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Integrated Full-Wave Modeling of ICRF Heating in Tokamak Plasmas

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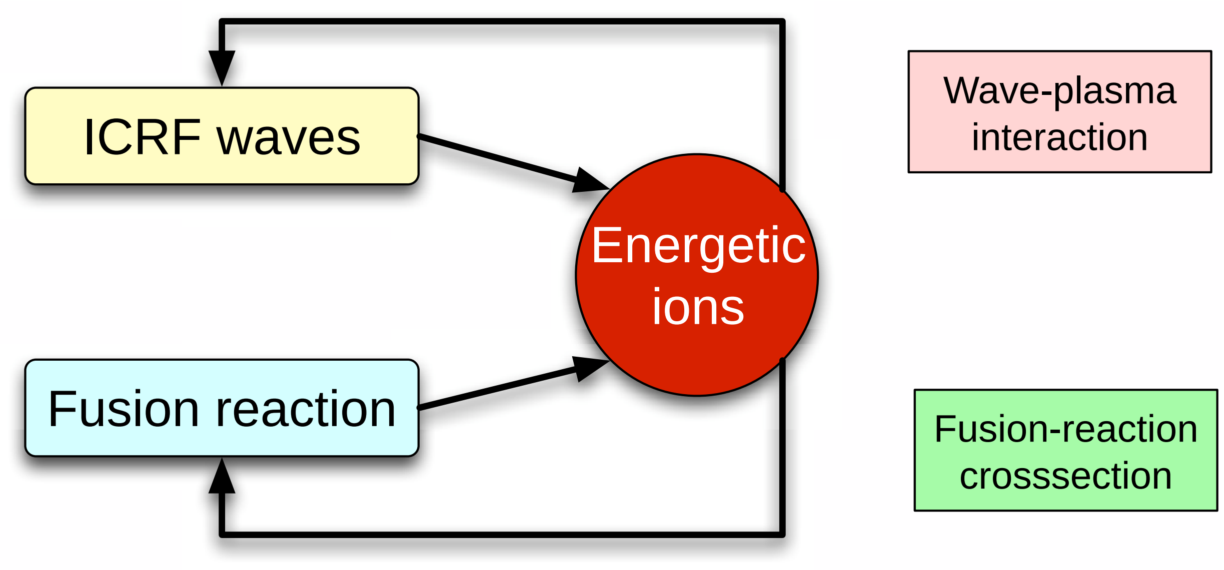
1. Introduction
2. Integrated tokamak modeling code: TASk
3. Modification of velocity distribution function
4. Integral formulation of full wave analysis
5. Summary

Introduction(1)

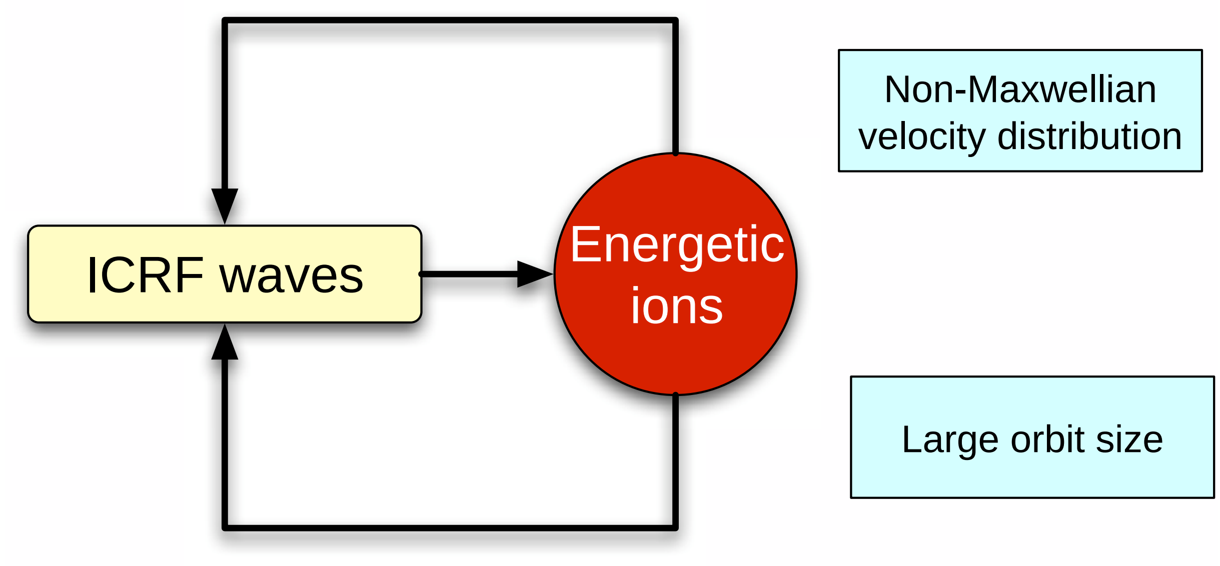
ICRF Waves in Burning Plasmas



- Both ICRF waves and fusion reaction generate energetic ions and are affected by the energetic ions.
- In the start up phase of ITER plasmas, the role of ICRF waves is important, time-evolving, and sensitive to the plasma conditions

Introduction(2)

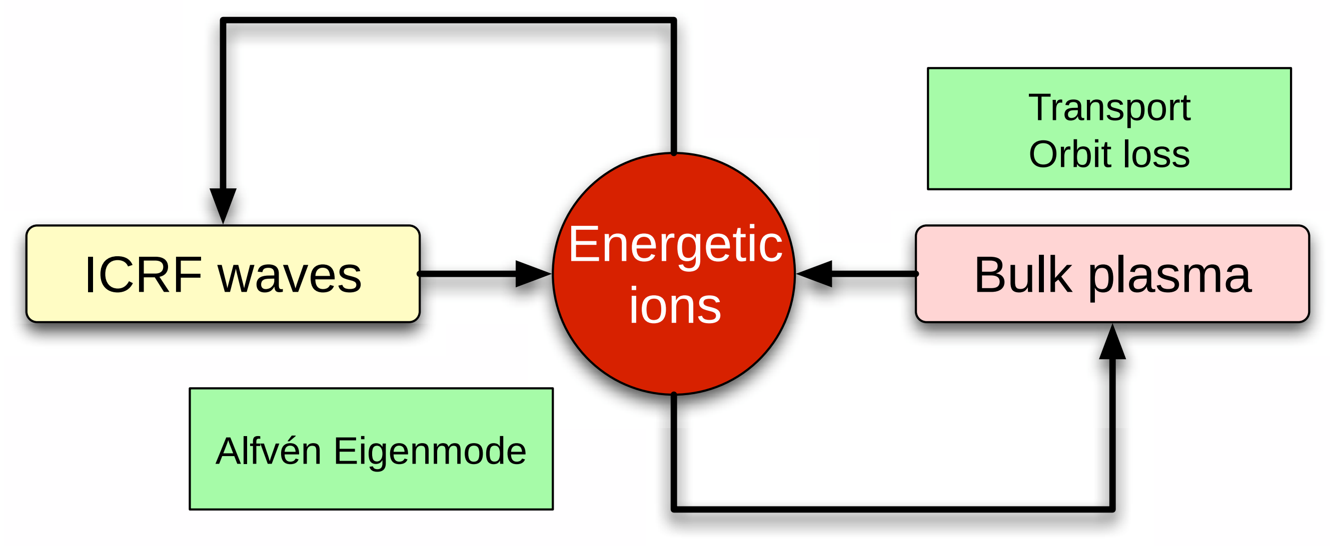
Gyrokinetic Behavior of Energetic Ions



- Self-consistent analysis of non-Maxwellian velocity distribution function is necessary.
- Finite gyroradius and finite orbit size affect the behavior of ICRF waves.

