17th Topical Conference on
Radio Frequency Power in Plasmas
Clearwater, FL, USA
2007/05/07

Integrated Full Wave Analysis of ICRF Waves in Burning Plasmas

A. Fukuyama
Department of Nuclear Engineering, Kyoto University,
Kyoto 606-8501, Japan

in collaboration with

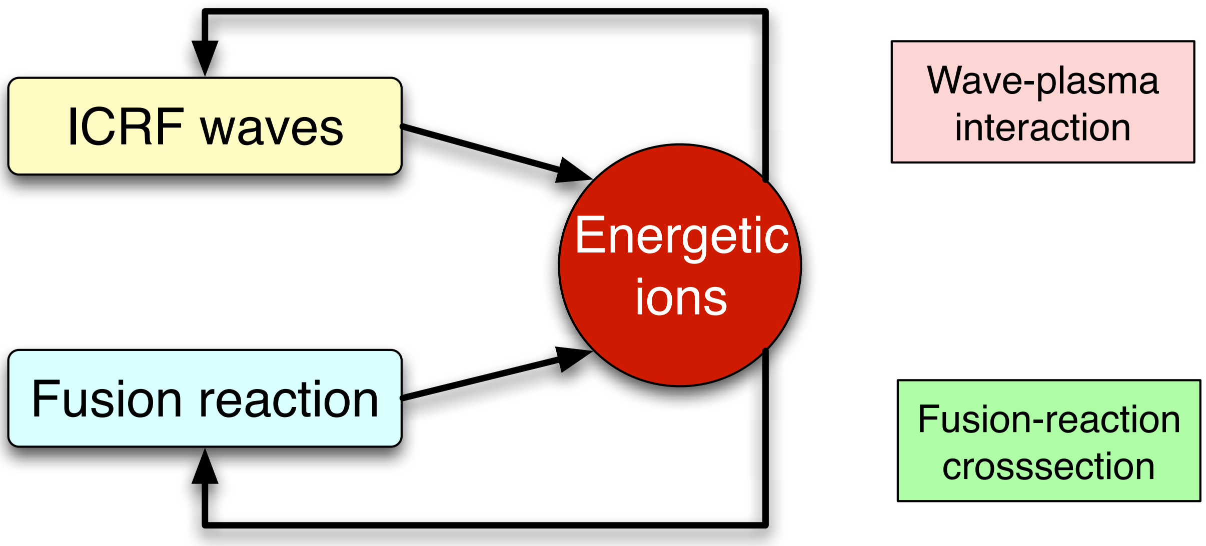
BPSI Working Group

Outline

1. Introduction
2. ICRF wave analysis in integrated modeling of tokamaks
3. Time evolution of velocity distribution function
4. Finite gyroradius effects
5. Integral formulation of full wave analysis
6. Summary

Introduction(1)

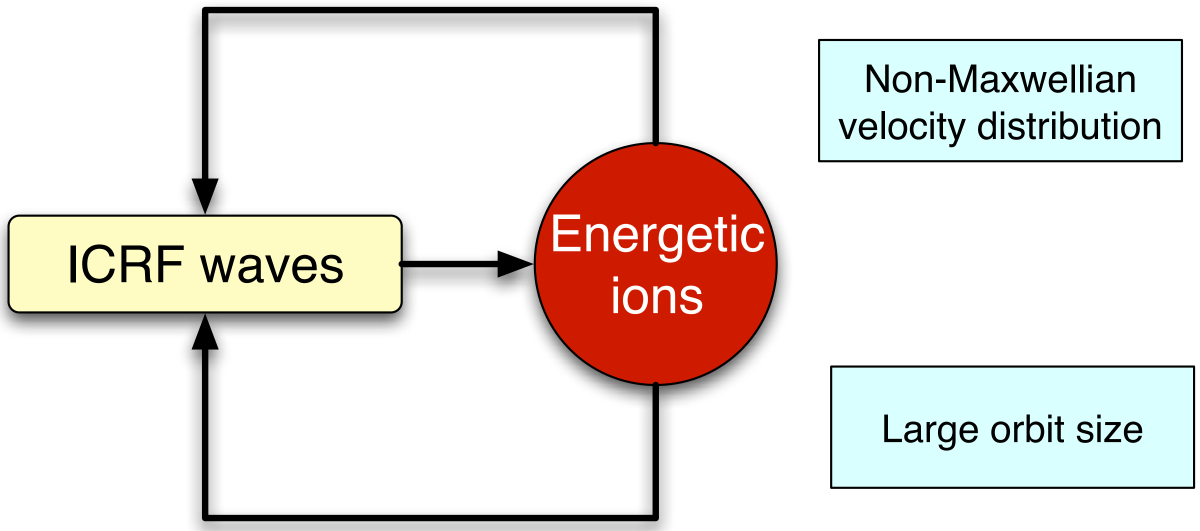
ICRF Waves in Burning Plasmas



- Both ICRF waves and fusion reaction generate energetic ions and are affected by the energetic ions.
- In the start up phase of ITER plasmas, the role of ICRF waves is important, time-evolving, and sensitive to the plasma conditions

Introduction(2)

