REAL-TIME CONTROL OF INTERNAL TRANSPORT BARRIERS IN JET: EXPERIMENTS AND SIMULATIONS


and contributors to the EFDA-JET workprogramme.

1EFDA-JET Close Support Unit, Culham Science Centre, Abingdon, OX14 3DB, U. K.
2Euratom-CEA Association, CEA-Cadarache, 13108, St Paul lez Durance, France
3Euratom-ENEA Association, C.R. Frascati, 00044 Frascati, Italy
4Euratom-ENEA Association, Consorzio RFX, 4-35127 Padova, Italy
5Euratom-Tekes Association, VTT Processes, FIN-02044 VTT, Finland
6Euratom-FOM Association, TEC Cluster, 3430 BE Nieuwegein, The Netherlands
7Euratom-UKAEA Association, Culham Science Centre, Abingdon, U. K.
OUTLINE

1. Dimensionless ITB temperature or pressure gradient characterizing ITB’s on JET, (also used on Tore Supra, FTU, Alcator C-Mod.

2. Technique for controlling the current and pressure profiles in high performance tokamak plasmas with ITB’s: a technique which offers the potentiality of retaining the distributed character of the plasma parameter profiles.

3. First experiments using the simplest, lumped-parameter, version of this technique for the current profile:

   3.1. Control of the q-profile with one actuator: LHCD Modelling with CRONOS

   3.2. Control of the q-profile with three actuators: LHCD, NBI, ICRH Modelling with JETTO

   *** PRELIMINARY***

4. Ongoing experiments on simultaneous real-time control of the current density + temperature gradient profiles.
Challenges of advanced profile control

Early experiments on JET were based on scalar measurements characterising the profiles ($\rho_T \times \text{max}$) and/or other global parameters ($I_i$)

HOWEVER

1. ITB = pressure and current (+ rotation ...) **profiles**
   Multiple-input multiple-output distributed parameter system (MIMO + DPS)

2. **Nonlinear interaction** between $p(r)$ and $j(r)$
   Feedback loop interaction

Need more information on the space-time structure of the system
Identify a high-order operator model around the target steady state
and try model-based DPS control using SVD techniques

*D. Moreau et al., Nucl. Fusion 43 (2003) 870*
ITB dimensionless gradient criterion

Stabilisation of drift wave microturbulence through flow shear

$$\rho_T^* = \frac{\rho_s}{L_T}$$

$$\rho_T^*(R,t) \geq \rho_{ITB}^* \iff \text{ITB at } (R,t)$$

$\rho_{ITB}^* (\Lambda_T, s, q, \beta, v^*, M_\phi, \ldots) \approx 1.4 \times 10^{-2}$

- 116 deuterium pulses
- $1.8 \, T \leq B_\phi \leq 4 \, T$
- $1.6 \, MA \leq I_p \leq 3.6 \, MA$
- $3.3 \leq q_{95} \leq 4.3$
- $2 \times 10^{19} \, m^{-3} \leq n_{e0} \leq 5.5 \times 10^{19} \, m^{-3}$
- $4.8 \, MW \leq P_{NBI} \leq 18.7 \, MW$
- $0 \, MW \leq P_{ICRH} \leq 8.7 \, MW$

G. Tresset et al, Nuclear Fusion 42 (2002) 520

D. Moreau Theory-Based Modeling and Integrated Simulation of Burning Plasmas, Kyoto (Japan) December 2003
Approximate Model and Singular Value Decomposition

\[ \mathcal{K} = \text{Linear response function} \quad (\mathbf{y} = \text{[current, pressure]} ; \mathbf{p} = \text{heating/CD power}) \]

\[ \mathbf{y}(x,t) = \int_0^t dt' \int_0^1 dx' \mathcal{K}(x,x', t - t') \mathbf{p}(x', t') \]

Laplace transform:

\[ \mathbf{y}(x,s) = \int_0^1 dx' \mathcal{K}(x,x',s) \mathbf{p}(x',s) \]

Kernel singular value expansion in terms of orthonormal right and left singular functions + System reduction through Truncated SVD (best least square approximation):

\[ \mathcal{K}(x,x',s) = \sum_{i=1}^{\infty} \mathbf{w}_i(x,s) \sigma_i(s) \mathbf{v}_i(x',s) \]
Set of output trial function basis

Output profiles:
\[ \mathcal{Y}(x,s) = \sum_{j=1}^{N} \mathcal{D}_j(x) \cdot Q_j(s) + \text{residual} \]

and

Output singular functions:
\[ \mathcal{W}_k(x,s) = \sum_{j=1}^{N} \mathcal{D}_j(x) \cdot \Omega_{kj}(s) + \text{residual} \]