Introduction(3)

Comprehensive Modeling of Burning Plasmas



- Energetic ions interact with bulk plasmas through, for example, transport processes and orbit loss.
- Alfvén eigenmodes may affect the energetic ions themselves.
- Integrated comprehensive modeling of burning plasmas is inevitable.

Introduction (4)

- **Analysis of ICRF waves in burning plasma requires**
  - **Full wave analysis**
  - **Non-Maxwellian velocity distribution function**
  - **Finite gyroradius effect**
  - **Integrated modeling**
- **Integrated approach using the TASK code**

TASK Code

- **Transport Analysing System for TokamaK**
- **Features**
  - **A Core of Integrated Modeling Code in BPSI**
    — Modular structure, Unified Standard data interface
  - **Various Heating and Current Drive Scheme**
    — Full wave analysis for IC and AW
    — Ray and beam tracing for EC and LH
    — 3D Fokker-Planck analysis
  - **High Portability**
  - **Development using CVS**
  - **Open Source**
  - **Parallel Processing using MPI Library**
  - **Extension to Toroidal Helical Plasmas**

Modules of TASK

PL	**Data Interface**	Data conversion, Profile database
EQ	**2D Equilibrium**	Fixed/Free boundary, Toroidal rotation
TR	**1D Transport**	Diffusive transport, Transport models
WR	**3D Geometr. Optics**	EC, LH: Ray tracing, Beam tracing
WM	**3D Full Wave**	IC, AW: Antenna excitation, Eigenmode
FP	**3D Fokker-Planck**	Relativistic, Bounce-averaged
DP	**Wave Dispersion**	Local dielectric tensor, Arbitrary $f(\boldsymbol{v})$
LIB	**Libraries**	LIB, MTX, MPI

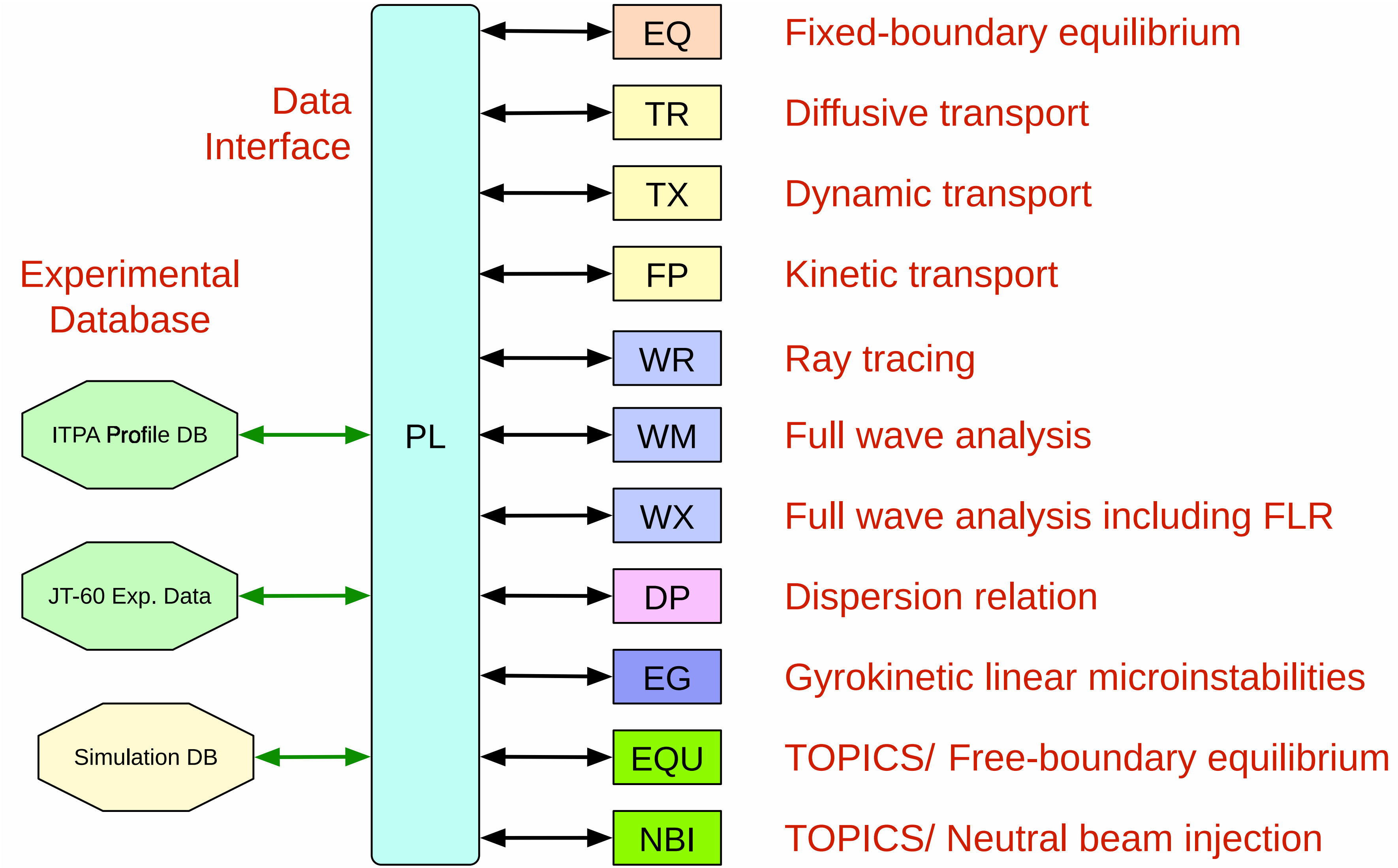
Under Development

TX	Transport analysis including plasma rotation and E_r
EG	Gyrokinetic linear stability analysis

Imported from TOPICS

EQU	Free boundary equilibrium
NBI	NBI heating

Modular Structure of TASK



Inter-Module Collaboration Interface: TASK/PL

- **Role of Module Interface**
  - **Data exchange between modules**:
    - — **Standard dataset**: Specify set of data (cf. ITPA profile DB)
    - — **Specification of data exchange interface**: initialize, set, get
  - **Execution control**:
    - — **Specification of execution control interface**: initialize, setup, exec, visualize, terminate
    - — **Uniform user interface**: parameter input, graphic output
- **Role of data exchange interface: TASK/PL**
  - **Keep present status of plasma and device**
  - **Store history of plasma**
  - **Save into file and load from file**
  - **Interface to experimental data base**

