12th ITPA CDBM TG Meeting
CRPP EPFL, Lausanne
2007/05/08

TASK experience with stiff and non-stiff transport models

A. Fukuyama[A] and M. Honda[B]

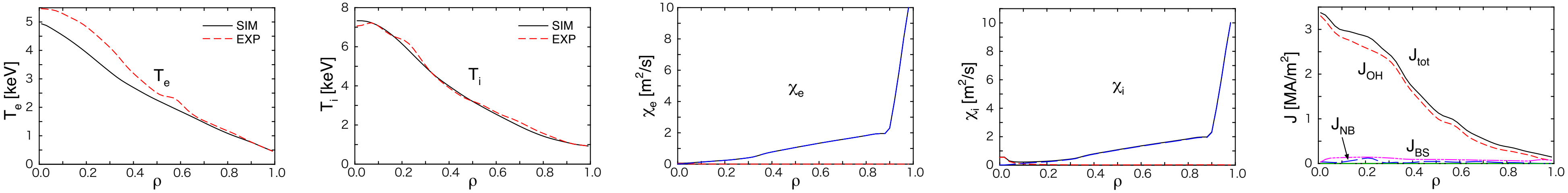
[A]Department of Nuclear Engineering, Kyoto University
[B]Naka Fusion Institute, Japan Atomic Energy Agency

Transport Solvers in TASK

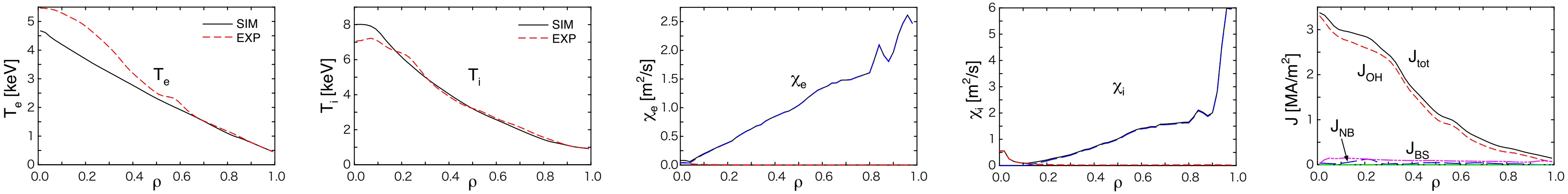
- **TASK/TR**: Diffusive transport equation
  - **FDM**: Finite difference method
  - **Full implicit method**:
    - Nonlinear equation solver: Simple Picard method
  - Typical time step for GLF23 model: 10^{-5} s
  - Typical time step for CDBM model: 10^{-2} s
- **TASK/TX**: Dynamic transport equation
  - Solving equations of motion (no derivative of D)
  - **FEM**: Finite element method (Linear and Hermite)
  - **Full implicit method**: Nonlinear equation: Simple Picard method
  - Typical time step for constant difuusivity model: 10^{-3} s

TFTR #88742 (L-mode)

