - **Magnetic coordinate system**: (ψ, χ, ζ)
  - **Local Cartesian coordinate system**: (s, p, h)
  - **Fourier expansion**: poloidal and toroidal mode numbers, m, n

- **Perturbed current**

$$\boldsymbol{J}(\boldsymbol{r}, t) = -\frac{q}{m} \int \mathrm{d}\boldsymbol{v}\, q\boldsymbol{v} \int_{-\infty}^{\infty} \mathrm{d}t' \left[\boldsymbol{E}(\boldsymbol{r}', t') + \boldsymbol{v}' \times \boldsymbol{B}(\boldsymbol{r}', t')\right] \cdot \frac{\partial f_0(\boldsymbol{v}')}{\partial \boldsymbol{v}'}$$

- **The time evolution of χ and ζ due to gyro-motion**

$$\begin{cases} \chi(t-\tau) = \chi(t) + \dfrac{\partial \chi}{\partial s}\dfrac{v_\perp}{\omega_c}\{\cos(\omega_c\tau + \theta_0) - \cos\theta_0\} + \dfrac{\partial \chi}{\partial p}\dfrac{v_\perp}{\omega_c}\{\sin(\omega_c\tau + \theta_0) - \sin\theta_0\} - \dfrac{\partial \chi}{\partial h} v_\parallel \tau \\ \zeta(t-\tau) = \zeta(t) + \dfrac{\partial \zeta}{\partial s}\dfrac{v_\perp}{\omega_c}\{\cos(\omega_c\tau + \theta_0) - \cos\theta_0\} + \dfrac{\partial \zeta}{\partial p}\dfrac{v_\perp}{\omega_c}\{\sin(\omega_c\tau + \theta_0) - \sin\theta_0\} - \dfrac{\partial \zeta}{\partial h} v_\parallel \tau \end{cases}$$

Variable Transformations

- **Transformation of Integral Variables**
  - Transformation from the velocity space variables $(v_\perp, \theta_0)$ to the particle position s' and the guiding center position s_0.
  - Jacobian: $J = \dfrac{\partial(v_\perp, \theta_0)}{\partial(s', s_0)} = -\dfrac{\omega_c^2}{v_\perp \sin \omega_c \tau}$.
  - Express $v_\perp$ and θ_0 by s' and s_0 using $\tau = t - t'$

$$v_\perp^2 = \left(\frac{s+s'}{2} - s_0\right)^2 \frac{\omega_c^2}{\cos^2 \frac{1}{2}\omega_c\tau} + \left(\frac{s-s'}{2}\right)^2 \frac{\omega_c^2}{\sin^2 \frac{1}{2}\omega_c\tau} \equiv V_0^2$$

$$v_\perp \sin\theta_0 = \frac{\omega_c}{v_\perp}\frac{s-s'}{2}\frac{1}{\tan\frac{1}{2}\omega_c\tau} - \frac{\omega_c}{v_\perp}\left(\frac{s+s'}{2} - s_0\right)\tan\frac{1}{2}\omega_c\tau \equiv V_1$$

$$v_\perp \sin(\omega_c\tau + \theta_0) = \frac{\omega_c}{v_\perp}\frac{s-s'}{2}\frac{1}{\tan\frac{1}{2}\omega_c\tau} + \frac{\omega_c}{v_\perp}\left(\frac{s+s'}{2} - s_0\right)\tan\frac{1}{2}\omega_c\tau \equiv V_2$$

Integration over τ and $v_{||}$

- **Maxwell distribution**
  - Anisotropic Maxwell distribution with $T_\perp$ and $T_{||}$:

$$f_0(s_0, \boldsymbol{v}) = n_0 \left(\frac{m}{2\pi T_\perp}\right)^{3/2} \left(\frac{T_\perp}{T_{||}}\right)^{1/2} \exp\left[-\frac{v_\perp^2}{2v_{T_\perp}^2} - \frac{v_{||}^2}{2v_{T_{||}}^2}\right]$$

- **Integration over τ**
  - Integral in time calculated by the Fourier series expansion with cyclotron period, $2\pi/\omega_c$

- **Integration over $v_{||}$**
  - Interaction between wave and particles along the magnetic field lines described by the plasma dispersion function.