Standard Dataset (interim)

Shot data

Machine ID, Shot ID, Model ID

Device data: (Level 1)

RR	R	m	Geometrical major radius
RA	a	m	Geometrical minor radius
RB	b	m	Wall radius
BB	B	T	Vacuum toroidal mag. field
RKAP	κ		Elongation at boundary
RDLT	δ		Triangularity at boundary
RIP	I_p	A	Typical plasma current

Equilibrium data: (Level 1)

PSI2D	$\psi_\mathrm{p}(R,Z)$	Tm^2	2D poloidal magnetic flux
PSIT	$\psi_\mathrm{t}(\rho)$	Tm^2	Toroidal magnetic flux
PSIP	$\psi_\mathrm{p}(\rho)$	Tm^2	Poloidal magnetic flux
ITPSI	$I_\mathrm{t}(\rho)$	Tm	Poloidal current: $2\pi B_\phi R$
IPPSI	$I_\mathrm{p}(\rho)$	Tm	Toroidal current
PPSI	$p(\rho)$	MPa	Plasma pressure
QINV	$1/q(\rho)$		Inverse of safety factor

Metric data

1D: $V'(\rho)$, $\langle\nabla V\rangle(\rho)$, $\cdots$

2D: g_{ij}, $\cdots$

3D: g_{ij}, $\cdots$

Fluid plasma data

NSMAX	s		Number of particle species
PA	A_s		Atomic mass
PZ0	Z_{0s}		Charge number
PZ	Z_s		Charge state number
PN	$n_s(\rho)$	m^3	Number density
PT	$T_s(\rho)$	eV	Temperature
PU	$u_{s\phi}(\rho)$	m/s	Toroidal rotation velocity
QINV	$1/q(\rho)$		Inverse of safety factor

Kinetic plasma data

FP	$f(p,\theta_p,\rho)$		momentum dist. fn at $\theta = 0$

Dielectric tensor data

CEPS	$\overleftrightarrow{\epsilon}(\rho,\chi,\zeta)$		Local dielectric tensor

Full wave field data

CE	$\boldsymbol{E}(\rho,\chi,\zeta)$	V/m	Complex wave electric field
CB	$\boldsymbol{B}(\rho,\chi,\zeta)$	$\mathrm{Wb/m}^2$	Complex wave magnetic field

Ray/Beam tracing field data

RRAY	$R(\ell)$	m	R of ray at length ℓ
ZRAY	$Z(\ell)$	m	Z of ray at length ℓ
PRAY	$\phi(\ell)$	rad	ϕ of ray at length ℓ
CERAY	$\boldsymbol{E}(\ell)$	V/m	Wave electric field at length ℓ
PWRAY	$\boldsymbol{P}(\ell)$	W	Wave power at length ℓ
DRAY	$\boldsymbol{d}(\ell)$	m	Beam radius at length ℓ
VRAY	$\boldsymbol{v}(\ell)$	1/m	Beam curvature at length ℓ

Example: Data Structure and Program Interface

- **Data structure**: **Derived type** (Fortran95)

```
type bpsd_plasmaf_data
   real(8) :: pn      ! Number density [m^-3]
   real(8) :: pt      ! Temperature [eV]
   real(8) :: ptpr    ! Parallel temperature [eV]
   real(8) :: ptpp    ! Perpendicular temperature [eV]
   real(8) :: pu      ! Parallel flow velocity [m/s]
end type bpsd_plasmaf_data
type bpsd_plasmaf_type
   real(8) :: time
   integer :: nrmax      ! Number of radial points
   integer :: nsmax      ! Number of particle species
   real(8), dimension(:), allocatable :: s
                         ! (rho^2) : normarized toroidal flux
   real(8), dimension(:), allocatable :: qinv
                         ! 1/q : inverse of safety factor
   type(bpsd_plasmaf_data), dimension(:,:), allocatable :: data
end type bpsd_plasmaf_type
```

- **Program interface**

Set data	`bpsd_set_data(plasmaf,ierr)`
Get data	`bpsd_get_data(plasmaf,ierr)`
Save data	`bpsd_save_data(filename,plasmaf,ierr)`
Load data	`bpsd_load_data(filename,plasmaf,ierr)`
Plot data	`bpsd_plot_data(plasmaf,ierr)`

Examples of sequence in a module

- **TR_EXEC(dt)**

```
call bpsd_get_data(plasmaf,ierr)
call bpsd_get_data(metric1D,ierr)
local data <- plasmaf,metric1D
advance time step dt
plasmaf <- local data
call bpsd_set_data(plasmaf,ierr)
```

- **EQ_CALC**

```
call bpsd_get_data(plasmaf,ierr)
local data <- plasmaf
calculate equilibrium
update plasmaf
call bpsd_set_data(plasmaf,ierr)
equ1D,metric1D <- local data
call bpsd_set_data(equ1D,ierr)
call bpsd_set_data(metric1D,ierr)
```

Wave Dispersion Analysis : TASK/DP

- **Various Models of Dielectric Tensor** $\overleftrightarrow{\epsilon}(\omega, \boldsymbol{k}; \boldsymbol{r})$:
  - **Resistive MHD** model
  - **Collisional cold** plasma model
  - **Collisional warm** plasma model
  - **Kinetic plasma** model (**Maxwellian**, non-relativistic)
  - **Kinetic plasma** model (**Arbitrary** $f(\boldsymbol{v})$, relativistic)
  - **Gyro-kinetic plasma** model (Maxwellian)
- **Numerical Integration in momentum space**: **Arbitrary** $f(\boldsymbol{v})$
  - Relativistic Maxwellian
  - Output of TASK/FP: Fokker-Planck code

Full wave analysis: TASK/WM

- **magnetic surface coordinate**: (ψ, θ, φ)
- Boundary-value problem of **Maxwell's equation**

$$\nabla \times \nabla \times \boldsymbol{E} = \frac{\omega^2}{c^2} \overleftrightarrow{\epsilon} \cdot \boldsymbol{E} + \mathrm{i}\,\omega\mu_0 \boldsymbol{j}_{\mathrm{ext}}$$

- Kinetic **dielectric tensor**: $\overleftrightarrow{\epsilon}$
  - Wave-particle resonance: $Z[(\omega - n\omega_{\mathrm{c}})/k_{\|} v_{\mathrm{th}}]$
  - Finite gyroradius effect: Reductive $\Longrightarrow$ Integral **(ongoing)**
- Poloidal and toroidal **mode expansion**
- FDM: $\Longrightarrow$ FEM **(onging)**
- Eigenmode analysis: **Complex eigen frequency** which maximize wave amplitude for fixed excitation proportional to electron density

Fokker-Planck Analysis : TASK/FP

- **Fokker-Planck equation**
  for **velocity distribution function** $f(p_{\|}, p_{\perp}, \psi, t)$

