Recent progress in simulations of kinetic-MHD instabilities

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Outline

1. Simulation of TAE bursts in the TFTR plasma

2. Simulation of energetic particle mode in the JT-60U plasma

3. Simulation of the precessional fishbone instability
Alfvén eigenmode bursts (1)


- Recurrent bursts of magnetic fluctuation.
- Drop in neutron emission suggests that energetic ion loss takes place associated with the TAE bursts.
Simulation model

- A Fokker-Planck-MHD hybrid simulation, where the MHD nonlinearity is taken into account, qualitatively reproduced TAE bursts [Y. Todo et al., NF 41 1153 (2001)]. No significant change in the mode profiles was observed when the modes are growing.

- Thus, for simulation with realistic parameters in energetic ion transport time scale, we employ a reduced model, where

1) TAE spatial profile is assumed to be independent of mode amplitude, and

2) the amplitude and phases and the nonlinear fast-ion dynamics are followed self-consistently [similar to H.L.Berk et al., Nucl. Fusion 35, 1713 (1995)].
The mode profile and frequency is obtained from a Fokker-Planck-MHD simulation [Y. Todo et al., NF 41 1153 (2001)].

The mode damping rate is assumed $\gamma_d = 4 \times 10^3 s^{-1}$. 

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(a) $n=1$, $\omega/\omega_A = 0.283$ (mode 1)  
(b) $n=2$, $\omega/\omega_A = 0.404$ (mode 2)  
(c) $n=2$, $\omega/\omega_A = 0.278$ (mode 3)  
(d) $n=2$, $\omega/\omega_A = 0.257$ (mode 4)  
(e) $n=3$, $\omega/\omega_A = 0.330$ (mode 5)
Co and Counter Particle Orbits in Tokamak

Co-injected particles shift outward and can stick out of plasma; Counter-injected particles immediately hit inner limiter.
Physics Condition based on the TFTR Experiment

1. **Particles are injected** with a constant energy 110keV which corresponds roughly the Alfven velocity parallel to the magnetic field.

2. The particles that are transported to the limiter are **removed**.

3. The injected beam ion has a uniform pitch angle distribution in the range of $0.7 \leq |l| \leq 1$ (balanced injection).

4. NBI heating power: 10MW

5. Slowing down time: 100ms

6. Pitch angle scattering rate: $\frac{\Delta d}{\Delta t} = \frac{V_c^3}{2V^3}$
Synchronized bursts take place recurrently at a burst interval that is roughly 2.9 ms which closely matches that of experimental value 2.2 ms in the TFTR experiment that we are comparing with.
Stored Energetic Particle Energy

Stored beam energy with TAE turbulence

Classically slowed down beam

40% of the classical distribution.

The drop in the stored beam energy is 9%.

Co-injected beam part

Counter injected beam part

Both the saturation level and the modulation depth of the drop closely match the experimental value.
Time evolution of the beam ion density

- The time evolution of the a) counter-injected and b) co-injected beam ion density as functions of the minor radius $r$ after they are averaged in the poloidal and toroidal directions.
Summary of TAE bursts

- We have closely reproduced many experimental characteristics (except for saturation amplitude):
  a) the synchronization of multiple TAEs,
  b) time intervals fairly close to the experimental value,
  c) the modulation depth of the drop in the stored energy,
  d) saturation level of the stored beam energy.
- We have analyzed the particle loss mechanism and found particle loss that is due to:
  1) the resonance overlap of different eigenmodes;
  2) the disappearance of KAM surfaces in phase space due to overlap of multiple nonlinear islands created by a single eigenmode.
- Only co-injected beam ions build up to a significant stored energy.
Fast Frequency Sweeping Mode observed in the JT-60U plasma with NNB injection


Frequency sweeping takes place both upward and downward by 10-20kHz in 1-5 ms.