Final Form of Induced Current

- **Induced current**:

$$\overleftrightarrow{\mu}^{-1} \cdot \begin{pmatrix} J_1^{mn}(\psi) \\ J_2^{mn}(\psi) \\ J_3^{mn}(\psi) \end{pmatrix} = \int \mathrm{d}u' \int \mathrm{d}u_0 \, \overleftrightarrow{\sigma}(u, u', u_0) \cdot \begin{pmatrix} E_1^{m'n}(\psi) \\ E_2^{m'n}(\psi) \\ E_3^{m'n}(\psi) \end{pmatrix}$$

- **Electrical conductivity**:

$$\overleftrightarrow{\sigma}(u, u', u_0) = -i \left(\frac{1}{2\pi}\right)^{\frac{7}{2}} n_0 \frac{q^2}{m} \sum_{m'n'} \sum_{l} \int_0^{2\pi} \mathrm{d}\chi \int_0^{2\pi} \mathrm{d}\zeta \exp i \left\{(m'-m)\chi + (n'-n)\zeta\right\} \overleftrightarrow{H}(u, u', \chi)$$

- **Matrix coefficients**:

$$H_{1i} = \left(Q_3\mu_{1i}^{-1} - u' Q_1 \mu_{2i}^{-1}\right) \frac{\sqrt{\pi}}{k_\parallel v_{T_\parallel}} Z(\eta_l) + \left[-\frac{Q_1 v_{T_\perp}}{v_{T_\parallel}^2}\mu_{3i}^{-1} + \left(1 - \frac{v_{T_\perp}^2}{v_{T_\parallel}^2}\right)\left\{Q_3\kappa_{2i} + Q_1 u' \kappa_{1i}\right\}\right] \sqrt{\frac{\pi}{2}} \frac{1}{k_\parallel} Z'(\eta_l)$$

$$H_{2i} = \left(-Q_2 u \mu_{1i}^{-1} + Q_0 u u' \mu_{2i}^{-1}\right) \frac{\sqrt{\pi}}{k_\parallel v_{T_\parallel}} Z(\eta_l) + \left[\frac{Q_0 u v_{T_\perp}}{v_{T_\parallel}^2}\mu_{3i}^{-1} - \left(1 - \frac{v_{T_\perp}^2}{v_{T_\parallel}^2}\right) u \left\{Q_2\kappa_{2i} + Q_0 u' \kappa_{1i}\right\}\right] \sqrt{\frac{\pi}{2}} \frac{1}{k_\parallel} Z'(\eta_l)$$

$$H_{3i} = \left(-\frac{Q_2}{v_{T_\perp}}\mu_{1i}^{-1} + \frac{Q_0 u'}{v_{T_\perp}}\mu_{2i}^{-1}\right) \sqrt{\frac{\pi}{2}} \frac{1}{k_\parallel} Z'(\eta_l) + \left[-\frac{Q_0}{v_{T_\parallel}}\mu_{3i}^{-1} + \frac{v_{T_\parallel}}{v_{T_\perp}}\left(1 - \frac{v_{T_\perp}^2}{v_{T_\parallel}^2}\right)\left\{Q_2\kappa_{2i} + Q_0 u' \kappa_{1i}\right\}\right] \frac{\sqrt{\pi}}{k_\parallel} \eta_l Z'(\eta_l)$$

Kernel Functions

- **Kernel functions**

$$Q_{\left\{\begin{matrix}0\\1\\2\\3\end{matrix}\right\}}(u,u',\chi_0,l) = \int_0^{2\pi} \mathrm{d}\lambda \frac{1}{|\sin\lambda|}\left\{\begin{matrix}1\\V_1\\V_2\\V_1V_2\end{matrix}\right\}$$

$$\times \exp i\left[\frac{k_\perp v_{T_\perp}}{\omega_\mathrm{c}}\left\{(V_2-V_1)\cos\alpha-(u-u')\sin\alpha\right\}+l\lambda-\frac{V_0^2}{2}\right]$$

$$V_0^2 = \left(\frac{u+u'}{2}\right)^2\frac{1}{\cos^2\frac{1}{2}\lambda}+\left(\frac{u-u'}{2}\right)^2\frac{1}{\sin^2\frac{1}{2}\lambda}$$

$$V_1 = \frac{u-u'}{2}\frac{1}{\tan\frac{1}{2}\lambda}-\frac{u+u'}{2}\tan\frac{1}{2}\lambda$$

$$V_2 = \frac{u-u'}{2}\frac{1}{\tan\frac{1}{2}\lambda}+\frac{u+u'}{2}\tan\frac{1}{2}\lambda$$

$$\overleftrightarrow{h} = \frac{1}{J\omega}\begin{pmatrix}0 & -n & m'\\ n & 0 & i\frac{\partial}{\partial\psi}\\ -m' & -i\frac{\partial}{\partial\psi} & 0\end{pmatrix}$$

$$u \equiv \frac{s-s_0}{v_{T_\perp}}\omega_\mathrm{c}$$

$$u' \equiv \frac{s'-s_0}{v_{T_\perp}}\omega_\mathrm{c}$$

where $\overleftrightarrow{\kappa} = \overleftrightarrow{\mu}^{-1}\cdot\overleftrightarrow{g}\cdot\overleftrightarrow{h}$, $\overleftrightarrow{\mu}$ is the transformation matrix for $(s,p,h)\to(\psi,\chi,\zeta)$, and $\overleftrightarrow{g}$ is the metric tensor.