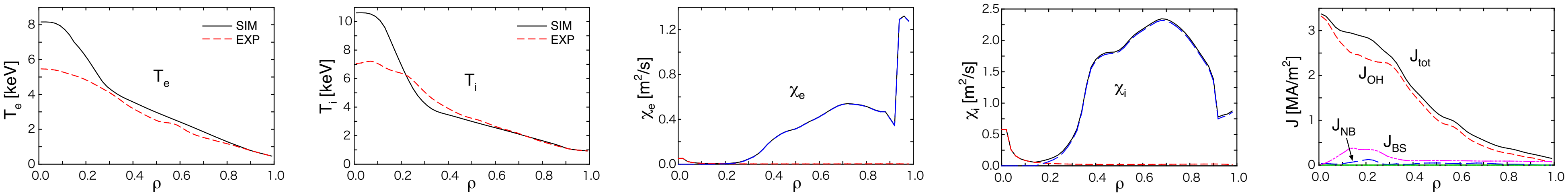
CDBM



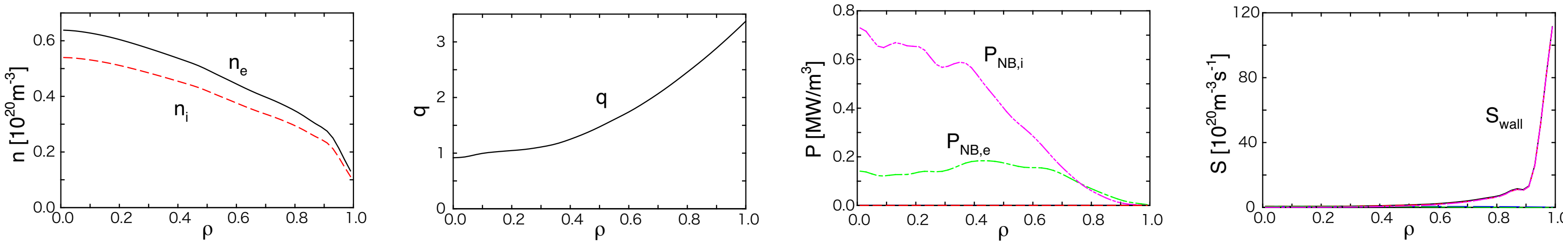
GLF23



Weiland

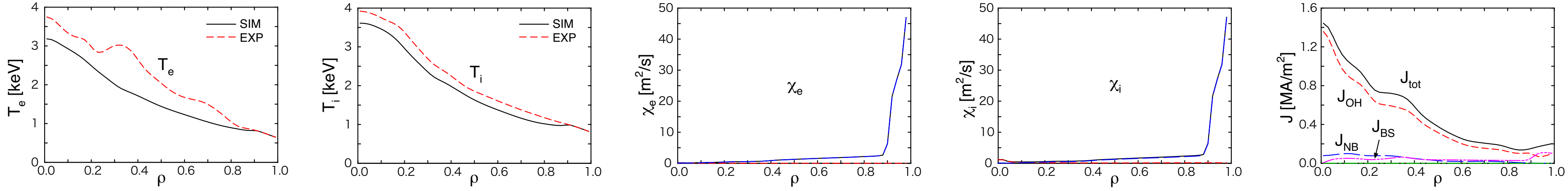


Common Profiles

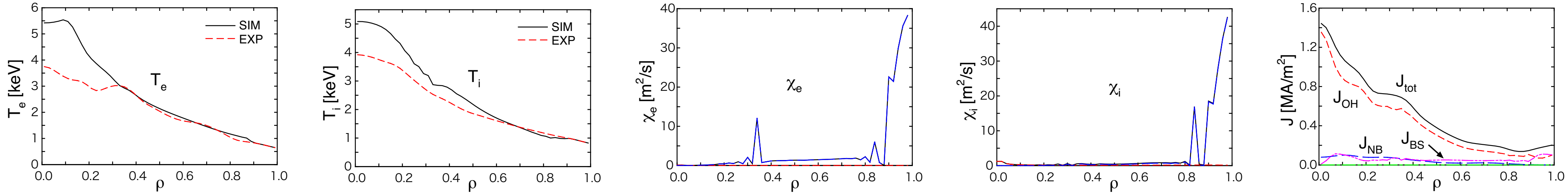


JET #38407 (H-mode with Giant ELMs)