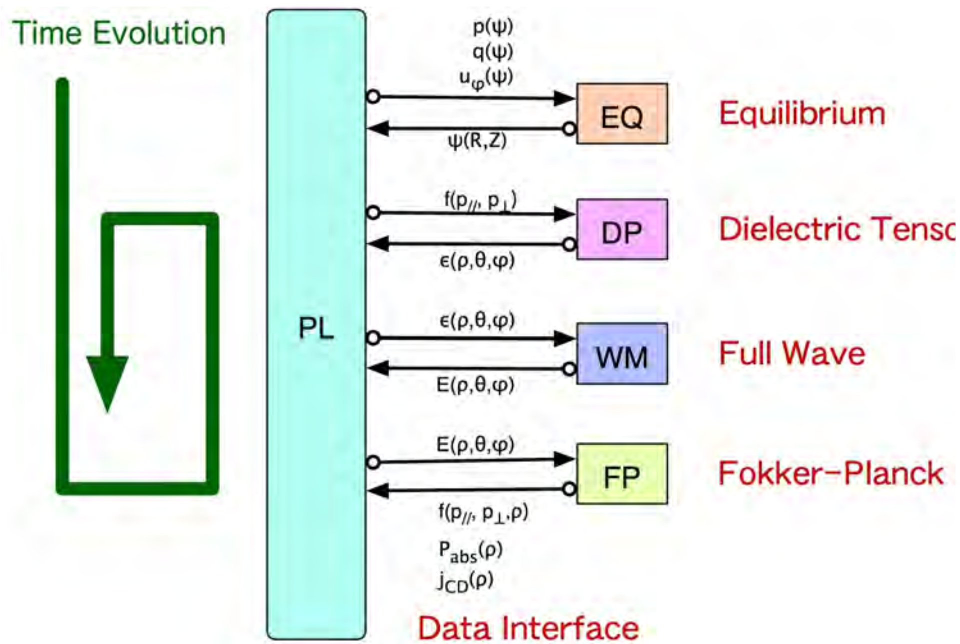
$$\frac{\partial f}{\partial t} = E(f) + C(f) + Q(f) + L(f)$$

  - $E(f)$: Acceleration term due to DC electric field
  - $C(f)$: Coulomb collision term
  - $Q(f)$: Quasi-linear term due to wave-particle resonance
  - $L(f)$: Spatial diffusion term

- **Bounce-averaged**: Trapped particle effect, zero banana width
- **Relativistic**: momentum $\boldsymbol{p}$, weakly relativistic collision term
- **Nonlinear collision**: momentum or energy conservation
- **Three-dimensional**: spatial diffusion (neoclassical, turbulent)

Self-Consistent Wave Analysis with Modified $f(v)$

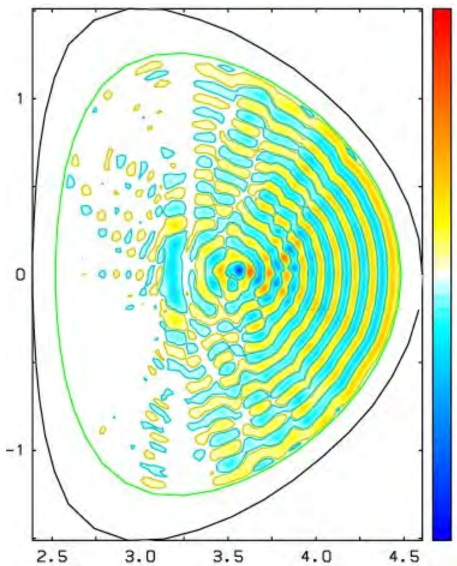
- **Modification of velocity distribution from Maxwellian**
  - Energetic ions generated by ICRF waves
  - Alpha particles generated by fusion reaction
  - Fast ions generated by NB injection
- **Self-consistent wave analysis including modification of $f(v)$**



