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Dynamic Transport Simulation Including Plasma Rotation and Radial Electric Field

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- TASK/TX: Dynamic transport simulation code
- Physics included in the TASK/TX code
- Numerical results
- Summary

Transport Modeling

- **Hierarchy of transport phenomena in toroidal plasmas**:
  - **TASK/TR**: Diffusive Transport Equations:
    - Gradient-flux relation: Stationary solution of eqs. of motion
    - Conventional way of transport simulations
  - **TASK/TX**: **Dynamic Transport Equations**: $\Longleftarrow$ **Main topic**
    - Flux-surface averaged multi-fluid equations
    - Including inertia terms in equations of motion
    - Coupling with Maxwell's equations
    - Transient analysis including plasma rotations and E_r
  - **TASK/FP**: Kinetic Transport Equations:
    - Bounce-averaged Fokker-Plank equations
    - Modification of momentum distribution functions
    - Integrating heating, current drive and kinetic stability analysis

Motivation of the TASK/TX Code

- **Transport Simulation Including Core and SOL Plasmas**
  - **Role of separatrix**
    - Closed magnetic surface $\Longleftrightarrow$ Open magnetic field line
    - Difference of dominant transport processes
- **Transient Behavior of Plasma Rotation**
  - **Radial electric field**: Radial force balance (Gauss's law = Poisson's equation)
  - **Poloidal rotation**: Equation of motion
  - **Toroidal rotation**: Equation of motion
  - Equation of motion rather than transport matrix
- **Analysis including Atomic Processes**

1D Dynamic Transport Code: TASK/TX

- **Dynamic Transport Equations** (TASK/TX)
  - **A set of flux-surface averaged equations**
  - **Two fluid equations for electrons and ions**
    - Continuity equations
    - Equations of motion (radial, poloidal and toroidal)
    - Energy transport equations
  - **Neoclassical transport**
    - Poloidal viscosity $\Longrightarrow$ Emerging all the neoclassical effects
  - **Turbulent transport**
    - Intrinsic ambipolar diffusion through poloidal momentum transf.
    - Thermal diffusivity and perpendicular viscosity
  - **Maxwell's equations including Poisson's equation**
  - **Slowing-down equations for beam ion component**
  - **Diffusion equations for two-groups (fast and slow) neutrals**

Model Equations

- **Fluid equations** for electrons and ions ($s = \mathrm{e}, \mathrm{i}$):

$$\frac{\partial n_s}{\partial t} = -\frac{1}{r}\frac{\partial}{\partial r}(r n_s u_{sr}) + S_s$$

$$\frac{\partial}{\partial t}(m_s n_s u_{sr}) = -\frac{1}{r}\frac{\partial}{\partial r}(r m_s n_s u_{sr}^2) + \frac{1}{r} m_s n_s u_{s\theta}^2 - \frac{\partial}{\partial r} n_s T_s + e_s n_s (E_r + u_{s\theta} B_\phi - u_{s\phi} B_\theta)$$

$$\frac{\partial}{\partial t}(m_s n_s u_{s\theta}) = -\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 m_s n_s u_{sr} u_{s\theta}) + \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^3 n_s m_s \mu_s \frac{\partial}{\partial r}\frac{u_{s\theta}}{r}\right) + e_s n_s (E_\theta - u_{sr} B_\phi)$$

$$+ F_{s\theta}^{\mathrm{NC}} + F_{s\theta}^{\mathrm{C}} + F_{s\theta}^{\mathrm{W}} + F_{s\theta}^{\mathrm{L}} + F_{s\theta}^{\mathrm{IN}} + F_{s\theta}^{\mathrm{CX}}$$

$$\frac{\partial}{\partial t}(m_s n_s u_{s\phi}) = -\frac{1}{r}\frac{\partial}{\partial r}(r m_s n_s u_{sr} u_{s\phi}) + \frac{1}{r}\frac{\partial}{\partial r}\left(r n_s m_s \mu_s \frac{\partial}{\partial r} u_{s\phi}\right) + e_s n_s (E_\phi + u_{sr} B_\theta)$$

$$+ F_{s\phi}^{\mathrm{C}} + F_{s\phi}^{\mathrm{W}} + F_{s\phi}^{\mathrm{L}} + F_{s\phi}^{\mathrm{IN}} + F_{s\phi}^{\mathrm{CX}}$$

$$\frac{\partial}{\partial t}\frac{3}{2} n_s T_s = -\frac{1}{r}\frac{\partial}{\partial r} r\left(\frac{5}{2} u_{sr} n_s T_s - \frac{3}{2} n_s \chi_s \frac{\partial}{\partial r} T_{\mathrm{e}}\right) + e_s n_s (E_\theta u_{s\theta} + E_\phi u_{s\phi})$$

$$+ P_s^{\mathrm{C}} + P_s^{\mathrm{L}} + P_s^{\mathrm{R}} + P_s^{\mathrm{RF}}$$

- **Slowing-down equations for beam ion component**

$$\frac{\partial n_\mathrm{b}}{\partial t} = S_\mathrm{b}^B - S_\mathrm{b}^\mathrm{C}$$

$$\frac{\partial}{\partial t}(m_\mathrm{b} n_\mathrm{b} u_{\mathrm{b}\theta}) = e_\mathrm{b} n_\mathrm{b} E_\theta + F_{\mathrm{b}\theta}^\mathrm{C} + F_{\mathrm{b}\theta}^\mathrm{IN} + F_{\mathrm{b}\theta}^\mathrm{CX}$$

$$\frac{\partial}{\partial t}\left(m_\mathrm{b} n_\mathrm{b} u_{\mathrm{b}\phi}\right) = e_\mathrm{b} n_\mathrm{b} E_\phi + F_{\mathrm{b}\phi}^\mathrm{C} + F_{\mathrm{b}\phi}^\mathrm{IN} + F_{\mathrm{b}\phi}^\mathrm{CX} + F_{\mathrm{b}\phi}^\mathrm{B}$$