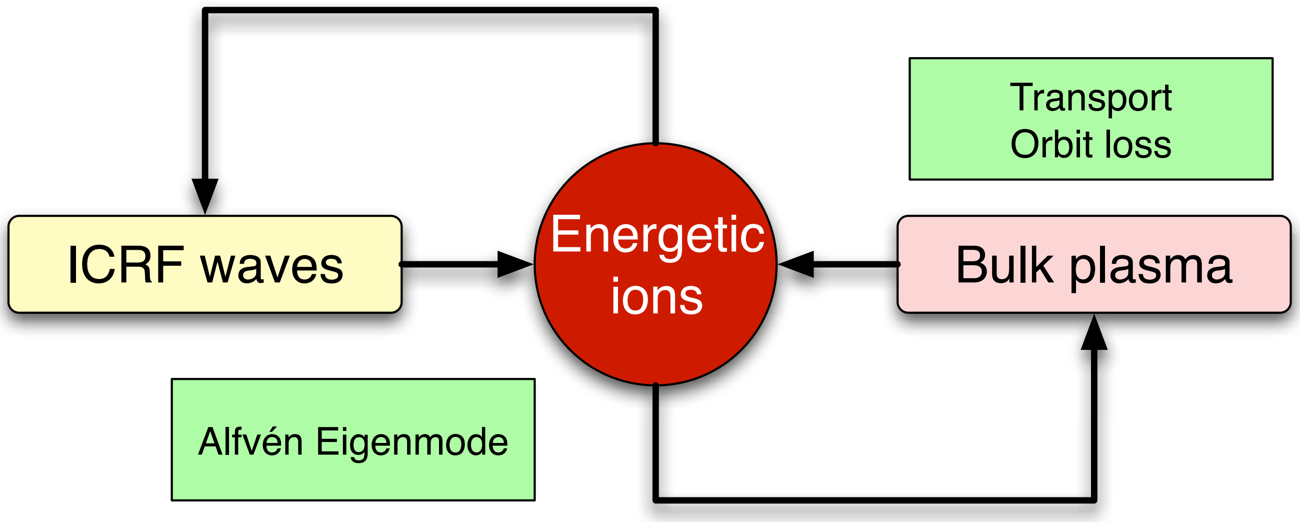
Gyrokinetic Behavior of Energetic Ions



- Self-consistent analysis of non-Maxwellian velocity distribution function is necessary.
- Finite gyroradius and finite orbit size affect the behavior of ICRF waves.

Introduction(3)

Comprehensive Modeling of Burning Plasmas



- Energetic ions interact with bulk plasmas through, for example, transport processes and orbit loss.
- Alfvén eigenmodes may affect the energetic ions themselves.
- Integrated comprehensive modeling of burning plasmas is inevitable.

Introduction (4)

- **Analysis of ICRF waves in burning plasma requires**
  - Full wave analysis
  - Non-Maxwellian velocity distribution function
  - Finite gyroradius effect
  - Integrated modeling
- **Integrated approach using the TASK code**

TASK Code

- **Transport Analysing System for TokamaK**
- **Features**
  - **A Core of Integrated Modeling Code in BPSI**
    - Modular structure, Unified Standard data interface
  - **Various Heating and Current Drive Scheme**
    - Full wave analysis for IC and AW
    - Ray and beam tracing for EC and LH
    - 3D Fokker-Planck analysis
  - **High Portability**
  - **Development using CVS**
  - **Open Source**
  - **Parallel Processing using MPI Library**
  - **Extension to Toroidal Helical Plasmas**

Modules of TASK

PL	**Data Interface**	Data conversion, Profile database
EQ	**2D Equilibrium**	Fixed/Free boundary, Toroidal rotation
TR	**1D Transport**	Diffusive transport, Transport models
WR	**3D Geometr. Optics**	EC, LH: Ray tracing, Beam tracing
WM	**3D Full Wave**	IC, AW: Antenna excitation, Eigenmode
FP	**3D Fokker-Planck**	Relativistic, Bounce-averaged
DP	**Wave Dispersion**	Local dielectric tensor, Arbitrary $f(\boldsymbol{v})$
LIB	**Libraries**	LIB, MTX, MPI

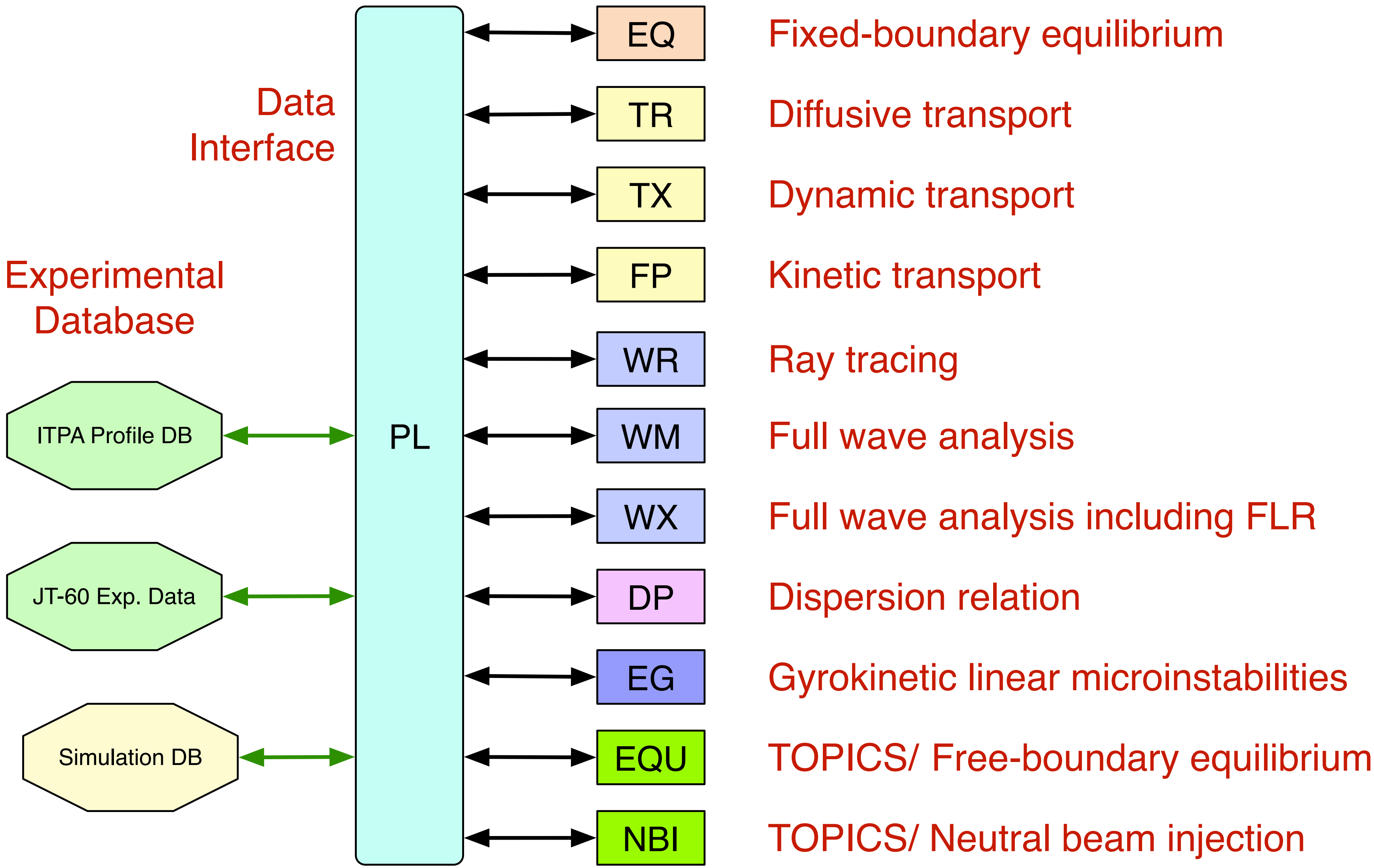
Under Development

TX	Transport analysis including plasma rotation and E_r
EG	Gyrokinetic linear stability analysis

