Global Kinetic Analysis of Alfven Eigenmode in Toroidal Plasmas

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• Motivation
• Alfven Eigenmodes in Toroidal Plasmas
• Analysis of Alfven Eigenmodes by TASK/WM
• Summary
Motivation

• Existence of Energetic Ions:
  ◦ ICRF heating generates energetic ions: High energy tail in $f(v_{\perp})$.
  ◦ Negative-ion-based neutral beam injection produces fast ions.
  ◦ Fusion reaction creates energetic alpha particles.

• Destabilization by Energetic Particles
  ◦ The least stable mode is destabilized by energetic ions.
  ◦ The stability depends on the radial profile of fast ion pressure.
  ◦ The stability is also sensitive to the $q$ profile.

• Nonlinear Interaction of Wave and Energetic Ions
  ◦ Radial diffusion and instantaneous loss of energetic ions
  ◦ Reduction of heating power and fusion reaction rate
  ◦ Localized damage of first wall
Alfvén Eigenmode Excited by Alpha Particles

- **DT Burning experiment on TFTR**
  
  (Nazikian et al., PRL 78 (1997) 2976)

**Decay phase after NBI**

**Excitation of AE**
Alfvén Eigenmode excited by ICRF and NBI

- **NNBI experiment on JT-60U** (Kusama et al., NF 39 (1999) 1837)
- **ICRF experiment on JT-60U** (Kimura et al., JPFR 71 (1993) 1147)
Burst Excitation of Alfvén Eigenmode

- NBI experiment on DIII-D (Duong et al., NF 33 (1993) 749)
- Nonlinear Simulation of TAE (Todo, JPFR 75 (1999) 567)

TFTR Simulation
• **Shear Alfvén Wave (SAW) and Compressional Alfvén Wave (CAW)**

![Diagram of Shear Alfvén Wave and Compressional Alfvén Wave](image)

- **Shear Alfvén Wave**: No coupling with adjacent line of force $\Rightarrow$ Frequency independent of $k_\perp$

  $$\omega = k_\parallel v_A,$$  
  Alfvén Velocity $$v_A^2 = \frac{c^2}{1 + \omega_{pi}^2/\omega_{ci}^2}$$
Alfvén Waves in Inhomogeneous Plasmas

- **Static magnetic field**: $z$-axis, **Density inhomogeneity**: $x$-axis

- **Maxwell’s Equation**: 
  
  \[-\nabla \times \nabla \times \mathbf{E} + \frac{\omega^2}{c^2} \epsilon \cdot \mathbf{E} = 0\]

  \[
  \begin{pmatrix}
  -k_y^2 - k_z^2 & -i k_y \frac{\partial}{\partial x} & -i k_z \frac{\partial}{\partial x} \\
  -i k_y \frac{\partial}{\partial x} & -k_z^2 + \frac{\partial^2}{\partial x^2} & +k_z k_y \\
  -i k_z \frac{\partial}{\partial x} & +k_z k_y & -k_y^2 + \frac{\partial^2}{\partial x^2}
  \end{pmatrix} \cdot \mathbf{E} + \frac{\omega^2}{c^2} \begin{pmatrix}
  S & -i D & 0 \\
  i D & S & 0 \\
  0 & 0 & P
  \end{pmatrix} \cdot \mathbf{E} = 0
  \]

- **Dielectric tensor**: $\epsilon$
  
  - **Local model**
    
    \[
    S \approx 1 + \frac{\omega_{pi}^2}{\omega_{ci}^2}, \quad D \approx \frac{\omega_{pi}^2}{\omega_{ci}^2} \frac{\omega}{\omega_{ci}}, \quad P \approx \left\{ \begin{array}{l}
    -\frac{\omega_{pe}^2}{\omega_{ci}^2} \\
    +\frac{\omega_{pe}^2}{k_{\parallel}^2 v_{Te}^2}
  \end{array} \right. \]
    
    (Cold plasma)

  - **Differential operator model**: Finite Larmor radius effect ($k_{\perp} \rho \ll 1$)