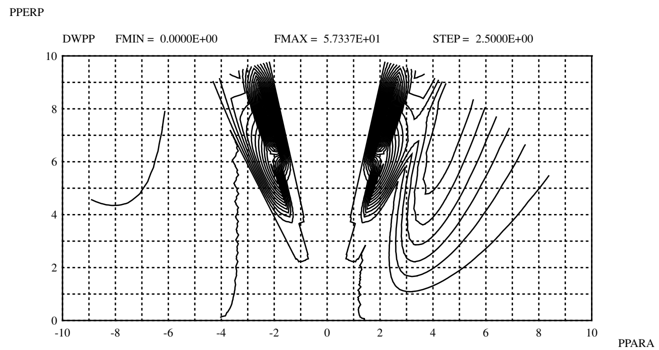
Preliminary Results

- **Tail formation by ICRF minority heating**

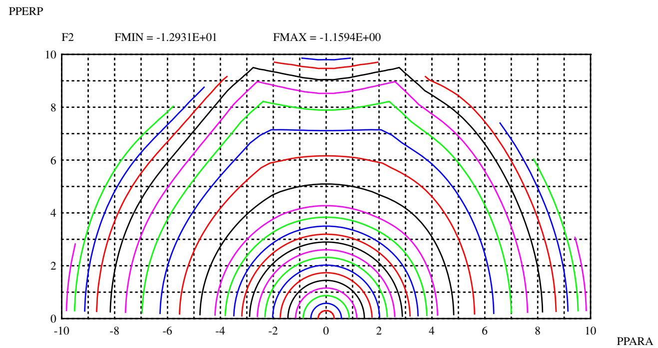
Wave pattern



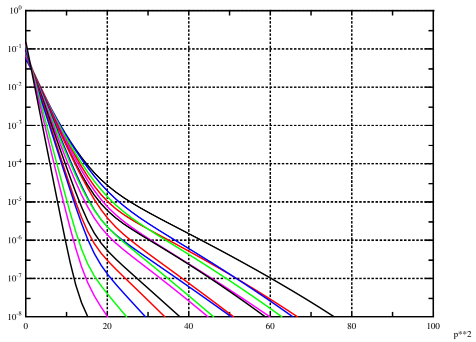
Quasi-linear Diffusion



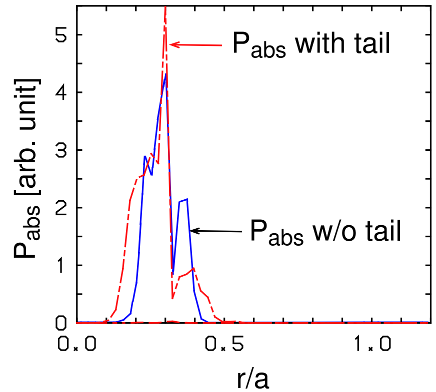
Momentum Distribution



Tail Formation



Power deposition



Finite Gyroradius Effects in Full Waves Analyses

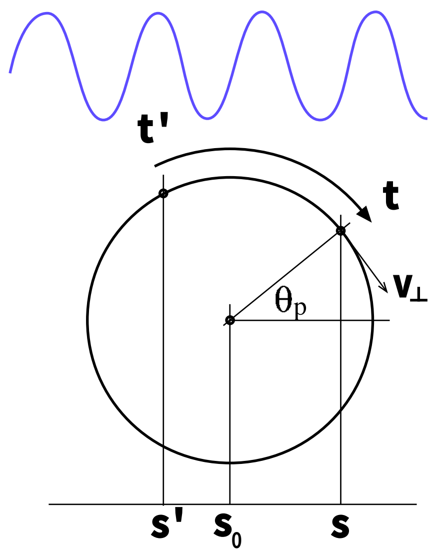
- Several approaches to describe the finite gyroradius effects.
- **Differential operators**: $k_\perp \rho \rightarrow i\rho\partial/\partial r_\perp$
  - This approach cannot be applied to the case $k_\perp \rho \gtrsim 1$.
  - Extension to the third and higher harmonics is difficult.
- **Spectral method**: Fourier transform in inhomogeneous direction
  - This approach can be applied to the case $k_\perp \rho > 1$.
  - All the wave field spectra are coupled with each other.
  - Solving a dense matrix equation requires large computer resources.
- **Integral operators**: $\int \epsilon(x - x') \cdot E(x')\mathrm{d}x'$
  - This approach can be applied to the case $k_\perp \rho > 1$
  - Correlations are localized within several gyroradii
  - Necessary to solve a large band matrix

Full Wave Analysis
Using an Integral Form of Dielectric Tensor

- **Maxwell's equation**:

$$\nabla \times \nabla \times \boldsymbol{E}(\boldsymbol{r}) + \frac{\omega^2}{c^2} \int \overleftrightarrow{\epsilon}(\boldsymbol{r}, \boldsymbol{r}') \cdot \boldsymbol{E}(\boldsymbol{r}') \mathrm{d}\boldsymbol{r} = \mu_0 \boldsymbol{j}_{ext}(\boldsymbol{r})$$

- **Integral form of dielectric tensor**: $\overleftrightarrow{\epsilon}(\boldsymbol{r}, \boldsymbol{r}')$
  - Integration along the unperturbed cyclotron orbit



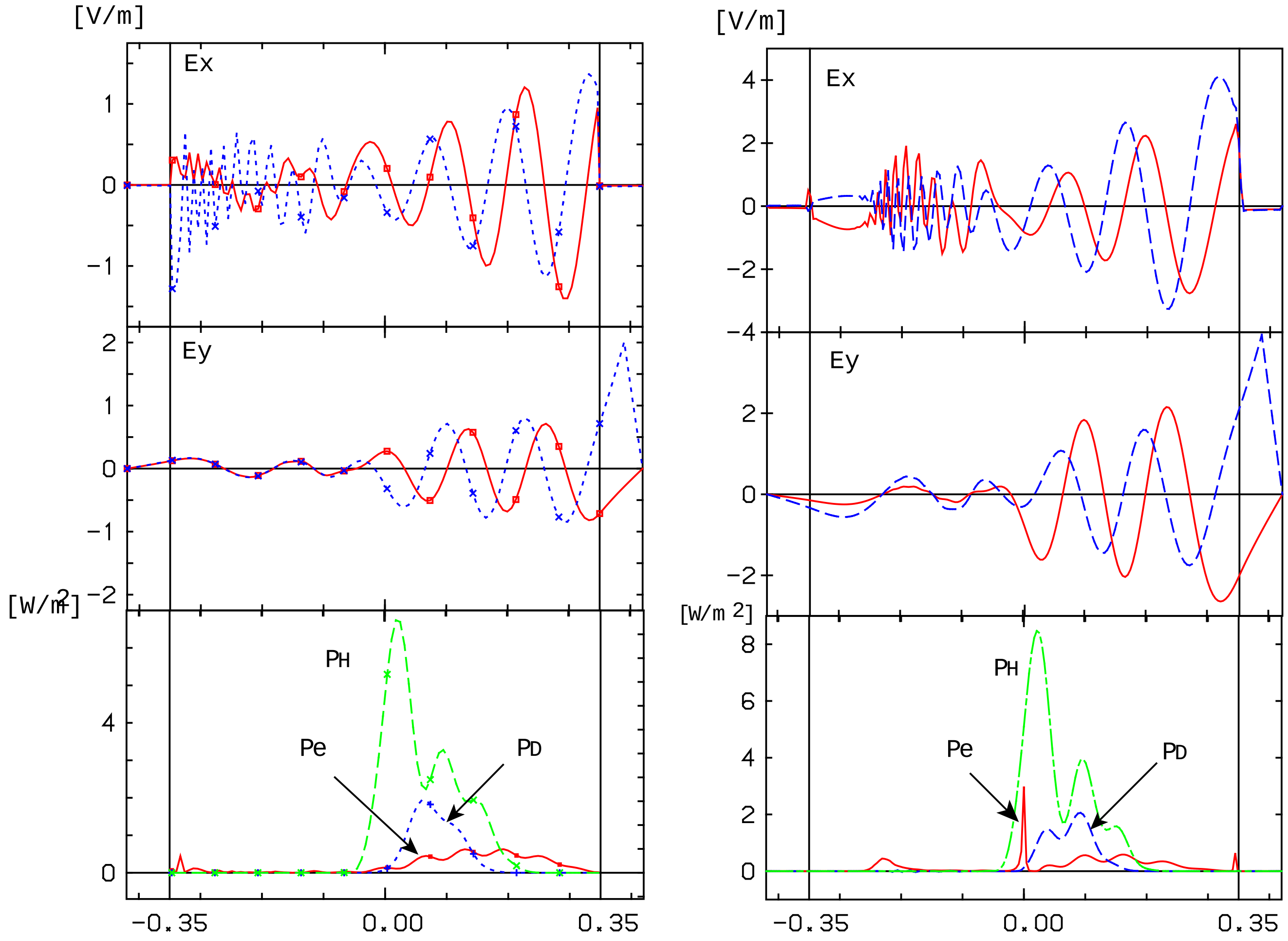
- **1D analysis in tokamaks**
  - To confirm the applicability
  - **Similar formulation in the lowest order of ρ/L**

  — Sauter O, Vaclavik J, Nucl. Fusion **32** (1992) 1455.