With 2 profiles (current, pressure):
\[ \mathcal{D}_j(x) = \begin{bmatrix} a_j(x) & 0 \\ 0 & b_j(x) \end{bmatrix} \]

Identification of the operator \( \mathcal{K} \)

Galerkin’s method: residuals spatially orthogonal to each basis function

\[ \int_{0}^{1} \text{residual. } \mathcal{D}_1(x) \, dx = 0 \]

\[ Q(s) = K_{\text{Galerkin}}(s) \cdot P(s) \]
Real time reconstruction of the safety factor profile (1)

The q-profile reconstruction uses the real-time data from the magnetic measurements and from the interfero-polarimetry, and a parameterization of the magnetic flux surface geometry

\[ q(x) = \frac{d\phi}{d\psi} = \frac{c_0 + c_1 x + c_2 x^2 + c_3 x^3 + c_4 x^4}{a_1 x + a_2 x^2 + a_3 x^3} \]

\[ 0 \leq x \leq 1 \]

\[ 0 \leq \psi \leq \pi \]

\[ P \leq \psi \leq Q \]

\[ P \leq x \leq Q \]

\[ P \leq \psi \leq Q \]

\[ P \leq x \leq Q \]

\[ P \leq \psi \leq Q \]

\[ P \leq x \leq Q \]

\[ P \leq \psi \leq Q \]

\[ P \leq x \leq Q \]

\[ P \leq \psi \leq Q \]

\[ P \leq x \leq Q \]

\[ P \leq \psi \leq Q \]

\[ P \leq x \leq Q \]

\[ P \leq \psi \leq Q \]

\[ P \leq x \leq Q \]

\[ P \leq \psi \leq Q \]

\[ P \leq x \leq Q \]

\[ P \leq \psi \leq Q \]

\[ P \leq x \leq Q \]

\[ P \leq \psi \leq Q \]

\[ P \leq x \leq Q \]

\[ P \leq \psi \leq Q \]

\[ P \leq x \leq Q \]

\[ P \leq \psi \leq Q \]

\[ P \leq x \leq Q \]

\[ P \leq \psi \leq Q \]

\[ P \leq x \leq Q \]

\[ P \leq \psi \leq Q \]

\[ P \leq x \leq Q \]

\[ P \leq \psi \leq Q \]

\[ P \leq x \leq Q \]

\[ P \leq \psi \leq Q \]

\[ P \leq x \leq Q \]

\[ P \leq \psi \leq Q \]

\[ P \leq x \leq Q \]

\[ P \leq \psi \leq Q \]

\[ P \leq x \leq Q \]

\[ P \leq \psi \leq Q \]

\[ P \leq x \leq Q \]

\[ P \leq \psi \leq Q \]

\[ P \leq x \leq Q \]

\[ P \leq \psi \leq Q \]

\[ P \leq x \leq Q \]

\[ P \leq \psi \leq Q \]

\[ P \leq x \leq Q \]

\[ P \leq \psi \leq Q \]

\[ P \leq x \leq Q \]

\[ P \leq \psi \leq Q \]

\[ P \leq x \leq Q \]

\[ P \leq \psi \leq Q \]

\[ P \leq x \leq Q \]

\[ P \leq \psi \leq Q \]

\[ P \leq x \leq Q \]

\[ P \leq \psi \leq Q \]

\[ P \leq x \leq Q \]

\[ P \leq \psi \leq Q \]

\[ P \leq x \leq Q \]

\[ P \leq \psi \leq Q \]

\[ P \leq x \leq Q \]

\[ P \leq \psi \leq Q \]

\[ P \leq x \leq Q \]

\[ P \leq \psi \leq Q \]

\[ P \leq x \leq Q \]

\[ P \leq \psi \leq Q \]

\[ P \leq x \leq Q \]

\[ P \leq \psi \leq Q \]

\[ P \leq x \leq Q \]

\[ P \leq \psi \leq Q \]

\[ P \leq x \leq Q \]

\[ P \leq \psi \leq Q \]

\[ P \leq x \leq Q \]
Trial function basis for $q(x)$ or $\tau(x)=1/q(x)$

If the real-time equilibrium reconstruction uses a particular set of trial functions, then one should take the same set for the controller design. Otherwise, the family of basis functions must be chosen as to reproduce as closely as possible the family of profiles assumed in the "measurements".

**EXAMPLE** (with the parameterization used in JET and $c_0 \approx 0$):

\[
q(x) = \frac{d\phi}{d\psi} = \frac{1+c'_1x+c'_2x^2+c'_3x^3}{a'_0+a'_1x+a'_2x^2}
\]

approximated by

\[
\delta q(x) \text{ or } \tau(x) \approx \sum_{i=1}^{N} Q_i \cdot b_i(x)
\]

with:

1. A set of $N = 6$ basis functions $b_i(x)$ can be obtained through differentiation of the rational fraction with respect to the coefficients.