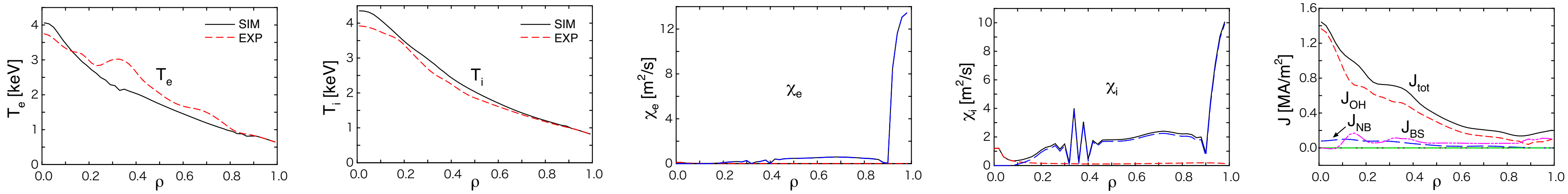
CDBM



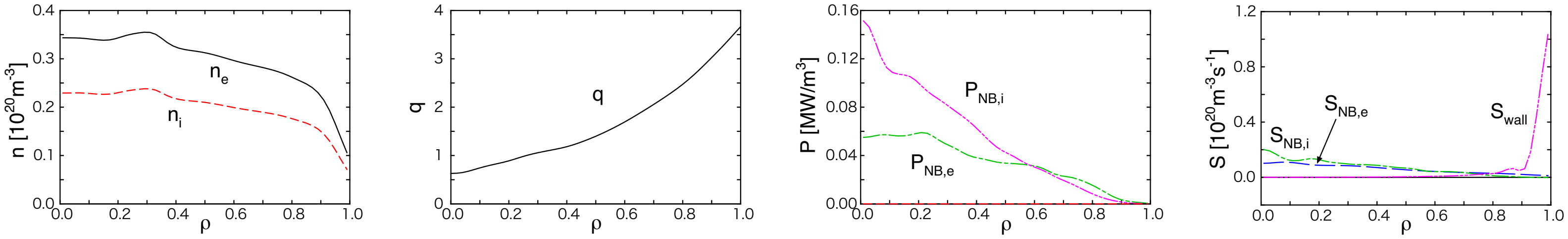
GLF23



Weiland



Common Profiles



Summary

- **The stiff models require very small time step**, typically less than 10^{-5} s, while **the CDBM models permit much longer time step**, 10^{-2} s.

- The **benchmark test of tranport codes** should be carried out with a non-stiff transport model.

- **Another approach**: The time step 10^{-5} s is the order of the inverse of the drift frequency. The assumption of stationarity

$$\frac{\mathrm{d}I}{\mathrm{d}t} = \gamma_L(\nabla p)\, I - k^2 D(I)\, I = 0, \qquad D(I) = \frac{\gamma_L(\nabla p)}{k^2}$$

  is not applicable. The time evolution of the turbulence amplitude

$$\frac{\mathrm{d}I}{\mathrm{d}t} = \gamma_L(\nabla p)\, I - k^2 D(I)\, I$$

  should be solved simultaneously with the transport equation.

CDBM Transport Model: CDBM05

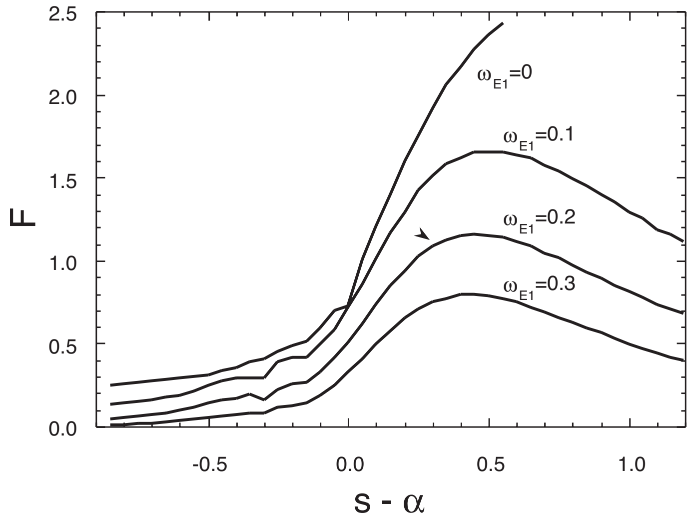
- **Thermal Diffusivity** (Marginal: $\gamma = 0$)

$$\chi_{\mathrm{TB}} = F(s, \alpha, \kappa, \omega_{\mathrm{E1}})\, \alpha^{3/2} \frac{c^2}{\omega_{\mathrm{pe}}^2} \frac{v_{\mathrm{A}}}{qR}$$

Magnetic shear	$s \equiv \dfrac{r}{q}\dfrac{\mathrm{d}q}{\mathrm{d}r}$
Pressure gradient	$\alpha \equiv -q^2 R \dfrac{\mathrm{d}\beta}{\mathrm{d}r}$
Elongation	$\kappa \equiv b/a$
$E \times B$ **rotation shear**	$\omega_{\mathrm{E1}} \equiv \dfrac{r^2}{s v_{\mathrm{A}}} \dfrac{\mathrm{d}}{\mathrm{d}r} \dfrac{E}{rB}$

- **Weak and negative magnetic shear, Shafranov shift, elongation, and $E \times B$ rotation shear reduce thermal diffusivity.**

$s - \alpha$ dependence of $F(s, \alpha, \kappa, \omega_{\mathrm{E1}})$



$$F(s, \alpha, \kappa, \omega_{\mathrm{E1}}) = \left(\frac{2\kappa^{1/2}}{1+\kappa^2}\right)^{3/2} \times \begin{cases} \dfrac{1}{1+G_1\omega_{\mathrm{E1}}^2} \dfrac{1}{\sqrt{2(1-2s')(1-2s'+3s'^2)}} & \text{for } s' = s - \alpha < 0 \\[2ex] \dfrac{1}{1+G_1\omega_{\mathrm{E1}}^2} \dfrac{1+9\sqrt{2}s'^{5/2}}{\sqrt{2}(1-2s'+3s'^2+2s'^3)} & \text{for } s' = s - \alpha > 0 \end{cases}$$