Imported from TOPICS

EQU	Free boundary equilibrium
NBI	NBI heating

Modular Structure of TASK



Wave Dispersion Analysis : TASK/DP

- **Various Models of Dielectric Tensor** $\overleftrightarrow{\epsilon}(\omega, \boldsymbol{k}; \boldsymbol{r})$:
  - **Resistive MHD** model
  - **Collisional cold** plasma model
  - **Collisional warm** plasma model
  - **Kinetic plasma** model (**Maxwellian**, non-relativistic)
  - **Kinetic plasma** model (**Arbitrary** $f(\boldsymbol{v})$, relativistic)
  - **Gyro-kinetic plasma** model (Maxwellian)
- **Numerical Integration in momentum space**: **Arbitrary** $f(\boldsymbol{v})$
  - Relativistic Maxwellian
  - Output of TASK/FP: Fokker-Planck code

Full wave analysis: TASK/WM

- **magnetic surface coordinate**: (ψ, θ, φ)
- Boundary-value problem of **Maxwell's equation**

$$\nabla \times \nabla \times \boldsymbol{E} = \frac{\omega^2}{c^2} \overleftrightarrow{\epsilon} \cdot \boldsymbol{E} + \mathrm{i}\, \omega \mu_0 \boldsymbol{j}_{\text{ext}}$$

- Kinetic **dielectric tensor**: $\overleftrightarrow{\epsilon}$
  - Wave-particle resonance: $Z[(\omega - n\omega_{\text{c}})/k_{||} v_{\text{th}}]$
  - Finite gyroradius effect: Reductive $\Longrightarrow$ Integral **(ongoing)**
- Poloidal and toroidal **mode expansion**
- FDM: $\Longrightarrow$ FEM **(onging)**
- Eigenmode analysis: **Complex eigen frequency** which maximize wave amplitude for fixed excitation proportional to electron density

Fokker-Planck Analysis : TASK/FP

- **Fokker-Planck equation**

  for **velocity distribution function** $f(p_{||}, p_{\perp}, \psi, t)$

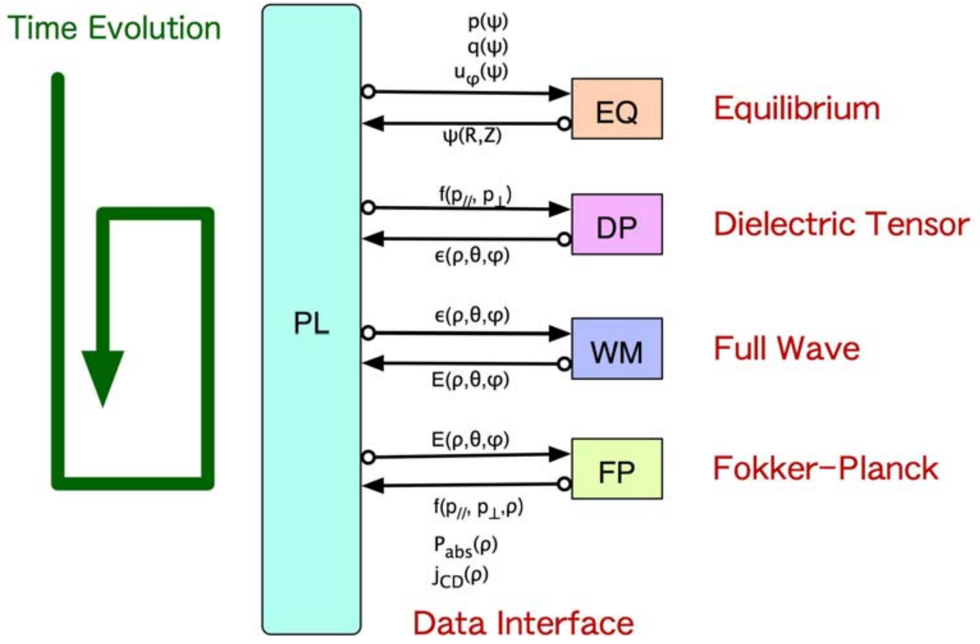
$$\frac{\partial f}{\partial t} = E(f) + C(f) + Q(f) + L(f)$$

  - $E(f)$: Acceleration term due to DC electric field
  - $C(f)$: Coulomb collision term
  - $Q(f)$: Quasi-linear term due to wave-particle resonance
  - $L(f)$: Spatial diffusion term

- **Bounce-averaged**: Trapped particle effect, zero banana width
- **Relativistic**: momentum $\boldsymbol{p}$, weakly relativistic collision term
- **Nonlinear collision**: momentum or energy conservation
- **Three-dimensional**: spatial diffusion (neoclassical, turbulent)

Self-Consistent Wave Analysis with Modified $f(v)$

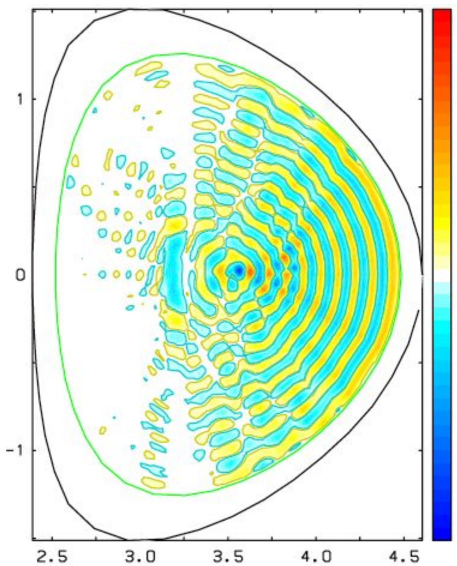
- **Modification of velocity distribution from Maxwellian**
  - Energetic ions generated by ICRF waves
  - Alpha particles generated by fusion reaction
  - Fast ions generated by NB injection
- **Self-consistent wave analysis including modification of $f(v)$**