2. Alternatively, one can choose $N \leq 6$ cubic splines for $b_i(x)$.
What does the controller minimize?

Output profiles: \[ \mathbf{V}(x,s) = \sum_{j=1}^{N} \mathbf{D}_j(x) \cdot \mathbf{Q}_j(s) + \text{residual} \]

Setpoint profiles: \[ \mathbf{V}_{\text{setpoint}}(x) = \sum_{j=1}^{N} \mathbf{D}_j(x) \cdot \mathbf{Q}_{j,\text{setpoint}} + \text{residual} \]

GOAL = minimize \[ [\mathbf{V}(s=0) - \mathbf{V}_{\text{setpoint}}] \cdot [\mathbf{V}(s=0) - \mathbf{V}_{\text{setpoint}}] \]

Define scalar product to minimize a least square quadratic form:

\[ \int_0^1 \mu_1(x) \left[ q(x) - q_{\text{setpoint}}(x) \right]^2 \, dx + \int_0^1 \mu_2(x) \left[ \rho_{f}(x) - \rho_{f,\text{setpoint}}(x) \right]^2 \, dx \]
Identification of the first singular values 
and singular functions of $\mathcal{Y}$ for the TSVD

\[
\mathcal{Y}(x,s) = \int_0^1 dx' \mathcal{Y}(x,x',s) \mathcal{Y}(x',s)
\]

The best approximation for $\sigma_k$, $W_k$ and $\mathcal{Y}_k$ in the Galerkin sense in 
the chosen trial function basis $b_i(x)$ is then obtained by performing the 
SVD of a matrix $\hat{K}(s)$ related by $K_{\text{Galerkin}}$ through:

\[
B_{i,j} = \int_0^1 b_i(x) \cdot b_j(x) \, dx \quad \Rightarrow \quad B = \Delta^+ \cdot \Delta \text{ (Cholesky decomposition)}
\]

\[
\hat{K}(s) = \Delta \cdot K_{\text{Galerkin}}(s) \quad \Rightarrow \quad \hat{K}(s) = \hat{W}(s) \cdot \Sigma(s) \cdot V^+(s) \quad \Rightarrow \quad W(s) = \Delta^{-1} \cdot \hat{W}(s)
\]
Pseudo-modal control scheme

SVD provides decoupled open loop relation between modal inputs $\alpha(s) = V^+P$ and modal outputs $\beta(s) = W^QB$

Truncated diagonal system ($\approx 2$ or $3$ modes) : $\beta(s) = \Sigma(s) \cdot \alpha(s)$

STEADY STATE DECOUPLING

Use steady state SVD ($s=0$) to design a Proportional-Integral controller

$$\alpha(s) = G(s) \cdot \delta\beta(s) = g_c [1+1/(\tau_i s)] \cdot \Sigma_0^{-1} \cdot \delta\beta(s)$$

Diagram:

- $Q_{\text{setpoint}}$ to $\delta Q$
- $\delta Q$ to $\text{G}(s) V_0 \Sigma_0^{(-1)} W_0^+B$
- $\text{P}$ to $\text{K}_{\text{plasma}}(s)$
- $Q$ output
Closed-loop transfer function (PI control)

To minimize the difference between the steady state profiles and the reference ones in the least square sense:

$$\min \int [q(x,s=0)-q_{\text{ref}}(x)]^2 \, dx \quad \Rightarrow \quad \min \left\{ (Q^+-Q_{\text{ref}}^+) \Delta^+ \Delta (Q-Q_{\text{ref}}) \right\}$$

$$Q = K_{\text{Galerkin}} \cdot P \quad \Rightarrow \quad \text{Solution: } P_{\text{optimal}} = [V_0 \Sigma_0^{-1} W_0^+ B] \cdot Q_{\text{ref}}$$

Proposed proportional-integral controller:

$$P(s) = g_c \left[ 1 + \frac{1}{(\tau_i \cdot s)} \right] \cdot [V_0 \Sigma_0^{-1} W_0^+ B] \cdot \delta Q = G_c \cdot \delta Q$$

- Closed loop transfer function ensures steady state convergence to the least square integral difference with no offset, i.e.

$$P(s=0) = P_{\text{optimal}}$$

- Choose $g_c$ and $\tau_i$ to ensure closed-loop stability [i.e. $\text{Im} \,(\text{poles } s_k) < 0$]
Initial experiments with the lumped-parameter version of the algorithm with 1 actuator q-profile control with LHCD power

The accessible targets are restricted to a one-parameter family of profiles

With 5 q-setpoints: no problem if the q-profile tends to "rotate" when varying the power.

With only the internal inductance some features of the q-profile shape could be missed (e.g. reverse or weak shear in the plasma core).