- **Diffusion equations for two-group neutrals (fast and slow)**

$$\frac{\partial n_0}{\partial t} = -\frac{1}{r}\frac{\partial}{\partial r}\left(-r D_0 \frac{\partial n_0}{\partial r}\right) + S_0$$

- **Maxwell's equations**

$$\frac{1}{r}\frac{\partial}{\partial r}(r E_r) = \frac{1}{\epsilon_0}\sum_s e_s n_s$$

$$\frac{\partial B_\theta}{\partial t} = \frac{\partial E_\phi}{\partial r}, \qquad \frac{\partial B_\phi}{\partial t} = -\frac{1}{r}\frac{\partial}{\partial r}(r E_\theta)$$

$$\frac{1}{c^2}\frac{\partial E_\theta}{\partial t} = -\frac{\partial}{\partial r}B_\phi - \mu_0 \sum_s e_s n_s u_{s\theta}, \qquad \frac{1}{c^2}\frac{\partial E_\phi}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}(r B_\theta) - \mu_0 \sum_s e_s n_s u_{s\phi}$$

Transport Model (1)

- **Neoclassical transport**
  - Parallel viscous force due to a poloidal plasma rotation
  - Valid for all three neoclassical regimes

$$F_{s\theta}^{\mathrm{NC}} \equiv -n_s m_s \nu_{\mathrm{NC}s} u_{s\theta} = -\frac{\langle B^2\rangle \hat{\mu}_{11}^{si}}{n_s m_s B_\theta^2} n_s m_s u_{s\theta}$$

$\hat{\mu}_{11}^{si}$: viscosity coefficient from the NCLASS module,
*W. A. Houlberg et al. PoP **4** (1997) 3230*

- **Due to the poloidal viscous force,**
  - Neoclassical diffusion and Ware pinch
  - Resistivity and bootstrap current

Transport Model (2)

- **Turbulent diffusion**
  - Poloidal momentum exchange between electrons and ions through turbulent fluctuating field
  - Intrinsic ambipolar flux
    (electron particle flux = ion particle flux)

$$F_{\mathrm{e}\theta}^{\mathrm{W}} = -F_{\mathrm{i}\theta}^{\mathrm{W}} = -\frac{e^2 B_\phi^2 D_\mathrm{e}}{T_\mathrm{e}} n_\mathrm{e} \left(u_{\mathrm{e}\theta} - \frac{B_\theta}{B_\phi} u_{\mathrm{e}\phi} \right)$$

$$F_{\mathrm{e}\phi}^{\mathrm{W}} = -F_{\mathrm{i}\phi}^{\mathrm{W}} = \frac{e^2 B_\phi^2 D_\mathrm{e}}{T_\mathrm{e}} \frac{B_\theta}{B_\phi} n_\mathrm{e} \left(u_{\mathrm{e}\theta} - \frac{B_\theta}{B_\phi} u_{\mathrm{e}\phi} \right)$$

- **Perpendicular viscosity**
  - Non-ambipolar particle flux
    (electron particle flux $\neq$ ion particle flux)

Modelling of SOL Plasma

- **Parallel losses in the SOL**
  - **Particle, momentum and ion heat losses: convection**

$$\nu_L = \frac{k_L C_s}{2\pi q R} \quad (a < r < b)$$

  - **Electron heat loss: conduction**

$$\nu_L = k_L \frac{\chi_{\parallel}}{(2\pi q R)^2} = k_L \frac{\kappa_0 T_e^{5/2}}{n_e (2\pi q R)^2} \quad (a < r < b)$$

- **Particle source**

$$S_e = n_0 \langle \sigma_{ion} v \rangle n_e - \nu_L (n_e - n_{e,div})$$

- **Recycling from divertor**
  - Recycling ratio: $\gamma_0 = 0.8$
- **Gas puff from wall**

Stationary Electron Flux (1)

- **Stationary Electron Flux**
  - Setting inertia terms to zero in the model equations
- **Radial velocity**

$$u_{\mathrm{er}} = -\frac{1}{1+\alpha}\frac{\bar{\nu}_{\mathrm{e}}+\nu_{\mathrm{eNC}}}{n_{\mathrm{e}}m_{\mathrm{e}}\Omega_{\mathrm{e}\phi}^2}\frac{\mathrm{d}p}{\mathrm{d}r} - \frac{\alpha}{1+\alpha}\frac{E_\phi}{B_\theta} + \frac{1}{1+\alpha}\frac{1}{n_{\mathrm{e}}m_{\mathrm{e}}\Omega_{\mathrm{e}\phi}}\left(F_{\mathrm{e}\theta}^{\mathrm{W}} + \frac{B_\phi}{B_\theta}\alpha F_{\mathrm{e}\phi}^{\mathrm{W}}\right)$$

$$+\frac{\alpha}{1+\alpha}\frac{1}{\Omega_{\mathrm{e}\phi}}\frac{B_\phi}{B_\theta}\left[\nu_{\mathrm{eb}}u_{\mathrm{b}\phi} - (\bar{\nu}_{\mathrm{e}}-\nu_{\mathrm{ei}})u_{\mathrm{i}\phi}\right] + \frac{1}{1+\alpha}\frac{\bar{\nu}_{\mathrm{e}}+\nu_{\mathrm{eNC}}-\nu_{\mathrm{ei}}}{\Omega_{\mathrm{e}\phi}}u_{\mathrm{i}\theta}$$