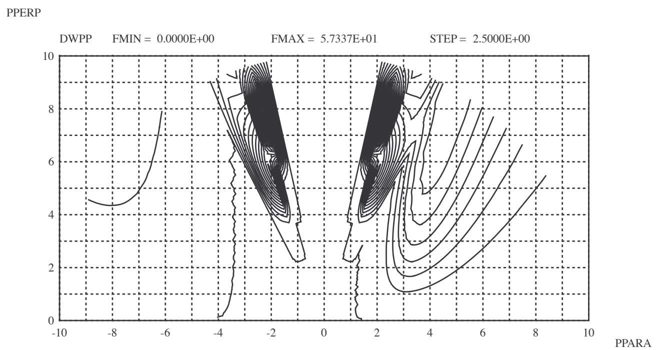
Preliminary Results

- **Tail formation by ICRF minority heating**

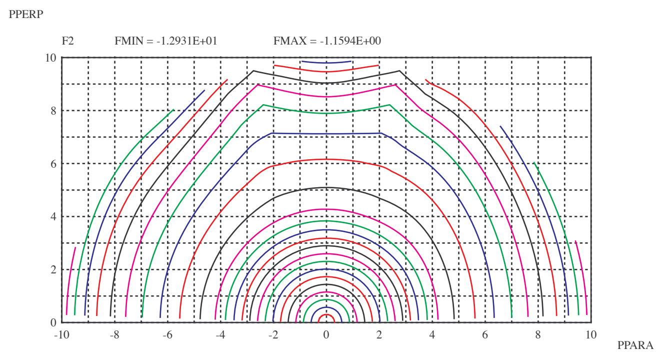
Wave pattern



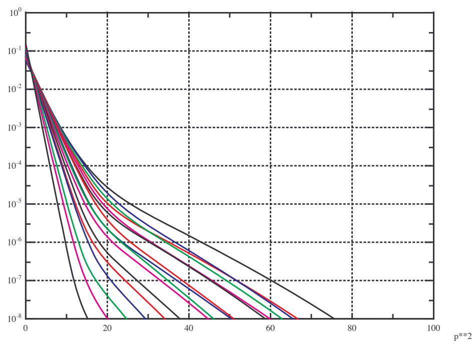
Quasi-linear Diffusion



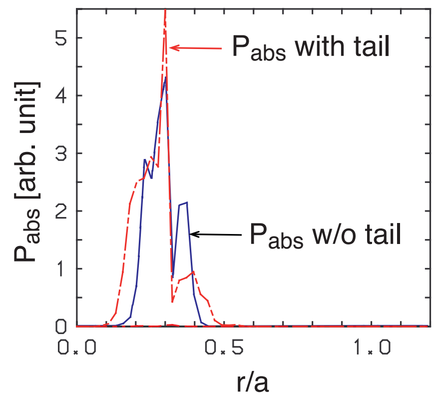
Momentum Distribution



Tail Formation



Power deposition



Finite Gyroradius Effects in Full Waves Analyses

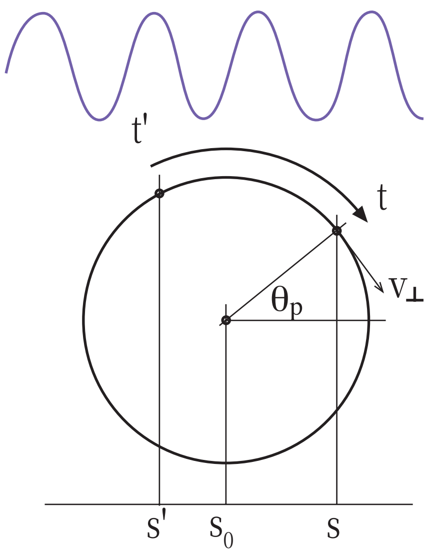
- Several approaches to describe the finite gyroradius effects.
- **Differential operators**: $k_\perp \rho \rightarrow i\rho\partial/\partial r_\perp$
  - This approach cannot be applied to the case $k_\perp \rho \gtrsim 1$.
  - Extension to the third and higher harmonics is difficult.
- **Spectral method**: Fourier transform in inhomogeneous direction
  - This approach can be applied to the case $k_\perp \rho > 1$.
  - All the wave field spectra are coupled with each other.
  - Solving a dense matrix equation requires large computer resources.
- **Integral operators**: $\int \epsilon(x - x') \cdot E(x')\mathrm{d}x'$
  - This approach can be applied to the case $k_\perp \rho > 1$
  - Correlations are localized within several gyroradii
  - Necessary to solve a large band matrix

Full Wave Analysis
Using an Integral Form of Dielectric Tensor

- **Maxwell's equation**:

$$\nabla \times \nabla \times \boldsymbol{E}(\boldsymbol{r}) + \frac{\omega^2}{c^2} \int \overleftrightarrow{\epsilon}(\boldsymbol{r}, \boldsymbol{r}') \cdot \boldsymbol{E}(\boldsymbol{r}')\mathrm{d}\boldsymbol{r} = \mu_0 \boldsymbol{J}_{ext}(\boldsymbol{r})$$

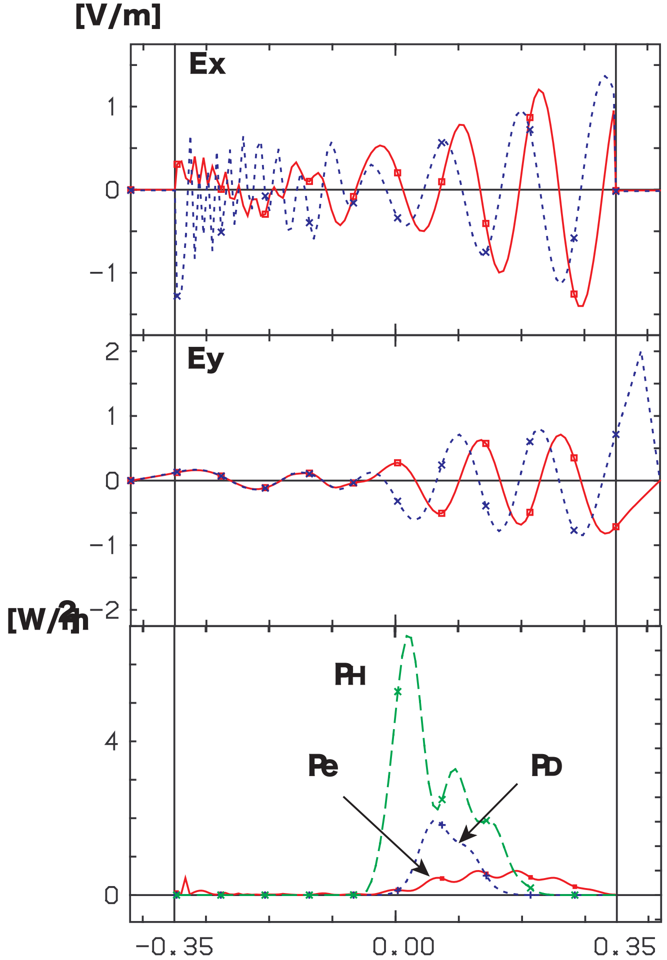
- **Integral form of dielectric tensor**: $\overleftrightarrow{\epsilon}(\boldsymbol{r}, \boldsymbol{r}')$
  - Integration along the unperturbed cyclotron orbit
- **1D analysis in tokamaks**
  - To confirm the applicability
  - **Similar formulation in the lowest order of ρ/L**
    - — Sauter O, Vaclavik J, Nucl. Fusion **32** (1992) 1455.