One-Dimensional Analysis (1)

ICRF minoring heating without energetic particles ($n_H/n_D = 0.1$)

Differential form | Integral form



$R_0 = 1.31\text{m}$

$a = 0.35\text{m}$

$B_0 = 1.4\text{T}$

$T_{e0} = 1.5\text{keV}$

$T_{D0} = 1.5\text{keV}$

$T_{H0} = 1.5\text{keV}$

$n_{s0} = 10^{20}\text{m}^{-3}$

$\omega/2\pi = 20\text{MHz}$

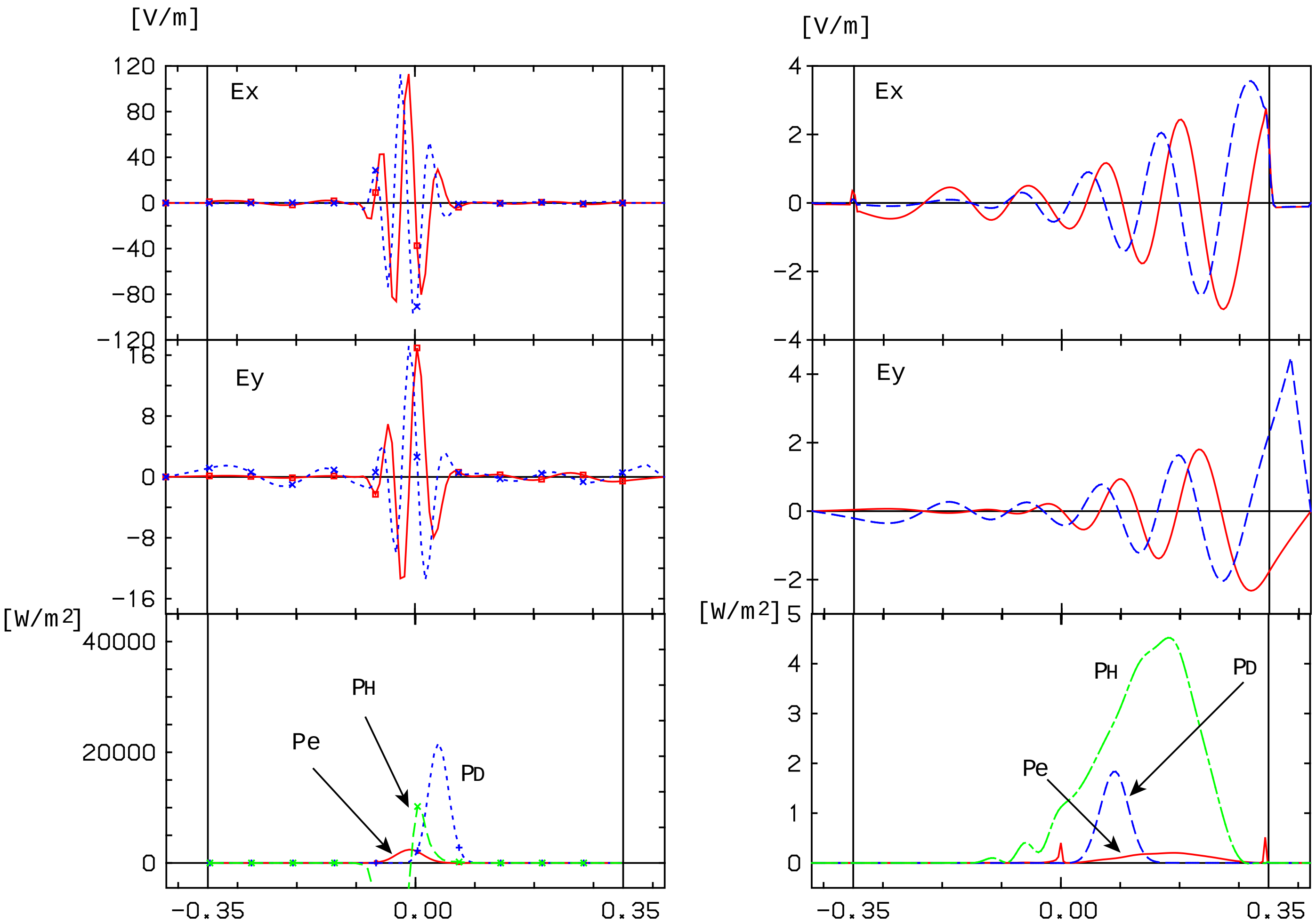
Differential approach is applicable

One-Dimensional Analysis (2)

ICRF minoring heating with energetic particles ($n_H/n_D = 0.1$)

Differential form

Integral form



$R_0 = 1.31\text{m}$

$a = 0.35\text{m}$

$B_0 = 1.4\text{T}$

$T_{e0} = 1\text{keV}$

$T_{D0} = 1\text{keV}$

$T_{H0} = 100\text{keV}$

$n_{s0} = 10^{20}\text{m}^{-3}$

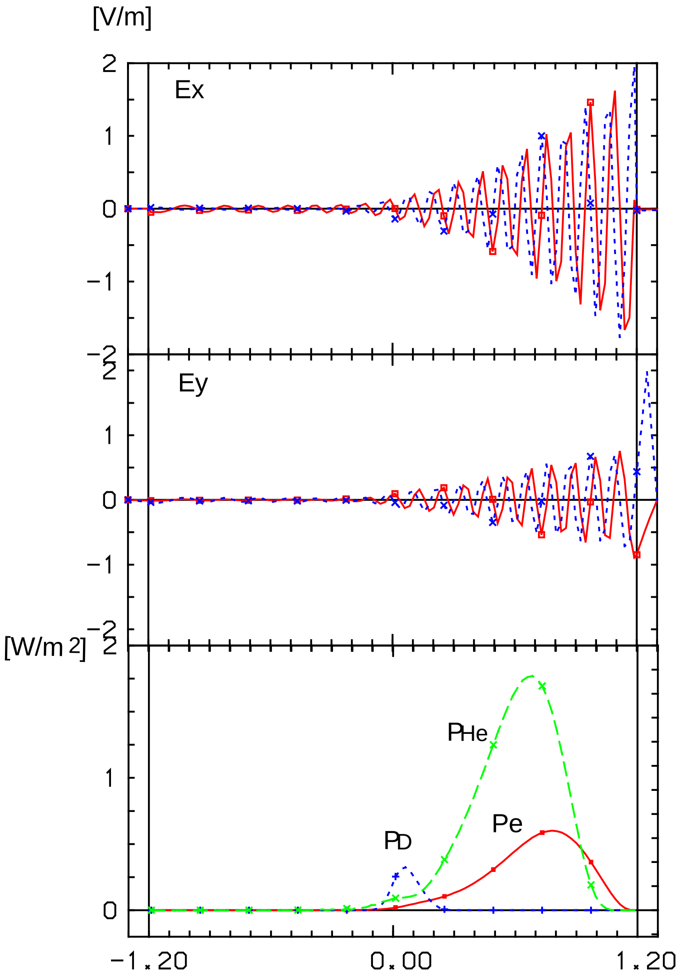
$\omega/2\pi = 20\text{MHz}$

Differential approach cannot be applied since $k_\perp \rho_i > 1$.