where $\bar{\nu}_{\mathrm{e}} \equiv \nu_{\mathrm{ei}} + \nu_{\mathrm{eb}} + \nu_{\mathrm{L}} + \nu_{0\mathrm{e}}$,

$$\alpha \equiv \frac{\bar{\nu}_{\mathrm{e}}+\nu_{\mathrm{eNC}}}{\bar{\nu}_{\mathrm{e}}}\frac{B_\theta^2}{B_\phi^2}, \quad \Omega_{\mathrm{e}\phi} \equiv \frac{eB_\phi}{m_{\mathrm{e}}}, \quad \text{and} \quad \nu_{\mathrm{eb}} \equiv \frac{n_{\mathrm{b}}m_{\mathrm{b}}}{n_{\mathrm{e}}m_{\mathrm{e}}}\nu_{\mathrm{be}}$$

  - Damping rate, $\bar{\nu}_{\mathrm{eNC}}$, due to the neoclassical viscosity
  - First, second and third terms in RHS are **neoclassical diffusion**, **Ware pinch** and **turbulent diffusion**, respectively.
  - Fourth term denotes a neoclassical pinch due to a momentum input from beam ions.

Stationary Electron Flux (2)

- **Toroidal velocity**

$$u_{e\phi} = -\frac{1}{\bar{\nu}_e}\left[\frac{1}{1+\alpha}\frac{e}{m_e}E_\phi - \frac{1}{1+\alpha}\frac{B_\theta}{B_\phi}\frac{\bar{\nu}_e + \nu_{eNC}}{n_e m_e \Omega_{e\phi}}\frac{dp}{dr} + \frac{1}{1+\alpha}\frac{1}{n_e m_e}\left(\frac{B_\theta}{B_\phi}F^W_{e\theta} - F^W_{e\phi}\right) - \frac{\nu_{eb}}{1+\alpha}u_{b\phi}\right.$$
$$\left. + \frac{1}{1+\alpha}\frac{B_\theta}{B_\phi}(\bar{\nu}_e + \nu_{eNC} - \nu_{ei})u_{i\theta} - \frac{\nu_{ei} + \alpha\bar{\nu}_e}{1+\alpha}u_{i\phi}\right],$$

  - First, second and third terms in RHS are **neoclassical resistivity**, **bootstrap current**, and **turbulent driven current**.

- **Poloidal velocity can be obtained in a similar way.**

Model equations include major neoclassical effects!

Numerical Schemes Used in TASK/TX Code

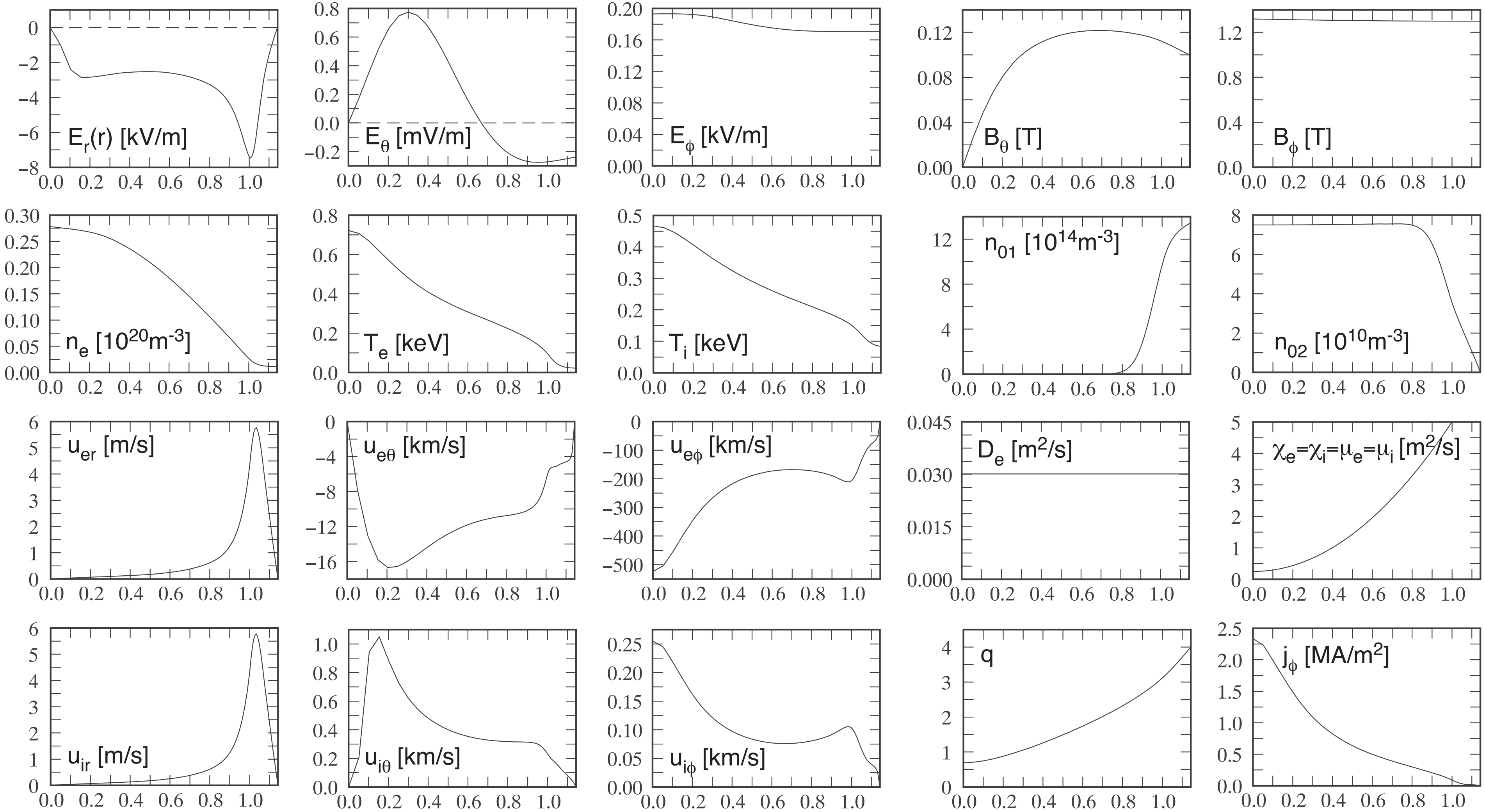
- **Finite element method (FEM)**
  - Linear interpolation function
  - **Streamline Upwind Petrov-Galerkin (SUPG) method**
    - Stabilizing spurious oscillation due to first-derivative terms
  - $s = r^2$ coordinate rather than r coordinate
  - **Achieving higher mesh resolution near the separatrix**
- **Time-advancing method**
  - **Full-implicit method**
    - Highly robust calculation
    - Time-consuming due to need of matrix equation solver
  - Mass lumping method
  - Picard method to solve nonlinear equations iteratively