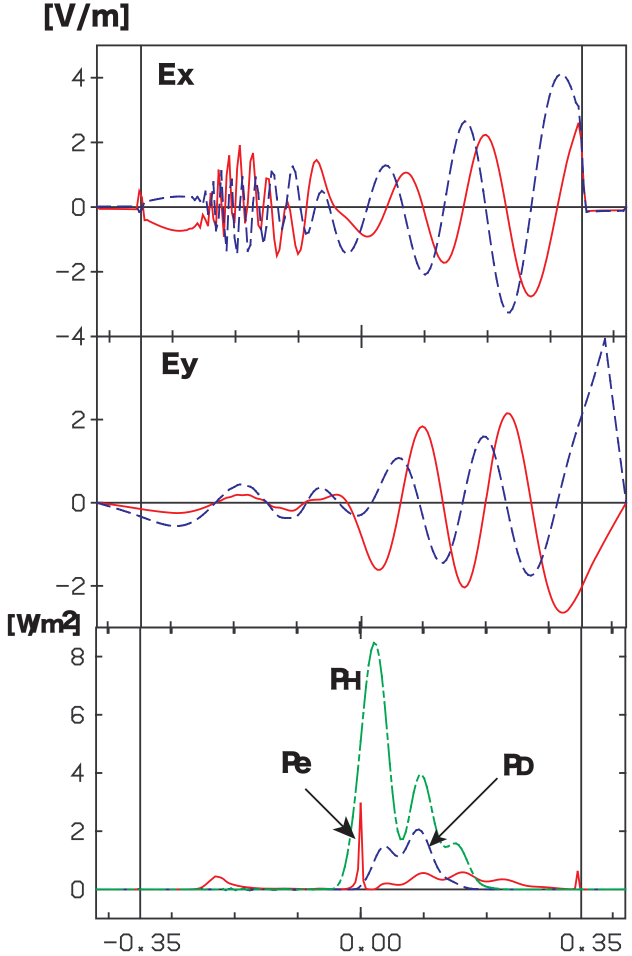
One-Dimensional Analysis (1)

ICRF minoring heating without energetic particles ($n_{\mathrm{H}}/n_{\mathrm{D}} = 0.1$)