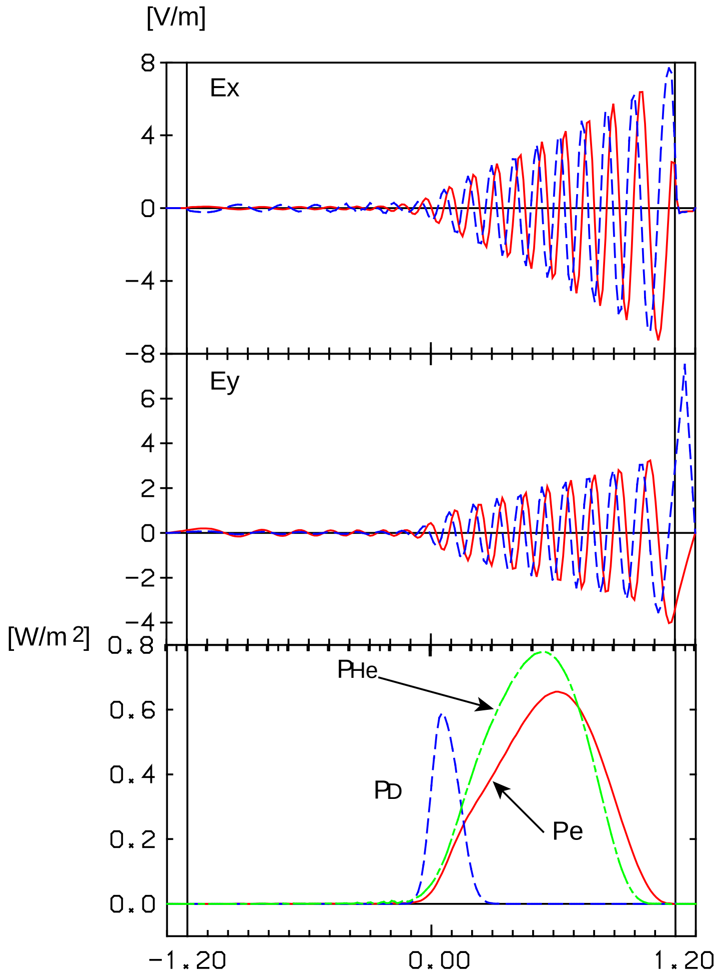
One-Dimensional Analysis (3)

ICRF minoring heating with α-particles ($n_{\mathrm{D}} : n_{\mathrm{He}} = 0.96 : 0.02$)

Differential form



Integral form



$R_0 = 3.0\mathrm{m}$

$a = 1.2\mathrm{m}$

$B_0 = 3\mathrm{T}$

$T_{e0} = 10\mathrm{keV}$

$T_{D0} = 10\mathrm{keV}$

$T_{\alpha 0} = 3.5\mathrm{MeV}$

$n_{s0} = 10^{20}\mathrm{m}^{-3}$

$\omega / 2\pi = 45\mathrm{MHz}$

Absorption by α may be over- or under-estimated by differential approach.

3D Formulation

- **Coordinates**
  - **Magnetic coordinate system**: (ψ, χ, ζ)
  - **Local Cartesian coordinate system**: (s, p, b)
  - **Fourier expansion**: poloidal and toroidal mode numbers, m, n
- **Perturbed current**

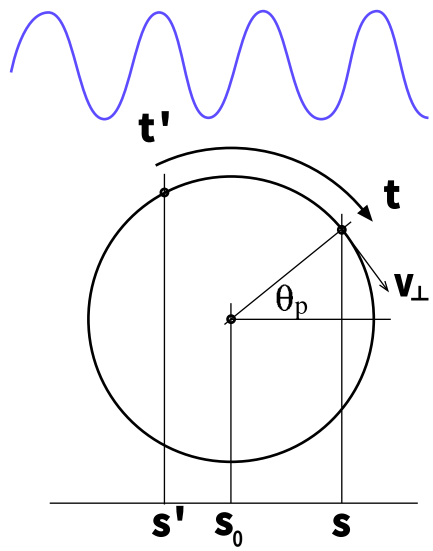
$$\boldsymbol{j}(\boldsymbol{r}, t) = -\frac{q}{m} \int \mathrm{d}\boldsymbol{v}\, q\boldsymbol{v} \int_{-\infty}^{\infty} \mathrm{d}t' \left[\boldsymbol{E}(\boldsymbol{r}', t') + \boldsymbol{v}' \times \boldsymbol{B}(\boldsymbol{r}', t')\right] \cdot \frac{\partial f_0(\boldsymbol{v}')}{\partial \boldsymbol{v}'}$$

- **Maxwell distribution function**
  - Anisotropic Maxwell distribution with $T_\perp$ and $T_\parallel$:

$$f_0(s_0, \boldsymbol{v}) = n_0 \left(\frac{m}{2\pi T_\perp}\right)^{3/2} \left(\frac{T_\perp}{T_\parallel}\right)^{1/2} \exp\left[-\frac{v_\perp^2}{2v_{T_\perp}^2} - \frac{v_\parallel^2}{2v_{T_\parallel}^2}\right]$$

Variable Transformations

- **Transformation of Integral Variables**
  - Transformation from the velocity space variables $(v_\perp, \theta_0)$ to the particle position s' and the guiding center position s_0.
  - Jacobian: $J = \dfrac{\partial(v_\perp, \theta_0)}{\partial(s', s_0)} = -\dfrac{\omega_c^2}{v_\perp \sin \omega_c \tau}$.



  - Express $v_\perp$ and θ_0 by s' and s_0 using $\tau = t - t'$, e.g.,

$$v_\perp \sin(\omega_c \tau + \theta_0) = \frac{\omega_c}{v_\perp} \frac{s - s'}{2} \frac{1}{\tan \frac{1}{2}\omega_c \tau} + \frac{\omega_c}{v_\perp} \left(\frac{s + s'}{2} - s_0 \right) \tan \frac{1}{2} \omega_c \tau$$

- **Integration over τ**: Fourier expansion with cyclotron motion
- **Integration over $v_\parallel$**: Plasma dispersion function

Final Form of Induced Current

- **Induced current**:

$$\cdot \begin{pmatrix} J_s^{mn}(s) \\ J_p^{mn}(s) \\ J_b^{mn}(s) \end{pmatrix} = \int \mathrm{d}s' \sum_{m'n'} \overleftrightarrow{\sigma}^{m'n'mn}(s,s') \cdot \begin{pmatrix} E_s^{m'n'}(s') \\ E_p^{m'n'}(s') \\ E_b^{m'n'}(s') \end{pmatrix}$$

- **Electrical conductivity**:

$$\overleftrightarrow{\sigma}^{m'n'mn}(s,s') = -in_0\frac{q^2}{m}\sum_{\ell}\int \mathrm{d}s_0 \int_0^{2\pi}\mathrm{d}\chi_0 \int_0^{2\pi}\mathrm{d}\zeta_0 \exp i\left\{(m'-m)\chi_0+(n'-n)\zeta_0\right\} \overleftrightarrow{H}_{\ell}(s,s',s_0,\chi_0,\zeta_0)$$

- **Matrix coefficients**: $\overleftrightarrow{H}_{\ell}(s,s',s_0,\chi_0,\zeta_0)$
  - Four kinds of **Kernel functions** including s, s', s_0 and harmonics number ℓ
    - — The kernel functions are localized within several thermal gyro-radii.
  - **Plasma dispersion function**