Typical Ohmic Plasma Profiles at $t = 50\,\mathrm{ms}$

- **JFT-2M like plasma** composed of electron and hydrogen

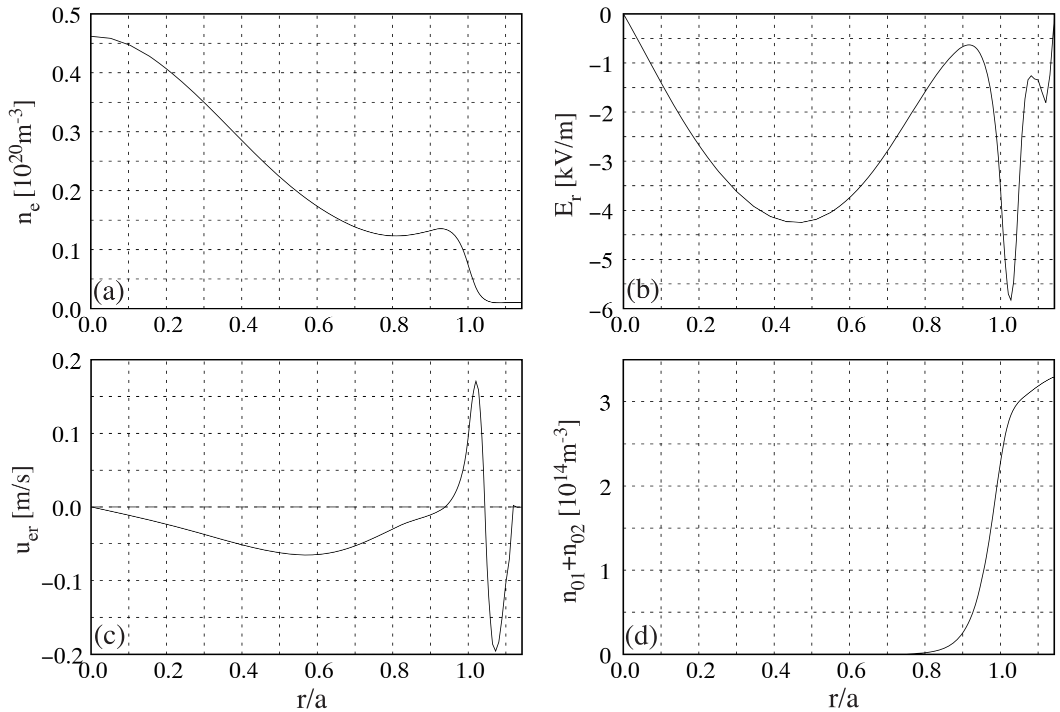
$R = 1.3\,\mathrm{m},\ a = 0.35\,\mathrm{m},\ b = 0.4\,\mathrm{m},\ B_{\phi b} = 1.3\,\mathrm{T},\ I_p = 0.2\,\mathrm{MA},\ S_{\mathrm{puff}} = 5.0 \times 10^{18}\,\mathrm{m^{-2}s^{-1}}$