  - **Integral operator model**: Finite orbit width effect (Arbitrary $k_{\perp} \rho$)
**MHD model**

- **Ideal MHD approximation**: \((S \approx \omega_{pi}^2/\omega_{ci}^2, D = 0, P = \infty)\)

- **Equation describing wave electric field** (normalized by \(c/\omega\)):
  \[
  \frac{\partial}{\partial x} \left( S - k_z^2 \right) \frac{\partial}{\partial x} E_y + (S - k_z^2)E_y = 0
  \]

- **Shear Alfvén Resonance**:
  \[S - k_z^2 \sim S'_r(x - x_0) + i S, \quad \delta = S_i/S'_r\]

- **Logarithmic singularity**: \(E_y = C \ln(x - x_0 + i \delta)\)
  \[
  \frac{\partial^2 E_y}{\partial x^2} + \frac{1}{x - x_0 + i \delta} \frac{\partial E_y}{\partial x} - k_y^2 E_y = 0
  \]

- **Power absorbed at the singularity**:
  \[
  P_{abs} = \frac{\omega \pi |C|^2}{2 \mu_0} \frac{S'_r}{k_y^2}
  \]
Propagation of Shear Alfvén Wave

• **Effect of finite frequency**:

\[ \frac{\omega}{\omega_{ci}} \neq 0 \implies D \neq 0 \]

• **Equation describing wave electric field**:

\[
\left( \frac{\partial}{\partial x} - \frac{D}{k_y} \right) \frac{k_y^2}{S - k_z^2 - k_y^2} \left( \frac{\partial}{\partial x} + \frac{D}{k_y} \right) E_y \\
+ \left( \frac{\partial^2}{\partial x^2} + S - k_z^2 \right) E_y = 0
\]

• **Propagation of SAW in the lower density side of Alfvén resonance**

![Graph showing Increase of density](image)
Mode conversion of Shear Alfvén Wave

- **In the vicinity of Alfvén resonance**
  - Short wave length $\Longrightarrow$ Electrostatic
  - Since $E_z \neq 0$, differential eq. of the 4th order
  - Propagation depends on the sign of $P$

- **Extremely low $\beta : \beta < m_e/m_i$**
  - Finite electron mass effect
    \[ \nu_{Te} < \omega/k_{||} \sim v_A, \quad P < 0, \]
  - Propagation in the **lower density side**

- **Finite $\beta : \beta > m_e/m_i$**
  - Finite temperature effect
    \[ \nu_{Te} > \omega/k_{||} \sim v_A, \quad P > 0, \]
  - Propagation in the **higher density side**
Alfvén Eigenmode in a Cylindrical Plasma

- **CA Eigenmode** ($|m| \neq 1$)
  \[
  \omega \gtrsim \left(\frac{\pi}{a}\right) v_A
  \]

- **CA Surface Eigenmode** ($m = 1$)
  
  - Nearly constant in plasma, discontinuous on surface

- **CA Surface Eigenmode** ($m = -1$)
  
  - Localize near surface with the increase of $k_\parallel$

- **Shear Alfvén Wave** : Strong damping
  \[
  k_{\parallel}v_{A_{\text{max}}} \gtrsim \omega \gtrsim k_{\parallel}v_{A_{\text{min}}}
  \]

- **Shear Alfvén Eigenmode**
  (GAE : Global Alfvén Eigenmode)
  \[
  \omega \sim k_{\parallel}v_{A_{\text{min}}}
  \]
Examples of Alfvén Eigenmode in a Cylindrical Plasma

• $B = 3 \, \text{T}$, $a = 1 \, \text{m}$, $n_e(0) = 10^{20} \, \text{m}^{-3}$, $m = -1$, $k_\parallel = 10 \, \text{m}^{-1}$

$f_r = 6.55 \, \text{MHz}$
GAE

$f_r = 8.21 \, \text{MHz}$
SAW

$f_r = 9.40 \, \text{MHz}$
SAW

$f_r = 10.31 \, \text{MHz}$
CAW
Alfvén Eigenmode in a Toroidal Plasma (I)