Differential form



Integral form



$$R_0 = 1.31\mathrm{m}$$

$$a = 0.35\mathrm{m}$$

$$B_0 = 1.4\mathrm{T}$$

$$T_{e0} = 1.5\mathrm{keV}$$

$$T_{D0} = 1.5\mathrm{keV}$$

$$T_{H0} = 1.5\mathrm{keV}$$

$$n_{s0} = 10^{20}\mathrm{m}^{-3}$$

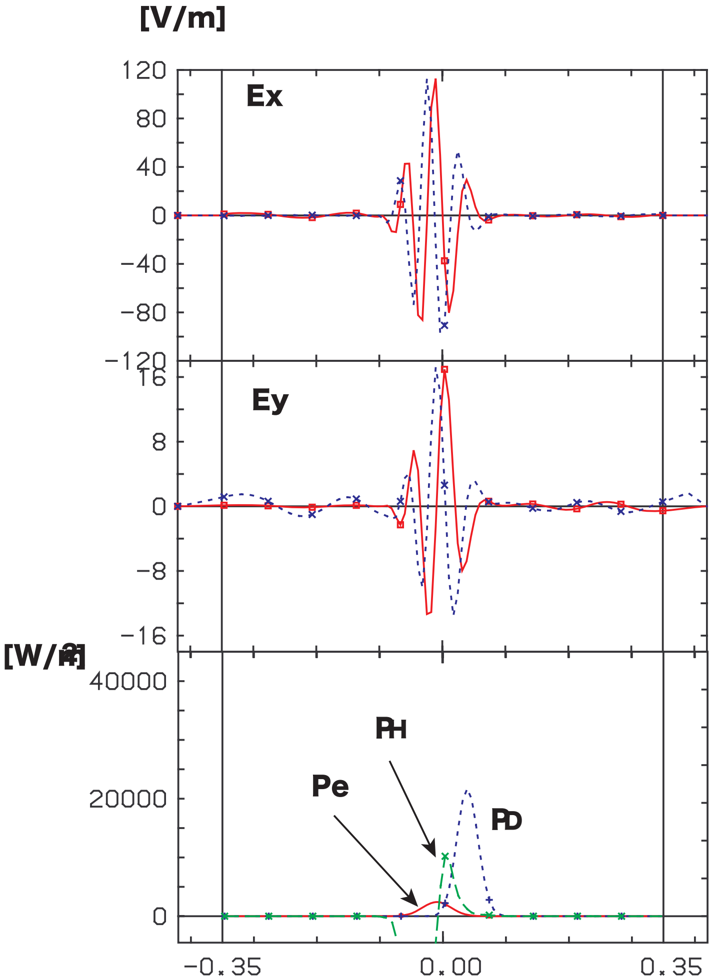
$$\omega/2\pi = 20\mathrm{MHz}$$

Differential approach is applicable

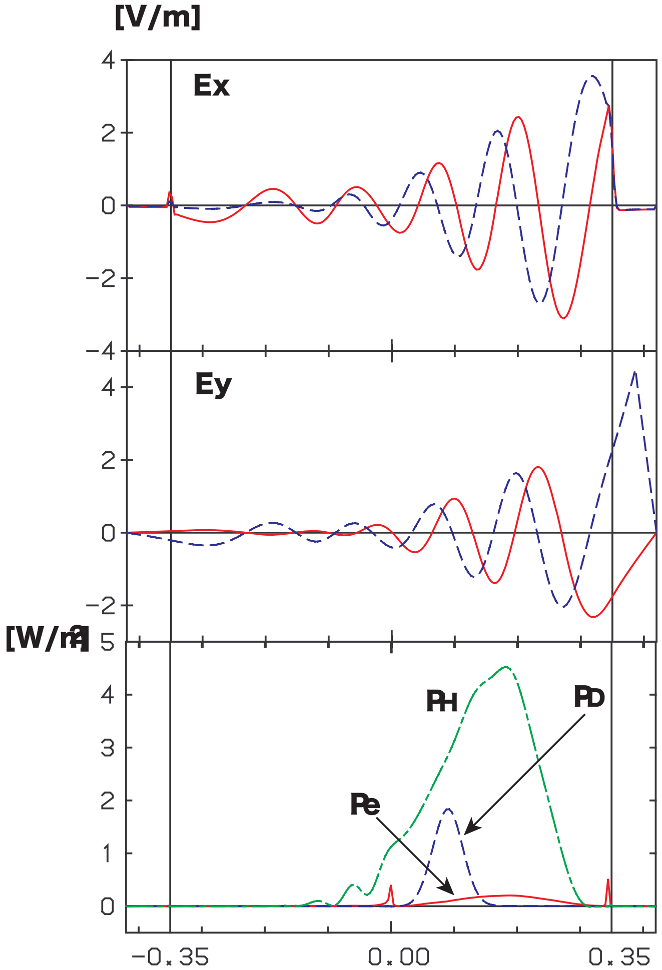
One-Dimensional Analysis (2)

ICRF minoring heating with energetic particles ($n_H/n_D = 0.1$)

Differential form



Integral form



$R_0 = 1.31\text{m}$

$a = 0.35\text{m}$

$B_0 = 1.4\text{T}$

$T_{e0} = 1\text{keV}$

$T_{D0} = 1\text{keV}$

$T_{H0} = 100\text{keV}$

$n_{s0} = 10^{20}\text{m}^{-3}$

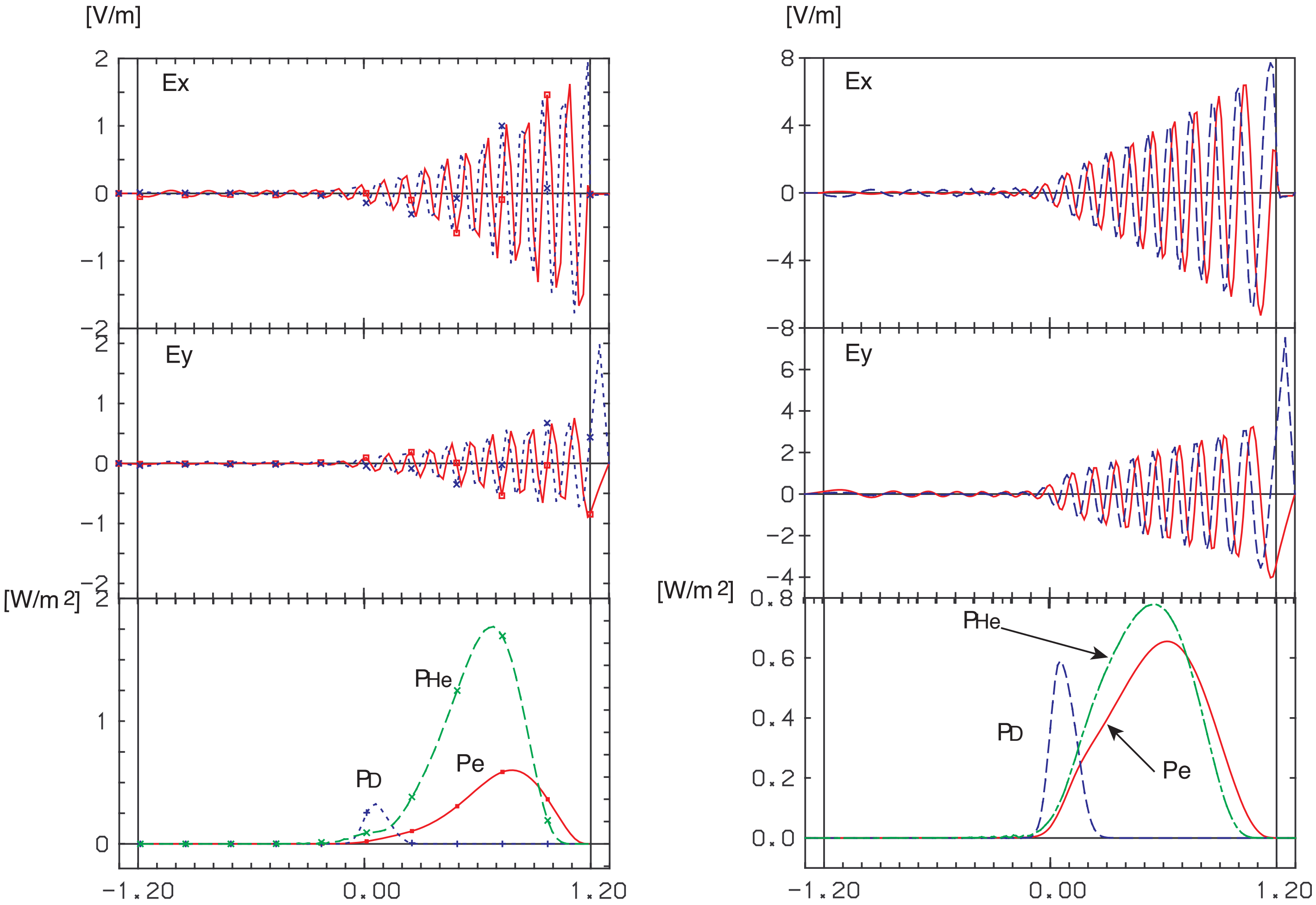
$\omega/2\pi = 20\text{MHz}$

Differential approach cannot be applied since $k_\perp \rho_i > 1$.