Consistent Formulation of Integral Full Wave Analysis

- **Analysis of wave propagation**
  - **Dielectric tensor**:

$$\nabla\times\nabla\times\boldsymbol{E}(\boldsymbol{r})-\frac{\omega^2}{c^2}\int\mathrm{d}\boldsymbol{r}_0\int\mathrm{d}\boldsymbol{r}'\,\frac{\boldsymbol{p}'}{m\gamma}\frac{\partial f_0(\boldsymbol{p}',\boldsymbol{r}_0)}{\partial\boldsymbol{p}'}\cdot\boldsymbol{K}_1(\boldsymbol{r},\boldsymbol{r}',\boldsymbol{r}_0)\cdot\boldsymbol{E}(\boldsymbol{r}')=\mathrm{i}\,\omega\mu_0\boldsymbol{j}_{\mathrm{ext}}$$

where $\boldsymbol{r}_0$ is the gyrocenter position.

- **Analysis of modification of momentum distribution function**
  - **Quasi-linear operator**

$$\frac{\partial f_0}{\partial t}+\left(\frac{\partial f_0}{\partial\boldsymbol{p}}\right)_{\boldsymbol{E}}+\frac{\partial}{\partial\boldsymbol{p}}\int\mathrm{d}\boldsymbol{r}\int\mathrm{d}\boldsymbol{r}'\boldsymbol{E}(\boldsymbol{r})\,\boldsymbol{E}(\boldsymbol{r}')\cdot\boldsymbol{K}_2(\boldsymbol{r},\boldsymbol{r}',\boldsymbol{r}_0)\cdot\frac{\partial f_0(\boldsymbol{p}',\boldsymbol{r}_0,t)}{\partial\boldsymbol{p}'}=\left(\frac{\partial f_0}{\partial\boldsymbol{p}}\right)_{\mathrm{col}}$$

- The kernels $\boldsymbol{K}_1$ and $\boldsymbol{K}_2$ are closely related and localized in the region $|\boldsymbol{r}-\boldsymbol{r}_0|\lesssim\rho$ and $|\boldsymbol{r}'-\boldsymbol{r}_0|\lesssim\rho$.

Summary

- **Toward an integrated full wave analysis in various range of frequencies, the integrated code, TASK, has been extended.**
- **Self-consistent analysis including modification of $f(p)$**
  - Full wave analysis with arbitrary velocity distribution was completed. Fokker-Planck analysis uses wave fields calculated by the full wave module. Preliminary result of self-consistent analysis was shown.
- **Full wave analysis of EC wave propagation in a small-size ST**
  - Tunneling through the cutoff layer and absorption on the upper hybrid layer were described.
- **2D full wave analysis including the FLR effects**:
  - 1D analysis elucidated the importance of the FLR effects. Formulation was extended to a 2D configuration. Implementation is under way.

Request for presentation

- **Please sign up you name and affiliation**:

(mail address from the List of Participants)

Future Plan of TASK Code

	Present Status	In 2 years	In 5 years
Equilibrium	Fixed/Free Boundary	Equilibrium Evolution	Start Up Analysis
Core Transport	1D Diffusive TR	Kinetic TR	2D Fluid TR
	1D Dynamic TR		
SOL Transport		2D Fluid TR	Plasma-Wall Interaction
Neutral Tranport	1D Diffusive TR	Orbit Following	
Energetic Ions	Kinetic Evolution	Orbit Following	
Wave Beam	Ray/Beam Tracing	Beam Propagation	
Full Wave	Kinetic ϵ	Gyro Integral ϵ	Orbit Integral ϵ
Stabilities	Sawtooth Osc.	Tearing Mode	Systematic Stability Analysis
	ELM Model	Resistive Wall Mode	
Turbulent Transport	CDBM Model	Linear GK + ZF	Nonlinear ZK + ZF
		Diagnostic Module	
		Control Module	

Future Plan of Integrated Full Wave Analysis

- **DP**: dielectric tensor
  - **2D integral operator for Maxwellian**: under way
  - **2D integral operator for arbitrary $f(v)$**: planned
  - **Gyrokinetic arbitrary $f(v)$**: planned
- **WM**: full wave analysis
  - **Update to FEM version**: under way
  - **Waveguide excitation**: under way
- **FP**: Fokker-Planck analysis
  - **Integral quasi-linear operator**: formulation
  - **Radial diffusion**: Re-installation