Model Equations

- **Fluid equations** for electrons and ions $(s = \mathrm{e}, \mathrm{i})$:

$$\frac{\partial n_s}{\partial t} = -\frac{1}{r}\frac{\partial}{\partial r}(r n_s u_{sr}) + S_s$$

$$\frac{\partial}{\partial t}(m_s n_s u_{sr}) = -\frac{1}{r}\frac{\partial}{\partial r}(r m_s n_s u_{sr}^2) + \frac{1}{r} m_s n_s u_{s\theta}^2 - \frac{\partial}{\partial r} n_s T_s + e_s n_s (E_r + u_{s\theta} B_\phi - u_{s\phi} B_\theta)$$

$$\frac{\partial}{\partial t}(m_s n_s u_{s\theta}) = -\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 m_s n_s u_{sr} u_{s\theta}) + \frac{1}{r^2}\frac{\partial}{\partial r} \quad r^3 n_s m_s \mu_s \frac{\partial}{\partial r}\frac{u_{s\theta}}{r} \quad + e_s n_s (E_\theta - u_{sr} B_\phi)$$

$$+ F_{s\theta}^{\mathrm{NC}} + F_{s\theta}^{\mathrm{C}} + F_{s\theta}^{\mathrm{W}} + F_{s\theta}^{\mathrm{L}} + F_{s\theta}^{\mathrm{IN}} + F_{s\theta}^{\mathrm{CX}}$$

$$\frac{\partial}{\partial t}\left(m_s n_s u_{s\phi}\right) = -\frac{1}{r}\frac{\partial}{\partial r}(r m_s n_s u_{sr} u_{s\phi}) + \frac{1}{r}\frac{\partial}{\partial r} \quad r n_s m_s \mu_s \frac{\partial}{\partial r} u_{s\phi} \quad + e_s n_s (E_\phi + u_{sr} B_\theta)$$

$$+ F_{s\phi}^{\mathrm{C}} + F_{s\phi}^{\mathrm{W}} + F_{s\phi}^{\mathrm{L}} + F_{s\phi}^{\mathrm{IN}} + F_{s\phi}^{\mathrm{CX}}$$

$$\frac{\partial}{\partial t}\frac{3}{2} n_s T_s = -\frac{1}{r}\frac{\partial}{\partial r} r \quad \frac{5}{2} u_{sr} n_s T_s - \frac{3}{2} n_s \chi_s \frac{\partial}{\partial r} T_{\mathrm{e}} \quad + e_s n_s (E_\theta u_{s\theta} + E_\phi u_{s\phi})$$

$$+ P_s^{\mathrm{C}} + P_s^{\mathrm{L}} + P_s^{\mathrm{R}} + P_s^{\mathrm{RF}}$$

The Way of Simulation

- **Neoclassical Transport Models**: **NCLASS**[6]
- **Turbulent Transport Models**: **CDBM, GLF23 v1.61 (retuned)**[6]**, Weiland**
  - CDBM: No $\boldsymbol{E} \times \boldsymbol{B}$ shearing (ω_{E1}) and magnetic curvature (κ_*) effects
  - GLF23: Using toroidal rotation velocity (V_{tor}) from exp. data
  - Weiland: Assuming $k_\theta \rho_s = 0.316$
- **Solve thermal transport equations**
  - **Fixed density profiles**
  - Taken from experimental analysis data in **ITPA profile database**
    - — 1D: R, a, I_p, B_t, κ, ϕ_a
    - — 2D: $T_{\mathrm{e,i}}$, $n_{\mathrm{e,bulk,imp}}$, Z_{eff}, j, Q_{heating}, $S_{\mathrm{NB,wall}}$, V_{rot}, Metrics
    - — $T_{\mathrm{e,i}}$ data used only for initial profiles and boundary conditions
    - — q data used only if j is not available.
  - **Boundary conditions** enforced at $\rho \leq 0.9$
  - **Particle flux calculated from $S_{\mathrm{NB,wall}}$ in thermal equations**
  - Diagonal turbulent transport coefficient set to zero if negative

[6] By courtesy of NTCC site (http://w3.pppl.gov/ntcc/)