**Figure 7.** (a) Time traces of the frequency spectrum of magnetic fluctuations during fast FS modes. (b) Time trace of a filtered magnetic probe signal by using a numerical band pass filter with a band frequency of 30–60 kHz.
Particle-MHD hybrid simulation

1. Plasma is divided into “energetic ions” + “MHD fluid”.

2. Electromagnetic fields are given by MHD equations.

3. Energetic ions are described by the drift-kinetic equation.
MHD Equations: current coupling model

\[ \frac{\partial \mathbf{B}}{\partial t} = \mathbf{E} \]

\[ \mathbf{E} = \mathbf{v} \times \mathbf{B} + \mathbf{B} \times \mathbf{B} \]

\[ \frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \frac{\partial \mathbf{v}}{\partial t} = \mathbf{v} \cdot \left( \frac{1}{\rho} \frac{\partial \mathbf{p}}{\partial t} + \frac{1}{\rho_0} \left( \mathbf{B} \cdot \mathbf{j}_{\parallel} \right) \mathbf{B} \right) + \frac{en_h}{B} E_\parallel \]

\[ + \mathbf{v} \cdot \left( \mathbf{v} \cdot \mathbf{B} \right) \mathbf{B} \cdot \mathbf{B} \mathbf{v} \]

\[ \frac{\partial p}{\partial t} = \mathbf{v} \cdot \left( p \mathbf{v} \right) \mathbf{v} \left( \frac{1}{\rho} \frac{\partial \mathbf{B}}{\partial t} \right) \mathbf{B} \cdot \mathbf{v} \]

\[ + \mathbf{B} \cdot \left( \mathbf{j}_{\parallel} \right) \mathbf{v} \cdot \left( \mathbf{v} \cdot \mathbf{B} \right) \mathbf{B} \cdot \mathbf{B} \mathbf{v} \]

\[ +(\frac{g-1}{\rho_0}) \mathbf{B} \cdot \left( \mathbf{j}_{\parallel} \right) \mathbf{B} \cdot \mathbf{B} \mathbf{B} \]

\[ \frac{\partial \rho}{\partial t} = \mathbf{v} \cdot \left( \rho \mathbf{v} \right) \mathbf{v} \left( \frac{1}{\rho} \frac{\partial \mathbf{B}}{\partial t} \right) \mathbf{B} \cdot \mathbf{v} \]

\[ + \mathbf{B} \cdot \left( \mathbf{j}_{\parallel} \right) \mathbf{v} \cdot \left( \mathbf{v} \cdot \mathbf{B} \right) \mathbf{B} \cdot \mathbf{B} \mathbf{v} \]
Condition (E36379, t=4.0s)

1. $R_0 = 3.4$ m, $a = 1.0$ m
2. $B = 1.2$ T
3. DD plasma
4. $n_e$ at the plasma center $1.6 \times 10^{19}$ m^{-3}
5. Alfvén velocity at the plasma center $4.6 \times 10^6$ m/s
6. Injection energy $346$ keV ($5.75 \times 10^6$ m/s)
7. Only parallel velocity is considered.
8. Maximum velocity of the slowing down distribution is assumed to be 80% of the injection velocity.
9. Finite viscosity and resistivity ($10^{-5}$ or $2 \times 10^{-5} R_0 v_A$) are assumed for nonlinear simulation.
Investigation for different energetic ion pressure profiles

- Red curve is a classical distribution based on the OFMC code calculation.
- The energetic ion beta value at the plasma center is 1.9% for the red curve.
- For the other curves central beta value is reduced.
- All the profiles are similar at r/a>0.6.
Mode spatial profile depends on energetic ion pressure profile. They do not peak at the TAE gap at $q=2.5$. 

$\text{------> Energetic Particle mode (EPM)}$
Mode spatial profile (toroidal electric field) depends on energetic ion pressure profile

Profile I: $\phi_h(0)=1.9\%$
Profile III: $\phi_h(0)=0.8\%$
A new kind of EPM

1. The EPMs are not on the shear Alfvén continuum. They are different from the local (resonant) EPM [L. Chen (1994), C.Z. Cheng et al. (1995)].