• Toroidal Plasma
  ○ Major radius dependence of $B \Rightarrow$ Poloidal angle dependence
  \[
  \frac{1}{v_A^2} \propto \frac{1}{B^2} \sim \frac{1 + 2\varepsilon \cos \theta}{B_0^2}
  \]

• SAW dispersion without toroidal effect
  \[k_{\parallel m} - \frac{\omega^2}{v_A^2} = 0\]

• SAW dispersion with toroidal effect
  \[
  \begin{vmatrix}
  k_{\parallel m-1}^2 - \frac{\omega^2}{v_A^2} & -\varepsilon\frac{\omega^2}{v_A^2} & 0 \\
  -\varepsilon\frac{\omega^2}{v_A^2} & k_{\parallel m}^2 - \frac{\omega^2}{v_A^2} & -\varepsilon\frac{\omega^2}{v_A^2} \\
  0 & -\varepsilon\frac{\omega^2}{v_A^2} & k_{\parallel m+1}^2 - \frac{\omega^2}{v_A^2}
  \end{vmatrix} = 0
  \]
Alfvén Eigenmode in a toroidal Plasma (II)

• Resonance frequency including only $m$ and $m + 1$ modes

$$\omega^2_{\pm} = \frac{k_{\parallel m}^2 + k_{\parallel m+1}^2 \pm \sqrt{(k_{\parallel m}^2 - k_{\parallel m+1}^2) + 4\varepsilon^2 k_{\parallel m}^2 k_{\parallel m+1}^2}}{2(1 - \varepsilon^2)}$$

• Condition for Alfvén frequency gap

$$k_{\parallel m}^2 = k_{\parallel m+1}^2 \implies k_{\parallel m} = -k_{\parallel m+1} \implies q = -\frac{m + 1/2}{n}$$

• Toroidicity-induced Alfvén Eigenmode : (TAE)
• Example of low $n$ TAE

\[
R = 3 \text{ m}, \quad a = 1 \text{ m}, \quad B_0 = 3 \text{ T}, \quad n_e = 0.5 \times 10^{20} \text{ m}^{-3}
\]

\[
q(0) = 1, \quad q(a) = 2, \quad n = 1, \quad f = 126.9 \text{ kHz}
\]
Kinetic Alfvén Eigenmode

• Low-\(\beta\) MHD equation including kinetic effects

\[
\left( \mathcal{L}_m + \bar{\rho}^2 \frac{d^4}{dr^4} \right) \phi_m + \bar{\epsilon}(r) \frac{\omega^2}{v_A^2} \frac{d^2}{dr^2} (\phi_{m+1} + \phi_{m-1}) = 0
\]

\[
\mathcal{L}_m = \frac{d}{dr} \left( \frac{\omega^2}{v_A^2} - k_{\parallel m}^2 \right) \frac{d}{dr} - \frac{m^2}{r^2} \left( \frac{\omega^2}{v_A^2} - k_{\parallel m}^2 \right), \quad \bar{\rho}^2 \equiv \rho_i^2 \left( \frac{3 \omega^2}{4 v_A^2} + \frac{T_e}{T_i} k_{\parallel m}^2 \right)
\]

• Frequency gap and kinetic Alfvén waves
Various Alfvén Eigenmodes

- **Non-circular tokamak**
  - Coupling between $m$ and $m + \ell$ modes
    \((\ell = 2$: Elongation, $\ell = 3$: Triangularity)\)
  - Coupling condition:
    \[ q = -\frac{m + \ell/2}{n} \]

- **Helical Plasma**
  - Coupling between $n$ and $n + \ell'N_h$ modes
    \((N_h$: Helical coil turn)\)
  - Coupling condition:
    \[ q = -\frac{m + \ell/2}{n + \ell'N_h/2} \]
Excitation of Alfvén Eigenmode by Energetic Ions

- Destabilization requires
  - Wave-particle resonance condition
    \[ v_{||} > \frac{\omega}{k_{||}} \sim v_A \]
    - Existence of energetic ions is required.
  - Diamagnetic drift velocity faster than poloidal phase velocity
    \[ v_{df} = \frac{T_f}{e_f B} \frac{d \ln n_f}{dr} > \frac{\omega r}{m} \]
    - Low frequency made can be easily excited.