$\gamma = 0.8,\ Z_{\mathrm{eff}} = 2.0$, Fixed turbulent coefficient profile



Neoclassical Transport without Turbulence

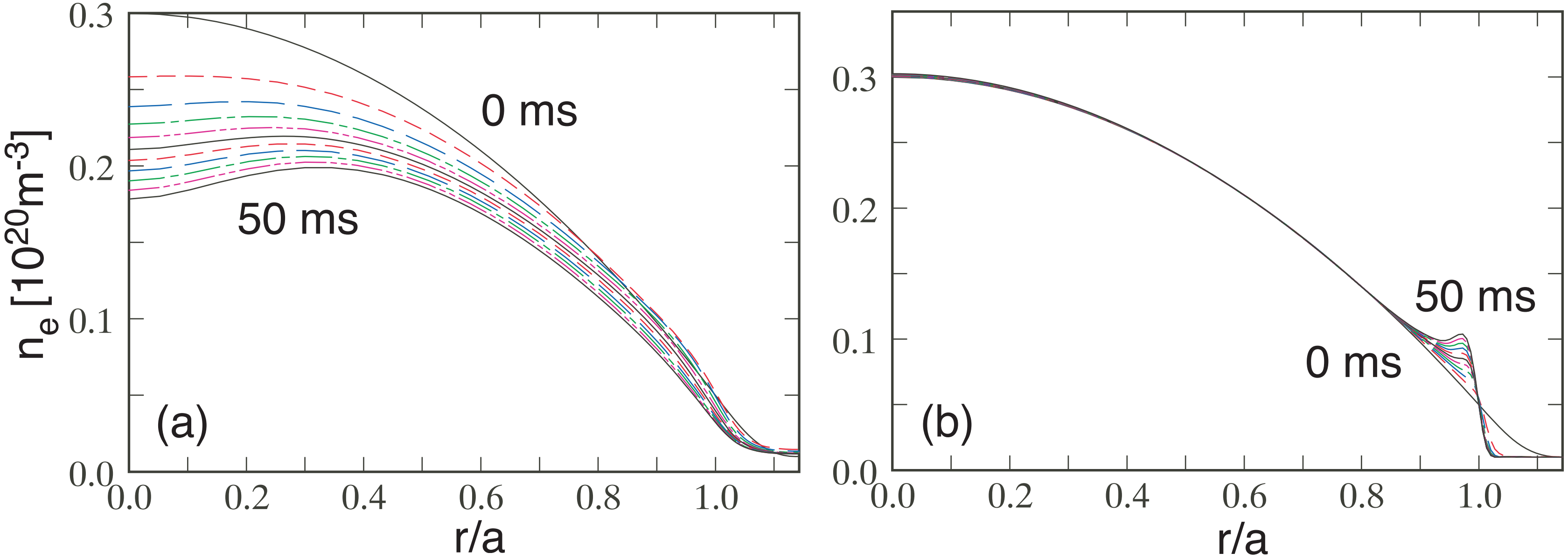
- **Clarifying neoclassical transport**
  - **No turbulent diffusivity and viscosity**
  - **Fixed temperature profiles**
- **Density peaking with steep gradient near the separatrix**
  - **Density peaking due to Ware pinch**
  - **Inward flux in the SOL due to ionization of neutrals**



Diffusion due to Turbulent Induced Force

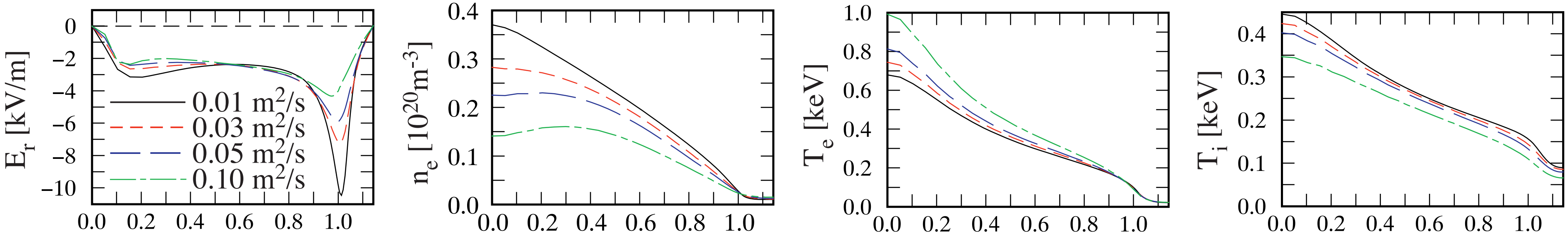
- **Confirming the validity of our particle diffusion model**
  - **No particle diffusion terms in the continuity equations**

    Turbulence-induced poloidal friction force $\Longrightarrow$ Change of radial velocity $\Longrightarrow$ Particle diffusion through convective term
  - No neoclassical viscosity assumed in this case
  - **Obvious particle diffusion (Left) and no diffusion (Right)**
- **Particle diffusion desribed properly in our model.**

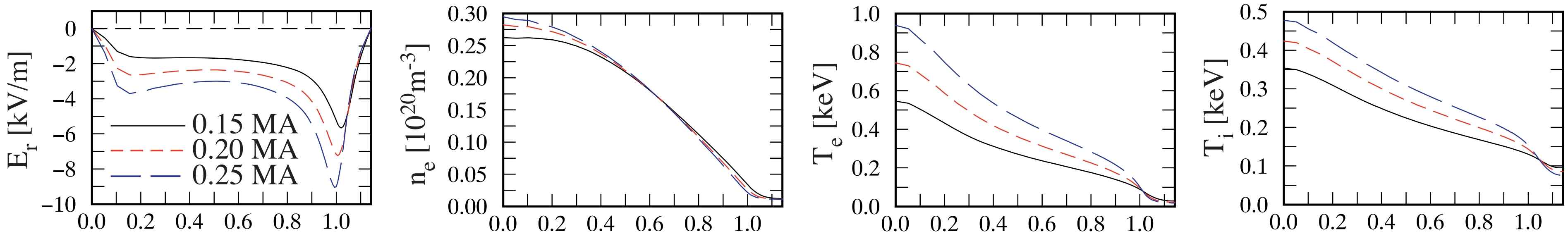


Parameter Dependence on D_e and I_p

- **Profile modifications due to the change of particle diffusivity**
  - **Density flattening with the increase of particle diffusion, D_e**
  - **Notch of E_r near the separatrix vanishes with the increase of D_e because of alleviation of $\frac{dp}{dr}$.**