Applying an SVD technique with 5 q-setpoints may not allow to reach any one of the setpoints exactly, but could minimize the error on the profile shape.
Lumped-parameter version of the algorithm with 1 actuator
q-profile control with LHCD power at 5 radii

Pulse No: 55873

\[ I_p \quad (\text{MA}) \]
\[ n_{eo} \quad \left(10^{19} \text{m}^{-3}\right) \]
\[ T_{eo} \quad \text{(keV)} \]
\[ P_{LHCD} \quad \text{(MW)} \]
\[ V_{loop} \quad \text{(V)} \]
5-point q-profile control with LHCD power steady state

\[ P_{\text{LH}} \ (\text{MW}) \]

\[ V_{\text{loop}} \ (\text{V}) \]

\[ I_i \]

\[ \rho = 0.8 \]
\[ \rho = 0.6 \]
\[ \rho = 0.5 \]
\[ \rho = 0.4 \]
\[ \rho = 0.2 \]

\[ (B_T = 3 \ T; I_p = 1.3 \text{ MA}) \]

Safety factor at \( r/a = 0.5 \)

Without LHCD control

Pulse No: 57324

Pulse No: 57329

Pulse No: 57322


D. Moreau  
Theory-Based Modeling and Integrated Simulation of Burning Plasmas, Kyoto (Japan) December 2003
CRONOS integrated modelling code

- Integrated suite of codes, fully modular.
- Solves transport equations (energy, current, density, ...), self-consistently with:
  - Plasma equilibrium (2D equilibrium solver HELENA)
  - Computation of the H/CD/particle sources (ICRH : PION, LHCD : DELPHINE, NBI : SINBAD, ECRH : REMA)
- Coupled to JET, Tore Supra, FTU databases, and ITPA Profile DBs
- Several transport models available: Bohm/gyro-Bohm, Weiland, GLF23, ...
- Post-processing: MHD stability (Mishka), diagnostic reconstruction (MSE, polarimetry, ...)
- Feedback control available

J.F. Artaud, V. Basiuk, F. Imbeaux, X. Litaudon
CRONOS Current Diffusion Simulation #57329

X. Litaudon, V. Basiuk

D. Moreau

Theory-Based Modeling and Integrated Simulation of Burning Plasmas, Kyoto (Japan) December 2003
CRONOS Current Diffusion simulation #57329

- CRONOS
- RT polarimetry
- Reference

$t=12s$

Normalised radius

Faraday angle (Rad.)

Chanel number

X. Litaudon, V. Basiuk
Initial experiments with the lumped-parameter version of the algorithm with 3 actuators
2-mode TSVD for 5-point q-profile control

\[ K_T = \sigma_1 W_1 \cdot V_1^+ + \sigma_2 W_2 \cdot V_2^+ \]
Initial experiments with the lumped-parameter version of the algorithm with 3 actuators
2-mode TSVD for 5-point q-profile control

D. Moreau et al., Nucl. Fusion 43 (2003) 870
F. Crisanti et al., EPS Conf. (2003)
JETTO simulations with the distributed-parameter version of the algorithm with 3 actuators
TSVD for 5-point q-profile control

The Predicted \( q \)-profile Evolution with RTC

Target \( q \)-profiles
- no RTC (ref.)
- flat
- weakly reversed
- reversed
- monotonic

T. Tala

T. Moreau

Theory-Based Modeling and Integrated Simulation of Burning Plasmas, Kyoto (Japan) December 2003
JETTO simulations with the distributed-parameter version of the algorithm with 3 actuators
TSVD for 5-point $q$-profile control

The Predicted $q$-profile Evolution with RTC

$r/a=0.2$

$r/a=0.4$

$r/a=0.5$

$r/a=0.6$

$r/a=0.8$
Conclusions and perspectives (1)

1. A **fairly successful control of the safety factor profile** was obtained with the **lumped-parameter version** of the proposed TSVD algorithm.

2. Preliminary results have just been obtained with the **distributed-parameter version including** $[q(r) + \rho_{T*}(r)]$.

These results provide an interesting basis and call for a larger integrated modeling and experimental programme on JET, aiming at the sustainment and control of ITB's in fully non-inductive plasmas and **with a large fraction of bootstrap current**.
Conclusions and perspectives (2)

The potential extrapolability of the proposed DPS/TSVD technique to strongly coupled distributed-parameter systems with a larger number of actuators and input/output parameters and with more flexibility in the deposition profiles, is an attractive feature for an INTEGRATED BURNING PLASMA CONTROL FOR STEADY STATE ADVANCED REACTOR OPERATION, including

- control of the plasma shape,
- of the safety factor profile (including plasma current, q_{edge}),
- of the temperature and density profiles,
- but also of the fusion and radiated powers,
- and of the primary flux consumption/recharging.