Conditions for Comparison

- Comparison of resulting $T_{\mathrm{e,i}}$ profiles with experimental data in each discharge
  - At a fully relaxed time (typically $0.5\ \mathrm{s}$)
  - Compared with fitted temperature profiles, not measured ones
- **55 discharges described** in **"ITER Physics Basis: Chapter 2[7]"**
  - 38 L-mode discharges
  - 14 H-mode discharges with small ELMs
  - 3 H-mode discharges with giant ELMs
- **Figures of merit**
  - **Relative RMS error, $\sigma_{\mathrm{T}}^{\mathrm{rel}}$, relative to the maximum experimental temperature for each temperature profile within the region of $0.2 \le \rho \le 0.9$**

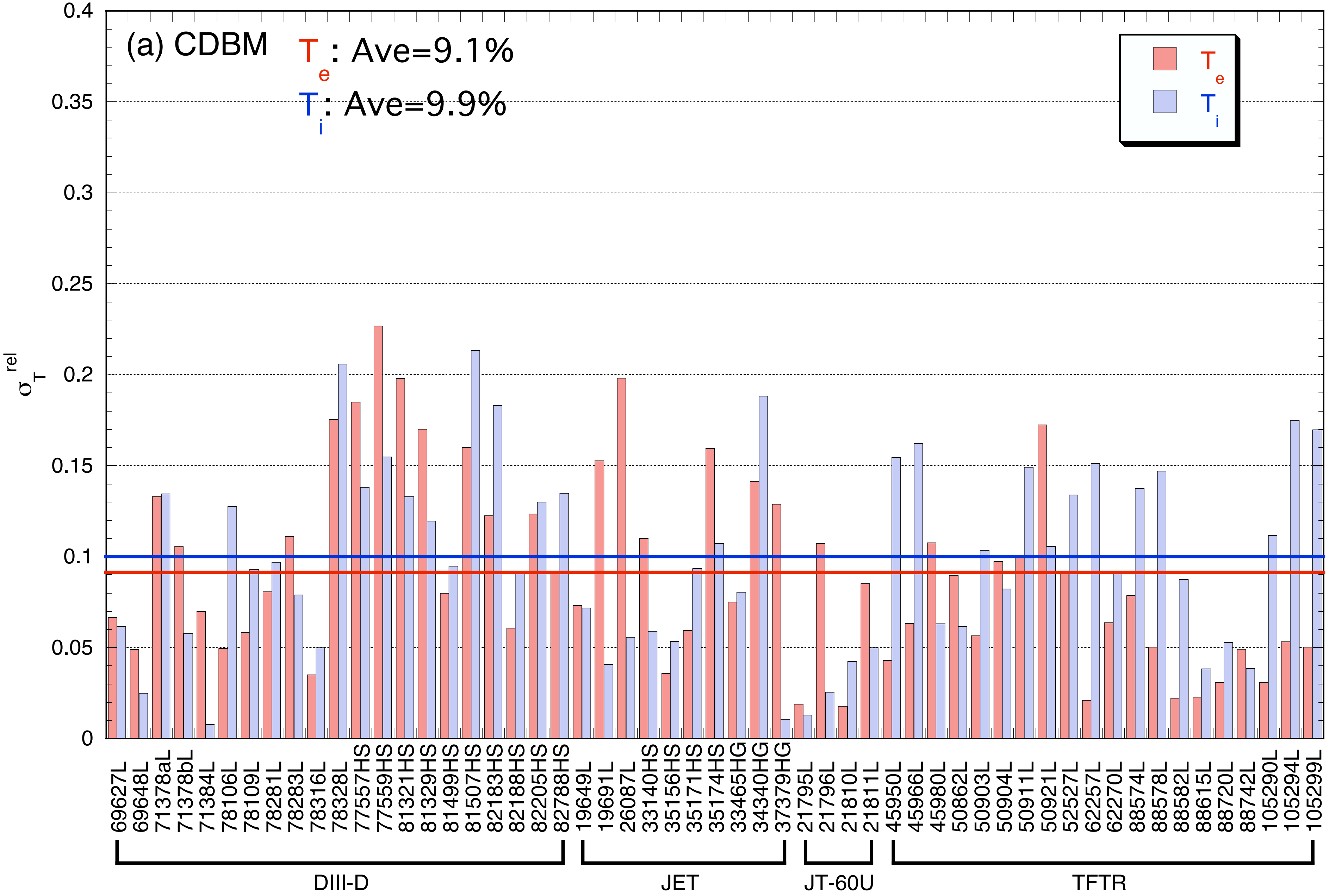
$$\sigma_{\mathrm{T}}^{\mathrm{rel}} = \sqrt{\frac{1}{N}\sum_{j=1}^{N} \epsilon_j^2}, \quad \epsilon_j = \frac{T_j^{\mathrm{sim}} - T_j^{\mathrm{exp}}}{T_{\mathrm{max}}^{\mathrm{exp}}}$$