One-Dimensional Analysis (3)

ICRF minoring heating with α-particles ($n_\mathrm{D} : n_\mathrm{He} = 0.96 : 0.02$)

Differential form　　Integral form



$R_0 = 3.0\mathrm{m}$

$a = 1.2\mathrm{m}$

$B_0 = 3\mathrm{T}$

$T_{e0} = 10\mathrm{keV}$

$T_{D0} = 10\mathrm{keV}$

$T_{\alpha 0} = 3.5\mathrm{MeV}$

$n_{s0} = 10^{20}\mathrm{m}^{-3}$

$\omega/2\pi = 45\mathrm{MHz}$

Absorption by α may be over- or under-estimated by differential approach.

3D Formulation

- **Coordinates**
  - **Magnetic coordinate system**: (ψ, χ, ζ)
  - **Local Cartesian coordinate system**: (s, p, b)
  - **Fourier expansion**: poloidal and toroidal mode numbers, m, n
- **Perturbed current**

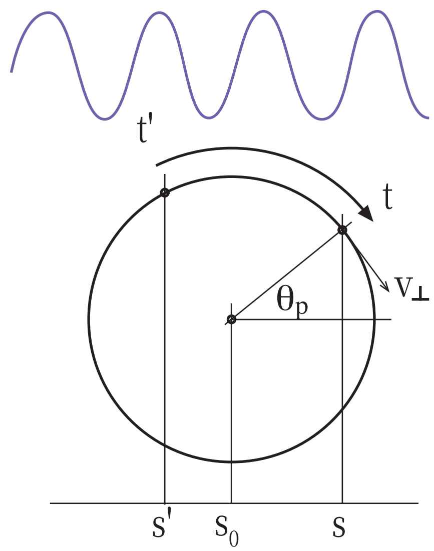
$$\boldsymbol{J}(\boldsymbol{r}, t) = -\frac{q}{m} \int \mathrm{d}\boldsymbol{v}\, q\boldsymbol{v} \int_{-\infty}^{\infty} \mathrm{d}t' \left[\boldsymbol{E}(\boldsymbol{r}', t') + \boldsymbol{v}' \times \boldsymbol{B}(\boldsymbol{r}', t')\right] \cdot \frac{\partial f_0(\boldsymbol{v}')}{\partial \boldsymbol{v}'}$$

- **Maxwell distribution function**
  - Anisotropic Maxwell distribution with $T_\perp$ and $T_\parallel$:

$$f_0(s_0, \boldsymbol{v}) = n_0 \left(\frac{m}{2\pi T_\perp}\right)^{3/2} \left(\frac{T_\perp}{T_\parallel}\right)^{1/2} \exp\left[-\frac{v_\perp^2}{2v_{T_\perp}^2} - \frac{v_\parallel^2}{2v_{T_\parallel}^2}\right]$$

Variable Transformations

- **Transformation of Integral Variables**
  - Transformation from the velocity space variables $(v_\perp, \theta_0)$ to the particle position s' and the guiding center position s_0.



  - Jacobian: $J = \dfrac{\partial(v_\perp, \theta_0)}{\partial(s', s_0)} = -\dfrac{\omega_c^2}{v_\perp \sin \omega_c \tau}$.
  - Express $v_\perp$ and θ_0 by s' and s_0 using $\tau = t - t'$, e.g.,

$$v_\perp \sin(\omega_c \tau + \theta_0) = \frac{\omega_c}{v_\perp} \frac{s - s'}{2} \frac{1}{\tan \frac{1}{2}\omega_c \tau} + \frac{\omega_c}{v_\perp} \left(\frac{s + s'}{2} - s_0 \right) \tan \frac{1}{2} \omega_c \tau$$

- **Integration over τ**: Fourier expansion with cyclotron motion
- **Integration over $v_\parallel$**: Plasma dispersion function

Final Form of Induced Current

- **Induced current**:

$$\cdot \begin{pmatrix} J_s^{mn}(s) \\ J_p^{mn}(s) \\ J_b^{mn}(s) \end{pmatrix} = \int \mathrm{d}s' \sum_{m'n'} \overleftrightarrow{\sigma}^{m'n'mn}(s,s') \cdot \begin{pmatrix} E_s^{m'n'}(s') \\ E_p^{m'n'}(s') \\ E_b^{m'n'}(s') \end{pmatrix}$$

- **Electrical conductivity**:

$$\overleftrightarrow{\sigma}^{m'n'mn}(s,s') = -in_0\frac{q^2}{m}\sum_{\ell}\int \mathrm{d}s_0 \int_0^{2\pi}\mathrm{d}\chi_0\int_0^{2\pi}\mathrm{d}\zeta_0 \exp i\left\{(m'-m)\chi_0+(n'-n)\zeta_0\right\}\overleftrightarrow{H}_{\ell}(s,s',s_0,\chi_0,\zeta_0)$$

- **Matrix coefficients**: $\overleftrightarrow{H}_{\ell}(s,s',s_0,\chi_0,\zeta_0)$
  - Four kinds of **Kernel functions** including s, s', s_0 and harmonics number ℓ
    - — The kernel functions are localized within several thermal gyro-radii.
  - **Plasma dispersion function**

Kernel Functions

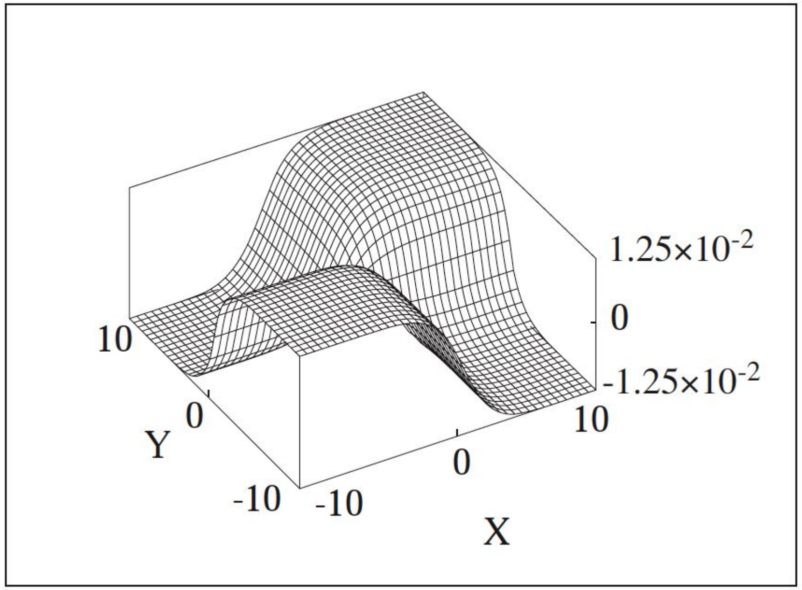
- **Kernel Function and its integrals**

$$F_n^{(i)}(X,Y) \equiv \frac{1}{2\pi^2}\int_0^{\pi} \mathrm{d}\theta \exp\left[-\frac{X^2}{1+\cos\theta} - \frac{Y^2}{1-\cos\theta}\right] f_n^{(i)}(\theta)$$