3. The EPMs found in this work is similar to the nonlocal EPM but in monotonic shear plasma and with passing energetic ions.

4. Theoretical exploration is needed!
Time evolution of the nonlocal EPM (energetic ion pressure profile I: classical)

Left: Time evolution of cosine part of the toroidal electric field. \( \frac{B_{\text{max}}}{B} \approx 10^{-2} \).

Right: Frequency shifts only downward by 33% (~17kHz) in 6 \( 10^2 \) Alfvén time (~0.5ms).
Consider reduced energetic ion pressure profiles

- In the experiments fast FS modes and abrupt large events take place with time intervals much shorter than the slowing down time.

- These activities lead to redistribution and loss of energetic ions.

- The classical distribution gives an overestimate of energetic ion pressure profile.

- We consider reduced pressure profiles whose shape is the same as the classical distribution.
Time evolution of the nonlocal EPM (classical distribution is reduced to 2/5)

Left: Time evolution of cosine part of the toroidal electric field. $\frac{B_{\text{max}}}{B} \sim 2 \times 10^{-3}$

Right: Frequency shifts upward by 14% ($\sim 7$kHz) and downward by 23% ($\sim 12$kHz) in $10^3$ Alfvén time ($\sim 0.8$ms). Close to the experiment.
Radial beta profiles

Initial: classical pressure profile.
40% reduction in central beta. Flattened at r/a<0.4.

Initial: reduced pressure profile.
15% reduction in central beta. Flattened only at r/a<0.2.
Time evolution of mode spatial profile
(toroidal electric field)

Classical pressure profile

Reduced pressure profile

Change in mode profile is small.
Summary for fast FS mode

1. **A new kind of nonlocal EPM** is found near the plasma center with monotonic magnetic shear and passing energetic ions.

2. When a classical distribution is taken for the initial condition, frequency shifts only downward.

3. When a reduced distribution is taken, frequency shifts both upward and downward. The rate of frequency sweeping is close to the fast FS modes in the experiments.

4. A great redistribution of energetic ions takes place when the classical distribution is taken for the initial condition.
Initial condition of precessional fishbone simulation

1: Major radius
   Minor radius
   1.43 [m] 
   0.43 [m] 

2: Magnetic field at axis
   1.5 [T] 

3: Bulk plasma component
   Beam plasma component
   H+ plasma 
   D+ plasma 

4: Density
   5\times10^{19} [m^{-3}] 

5: Velocity distribution of NBI fast ion
   isotropic slowing down distribution, injection energy 50keV 

6: Spatial profile of fast ion pressure
   Gaussian distribution 
   Pressure gradient scale at q=1 surface 
   0.15 [m] 

Mode Structure of kink mode

Radial velocity of (1,1) mode is well-known top-hat structure.

Mode Structure of fishbone

Across the q=1 surface phase of the mode profile changes (spiral structure).
Temporal evolution of poloidal magnetic field and frequency of the fishbone mode, $\delta_h=3\%$.

Saturation level $|B|/B_{\infty}\sim 5\times 10^{-3}$, $v/v_A\sim 2\times 10^{-3}$. This corresponds to $\delta\sim v/\omega_{ci}\sim 0.15a/0.5r_{q=1}$.

Frequency shifts downward.
Summary for fishbone mode

1. Linear stability and nonlinear evolution of fishbone mode is investigated. Frequency shifts downward at saturation. Saturation level is \( \frac{B}{B_{\text{sat}}} \approx 5 \times 10^{-3}, \ \frac{v}{v_A} \approx 2 \times 10^{-3} \). This corresponds to \( \frac{v}{v_A} \approx 0.15a^{0.5r_{q=1}} \).

2. Future work:

   Investigate saturation and frequency sweeping mechanism.