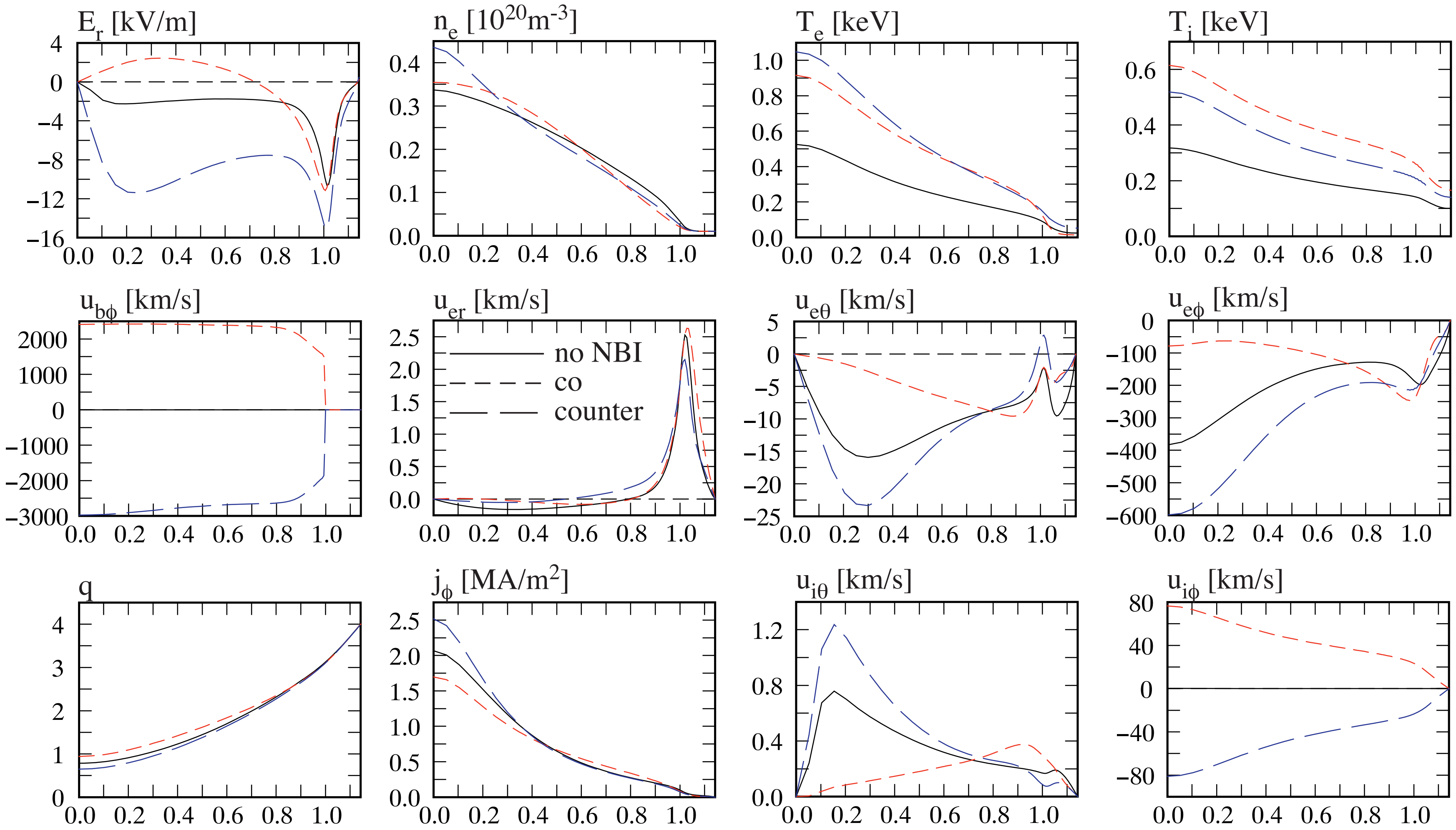


- **Profile modifications due to the change of plasma current**
  - **Increase in n and T near the axis and decrease in E_r with the increase of I_p**