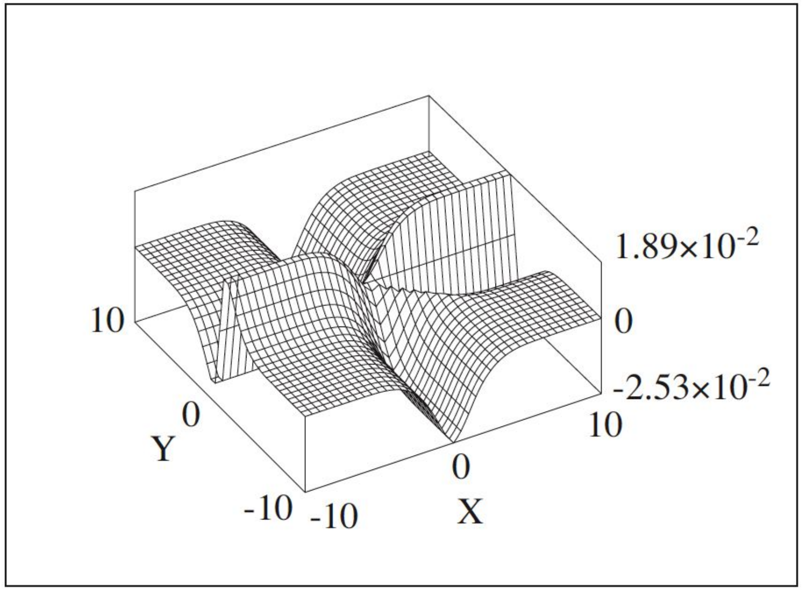
$$\mathcal{F}_n^{(ijk)}(X,Y) \equiv \int_0^{Y} \mathrm{d}Y' \int_0^{X+Y'} \mathrm{d}X' X'^j Y'^k F_n^{(i)}(X',Y')$$

$$f_n^{(i)}(\theta) = \begin{cases} \dfrac{\cos n\theta}{\sin\theta} & (i=1) \\ \sin n\theta & (i=2) \\ \dfrac{\sin n\theta}{\sin^2\theta} & (i=3) \\ \dfrac{\cos\theta \sin n\theta}{\sin^2\theta} & (i=4) \end{cases}$$

$F_0^{(100)}$



$F_1^{(100)}$



Status of extension to 3D configuration

- In a homogeneous plasma, usual formula including th Bessel functions can be recovered.
- Kernel functions are the same as the 1D case,
- **FEM formulation is required for convolution integral.**
- **Development of the FEM version of TASK/WM is ongoing (almost complete).**
- Integral operator code in 3D configuration is waiting for the FEM version of TASK/WM.

Consistent Formulation of Integral Full Wave Analysis

- **Full wave analysis for arbitrary velocity distribution function**
  - **Dielectric tensor**:

$$\nabla\times\nabla\times\boldsymbol{E}(\boldsymbol{r})-\frac{\omega^2}{c^2}\int \mathrm{d}\boldsymbol{r}_0\int \mathrm{d}\boldsymbol{r}'\,\frac{\boldsymbol{p}'}{m\gamma}\frac{\partial f_0(\boldsymbol{p}',\boldsymbol{r}_0)}{\partial \boldsymbol{p}'}\cdot\boldsymbol{K}_1(\boldsymbol{r},\boldsymbol{r}',\boldsymbol{r}_0)\cdot\boldsymbol{E}(\boldsymbol{r}')=\mathrm{i}\,\omega\mu_0\boldsymbol{j}_{\mathrm{ext}}$$

  where $\boldsymbol{r}_0$ is the gyrocenter position.

- **Fokker-Planck analysis including finite gyroradius effects**
  - **Quasi-linear operator**

$$\frac{\partial f_0}{\partial t}+\left(\frac{\partial f_0}{\partial \boldsymbol{p}}\right)_{\boldsymbol{E}}+\frac{\partial}{\partial \boldsymbol{p}}\int \mathrm{d}\boldsymbol{r}\int \mathrm{d}\boldsymbol{r}'\,\boldsymbol{E}(\boldsymbol{r})\,\boldsymbol{E}(\boldsymbol{r}')\cdot\boldsymbol{K}_2(\boldsymbol{r},\boldsymbol{r}',\boldsymbol{r}_0)\cdot\frac{\partial f_0(\boldsymbol{p}',\boldsymbol{r}_0,t)}{\partial \boldsymbol{p}'}=\left(\frac{\partial f_0}{\partial \boldsymbol{p}}\right)_{\mathrm{col}}$$

- The kernels $\boldsymbol{K}_1$ and $\boldsymbol{K}_2$ are closely related and localized in the region $|\boldsymbol{r}-\boldsymbol{r}_0|\lesssim\rho$ and $|\boldsymbol{r}'-\boldsymbol{r}_0|\lesssim\rho$.

- **To be challenged**

Summary

- **For comprehensive analyses of ICRF heating in burning plasmas, time-evolution of the velocity distribution functions and the finite gyroradius effects have to be consistently included. For this purpose, the extension of the integrated code TASK is ongoing.**
- **Self-consistent analysis including modification of $f(p)$**
  - Full wave analysis with arbitrary velocity distribution function and Fokker-Planck analysis using full wave field are available. Preliminary result of self-consistent analysis was obtained.
- **3D full wave analysis including the finite gyroradius effects**:
  - 1D analysis elucidated the importance of the gyroradius effects of energetic ions. Formulation was extended to a 2D configuration. Implementation is waiting for the FEM version of TASK/WM.