T_j: jth point of experimental data and simulation result for each temperature
N: the number of experimental data points in a profile

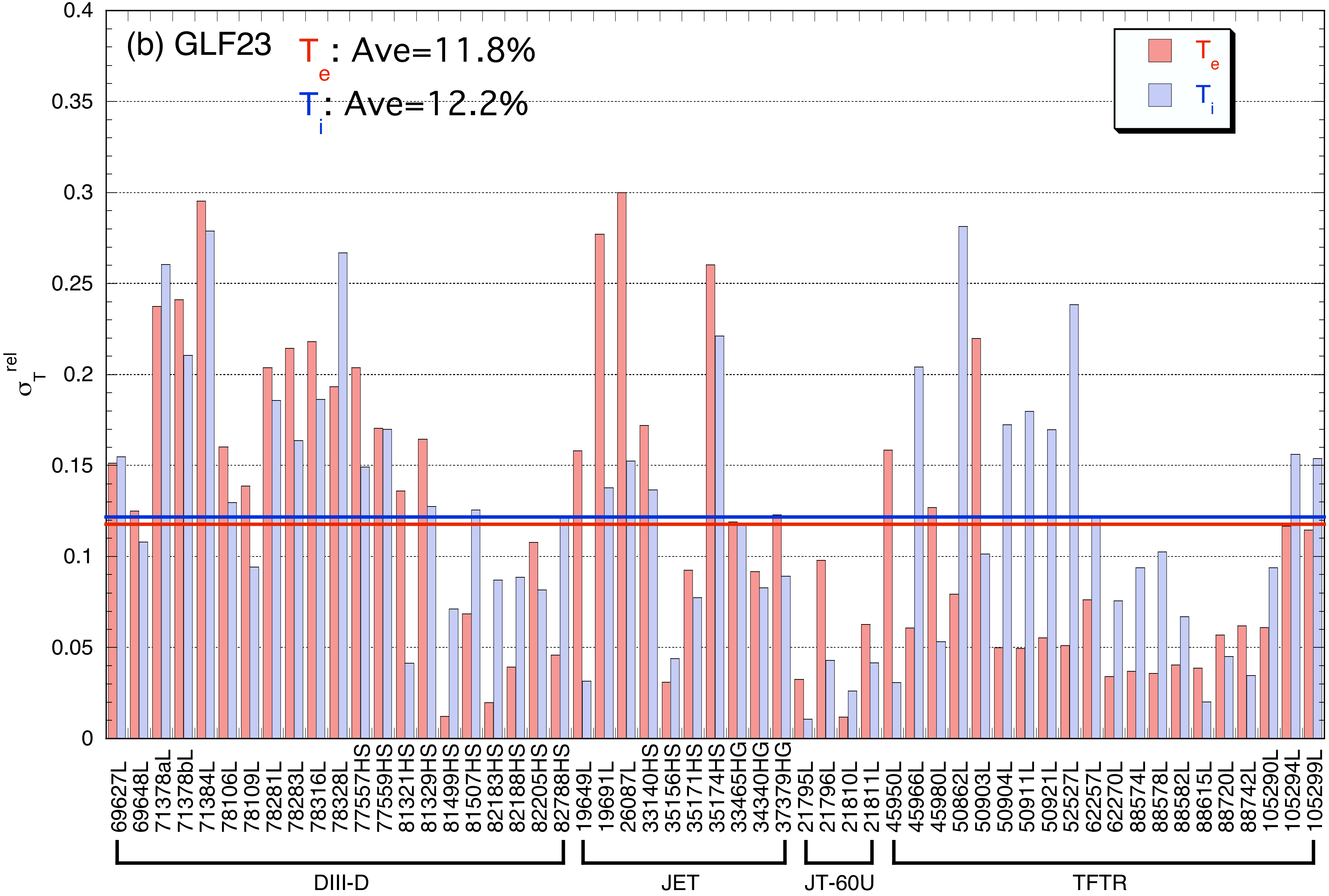
  - **Six figures of merit defined in ITER Physics Basis as described later**

[7] ITER Physics Basis Expert Groups 1999 *Nucl. Fusion* **39** 2175-2249

Relative RMS Error for Temperature Profiles (CDBM)



Relative RMS Error for Temperature Profiles (GLF23)



Relative RMS Error for Temperature Profiles (Weiland)