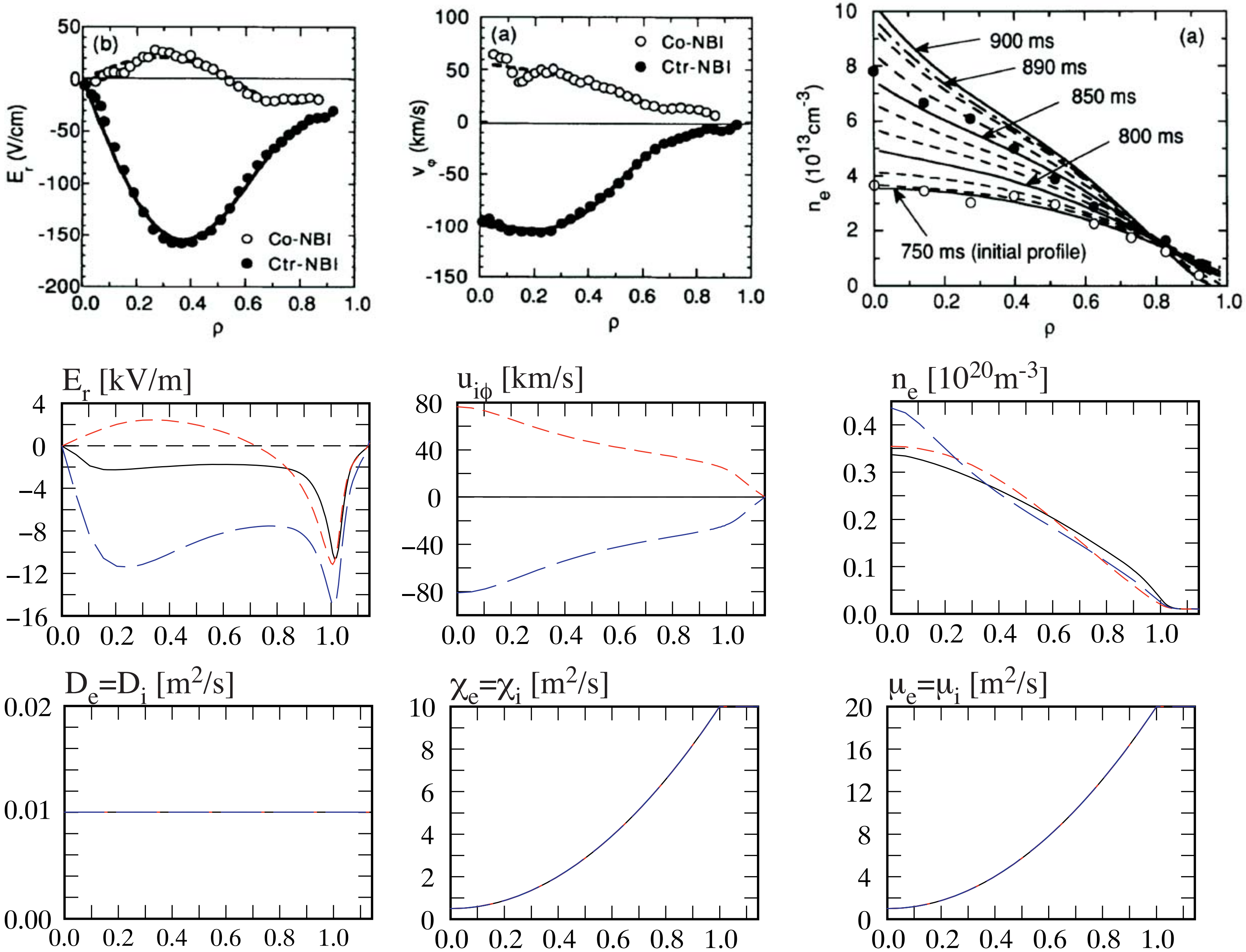
NBI of P_{NB} = 0.5 MW at t = 100 ms

- **The cases of before-, co- and ctr-NBIs**
  - **Modification of E_r profile depending on the direction of NBI, viz. $u_{\mathrm{i}\phi}$**
  - **Co: $u_{\mathrm{b}\phi} \nearrow \Rightarrow u_{\mathrm{i}\phi} \nearrow \Rightarrow E_r \nearrow$, $u_{\mathrm{b}\phi} \nearrow \Rightarrow u_{\mathrm{i}\phi} \nearrow \Rightarrow u_{\mathrm{e}\theta} \nearrow$ & $u_{\mathrm{e}\phi} \nearrow \Rightarrow u_{\mathrm{er}} \Rightarrow$ density flattening**
  - **Ctr: $u_{\mathrm{b}\phi} \searrow \Rightarrow u_{\mathrm{i}\phi} \searrow \Rightarrow E_r \searrow$, $u_{\mathrm{b}\phi} \searrow \Rightarrow u_{\mathrm{i}\phi} \searrow \Rightarrow u_{\mathrm{e}\theta} \searrow$ & $u_{\mathrm{e}\phi} \searrow \Rightarrow u_{\mathrm{er}} \Rightarrow$ density peaking**



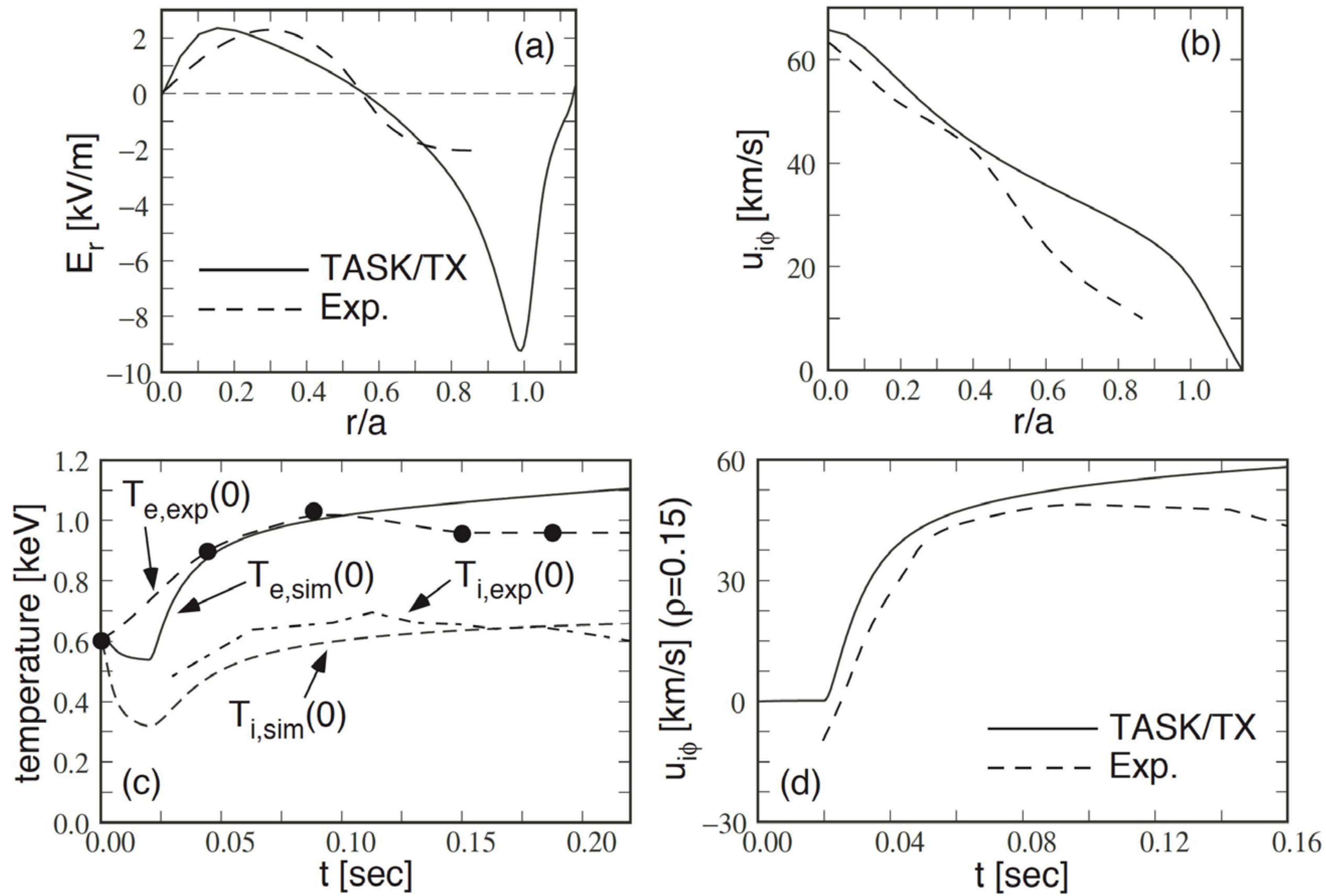
Comparison with JFT-2M experiment (1)