Kernel Functions

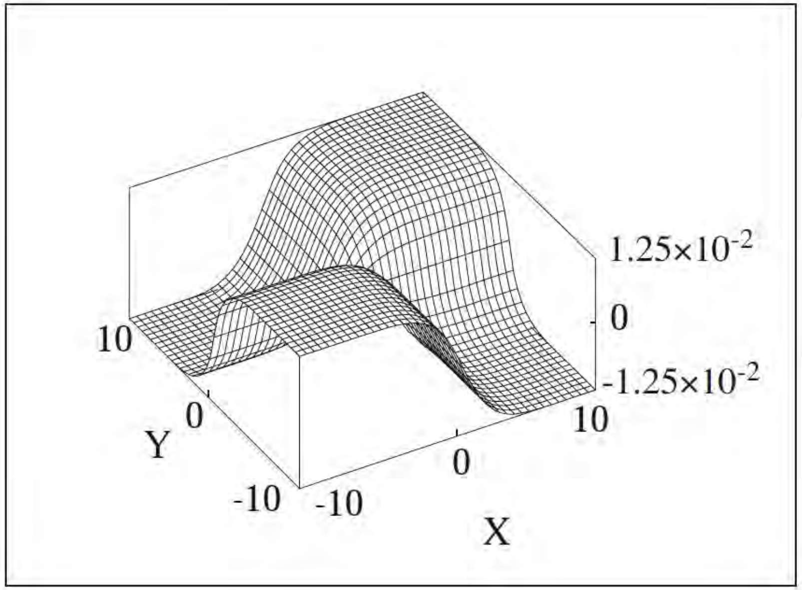
- **Kernel Function and its integrals**

$$F_n^{(i)}(X,Y) \equiv \frac{1}{2\pi^2}\int_0^{\pi} \mathrm{d}\theta \exp\left[-\frac{X^2}{1+\cos\theta} - \frac{Y^2}{1-\cos\theta}\right] f_n^{(i)}(\theta)$$

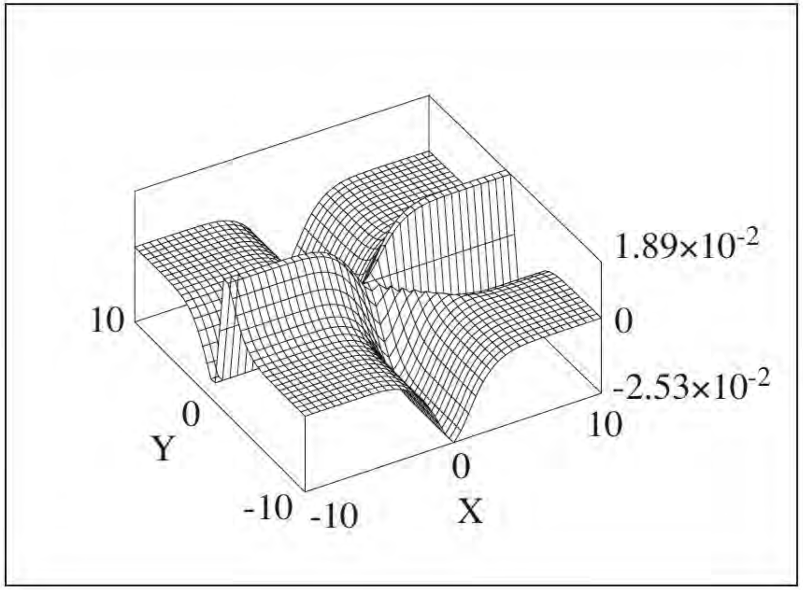
$$\mathcal{F}_n^{(ijk)}(X,Y) \equiv \int_0^{Y} \mathrm{d}Y' \int_0^{X+Y'} \mathrm{d}X' X'^j Y'^k F_n^{(i)}(X',Y')$$

$$f_n^{(i)}(\theta) = \begin{cases} \dfrac{\cos n\theta}{\sin\theta} & (i=1) \\ \sin n\theta & (i=2) \\ \dfrac{\sin n\theta}{\sin^2\theta} & (i=3) \\ \dfrac{\cos\theta \sin n\theta}{\sin^2\theta} & (i=4) \end{cases}$$

$F_0^{(100)}$



$F_1^{(100)}$



Status of extension to 3D configuration

- In a homogeneous plasma, usual formula including th Bessel functions can be recovered.
- Kernel functions are the same as the 1D case,
- **FEM formulation is required for convolution integral.**
- **Development of the FEM version of TASK/WM is ongoing (almost complete).**
- Integral operator code in 3D configuration is waiting for the FEM version of TASK/WM.

Consistent Formulation of Integral Full Wave Analysis

- **Full wave analysis for arbitrary velocity distribution function**
  - **Dielectric tensor**:

$$\nabla\times\nabla\times\boldsymbol{E}(\boldsymbol{r})-\frac{\omega^2}{c^2}\int \mathrm{d}\boldsymbol{r}_0\int \mathrm{d}\boldsymbol{r}'\,\frac{\boldsymbol{p}'}{m\gamma}\frac{\partial f_0(\boldsymbol{p}',\boldsymbol{r}_0)}{\partial \boldsymbol{p}'}\cdot\boldsymbol{K}_1(\boldsymbol{r},\boldsymbol{r}',\boldsymbol{r}_0)\cdot\boldsymbol{E}(\boldsymbol{r}')=\mathrm{i}\,\omega\mu_0\boldsymbol{j}_{\mathrm{ext}}$$

  where $\boldsymbol{r}_0$ is the gyrocenter position.

- **Fokker-Planck analysis including finite gyroradius effects**
  - **Quasi-linear operator**

$$\frac{\partial f_0}{\partial t}+\left(\frac{\partial f_0}{\partial \boldsymbol{p}}\right)_{\boldsymbol{E}}+\frac{\partial}{\partial \boldsymbol{p}}\int \mathrm{d}\boldsymbol{r}\int \mathrm{d}\boldsymbol{r}'\,\boldsymbol{E}(\boldsymbol{r})\,\boldsymbol{E}(\boldsymbol{r}')\cdot\boldsymbol{K}_2(\boldsymbol{r},\boldsymbol{r}',\boldsymbol{r}_0)\cdot\frac{\partial f_0(\boldsymbol{p}',\boldsymbol{r}_0,t)}{\partial \boldsymbol{p}'}=\left(\frac{\partial f_0}{\partial \boldsymbol{p}}\right)_{\mathrm{col}}$$

- The kernels $\boldsymbol{K}_1$ and $\boldsymbol{K}_2$ are closely related and localized in the region $|\boldsymbol{r}-\boldsymbol{r}_0|\lesssim\rho$ and $|\boldsymbol{r}'-\boldsymbol{r}_0|\lesssim\rho$.

- **To be challenged**

Summary

- **Comprehensive analyses of ICRF heating in tokama plasmas**
  - Time-evolution of the velocity distribution functions and the finite gyroradius effects have to be consistently included. For this purpose, the extension of the integrated code TASK is ongoing.

- **Self-consistent analysis including modification of $f(p)$**
  - Full wave analysis with arbitrary velocity distribution function and Fokker-Planck analysis using full wave field are available. Preliminary result of self-consistent analysis was obtained.

- **3D full wave analysis including the finite gyroradius effects**:
  - 1D analysis elucidated the importance of the gyroradius effects of energetic ions. Formulation was extended to a 2D configuration. Implementation is waiting for the FEM version of TASK/WM.