Ref. K. Ida et al., PRL ***68*** *(1992) 182*



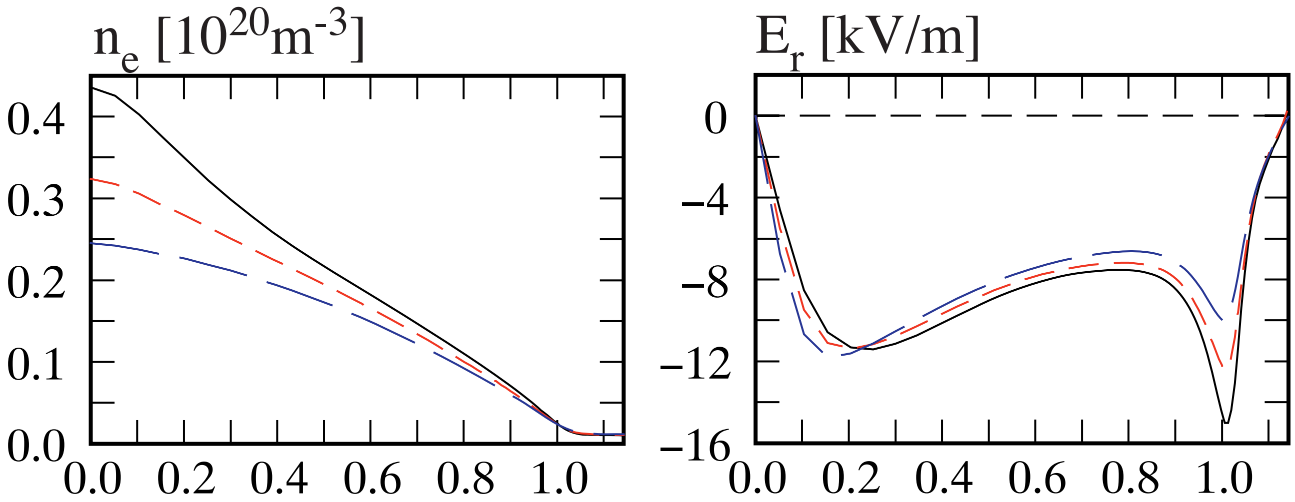
Comparison with JFT-2M experiment (2)

*Ref. K. Ida et al., PRL **68** (1992) 182*



D_e dependence in the case of counter NBI

- **Density profile is determined by the balance between turbulent-driven and neoclassical particle fluxes during NBI.**
- **Increase particle diffusivity from 0.01 to 0.03 for counter NBI**
  - $D_e = 0.01$ (Black): **Density peaking**
  - $D_e = 0.02$ (Red): Almost the same profile before NBI
  - $D_e = 0.03$ (Blue): **No peaking due to strong particle diffusion**
  - E_r **is not significantly affected except near the separatrix.**



Summary

- **TASK/TX**: We are developing the TASK/TX code in order to describe poloidal and toroidal rotations and radial electric field formation. The code simultaneously solves the two-fluid equations of motion; continuity equations and heat transport equations coupled with Maxwell's equations.
- **Transport modeling**: The validity of our approach of neo-classical transport and particle transport was confirmed.
- **Ohmic**: We have examined the I_{p} dependence of the density, temperature and radial electric field profiles.
- **NBI**: We have analyzed the modifications of density profiles during NBI in JFT-2M like plasma. Density flattening in the case of co-NBI and density peaking in the case of ctr-NBI are qualitatively reproduced.

Future Works

- Precise surface average including toroidal metrics
- Equation for heat flux to complete neoclassical transport
- Theory-based turbulent transport model
- Multi-spices ion transport
- Dynamic simulation of transport barrier formation
- Long time simulation with more numerical robustness