- Growth rate of low \( n \) TAE:
  \[ \frac{\gamma}{\omega} \sim \frac{9}{4} \left[ \beta_f \left( \frac{\omega_{*f}}{\omega_0} - \frac{1}{2} \right) F \left( \frac{v_A}{v_f} \right) - \beta_e \frac{v_A}{v_e} \right] \]
Damping Mechanism of Alfvén Eigenmodes

• MHD model
  ◦ Absorption near Alfvén resonance
    (Continuous spectrum damping)

• Perturbative treatment of kinetic Alfvén waves
  (Eigen function: MHD, Damping: Kinetic)
  ◦ Radiative damping
    (power propagating outward)
  ◦ Landau damping
    (Estimation of parallel wave electric field)

• Kinetic absorption mechanism
  ◦ Electron Landau damping
  ◦ Landau damping of energetic ions
• Magnetic flux coordinates: \((\psi, \theta, \varphi)\)
  ○ Non-orthogonal system (including 3D helical configuration)

• Maxwell’s equation for stationary wave electric field \(E\)

\[
\nabla \times \nabla \times E = \frac{\omega^2}{c^2} \epsilon \cdot E + i \omega \mu_0 j_{\text{ext}}
\]

○ \(\epsilon\) : Dielectric tensor with kinetic effects: \(Z[(\omega - n\omega_c)/k_\parallel]\)

• Fourier expansion in poloidal and toroidal directions
  ○ Exact parallel wave number: \(k_{\parallel, m,n} = (mB^\theta + nB^\varphi)/B\)

• Destabilization by energetic ions included in \(\epsilon\)
  ○ Drift kinetic equation

\[
\left[ \frac{\partial}{\partial t} + v_\parallel \nabla_\parallel + (v_d + v_E) \cdot \nabla + \frac{e_\alpha}{m_\alpha} (v_\parallel E_\parallel + v_d \cdot E) \frac{\partial}{\partial \epsilon} \right] f_\alpha = 0
\]

• Eigenvalue problem for complex wave frequency
  ○ Maximize wave amplitude for finite excitation proportional to \(n_e\)
Typical TAE with Positive Magnetic Shear

- **Configuration**
  - \( q(\rho) = q_0 + (q_a - q_0)\rho^2 \), \( q_0 = 1 \), \( q_a = 2 \)
  - Flat Density Profile

**Contour of \( |E|^2 \) in Complex Frequency Space**

![Graph of Alfvén Frequency](image1)

**Alfvén Frequency**

- Eigen function
  - \( f_r = 81.95 \text{ kHz} \)
  - \( f_i = -20.32 \text{ Hz} \)
AE in the Reversed Magnetic Shear Configuration (JT-60U)

- Takechi et al. IAEA 2002 (Lyon) EX/W-6

Fluctuation Amplitude

Observed frequency  calculated frequency
Analysis of AE in Reversed Shear Configuration

Assumed $q$ profile

Plasma Parameters

- $R_0 = 3$ m
- $a = 1$ m
- $B_0 = 3$ T
- $n_e(0) = 10^{20}$ m\(^{-3}\)
- $T(0) = 3$ keV
- $q(0) = 3$
- $q(a) = 5$
- $\rho_{\text{min}} = 0.5$
- $n = 1$

Flat density profile

$\bullet$ RSAE (reversed-shear-induced Alfvén eigenmode) for $\ell + \frac{1}{2} < q_{\text{min}} < \ell + 1$
$q_{min}$ Dependence of Radial Structure of Alfvén resonance

$\begin{align*}
q_{min} &= 2.0 \\
q_{min} &= 2.1 \\
q_{min} &= 2.2 \\
q_{min} &= 2.3 \\
q_{min} &= 2.4 \\
q_{min} &= 2.5 \\
q_{min} &= 2.6 \\
q_{min} &= 2.7 \\
q_{min} &= 2.8 \\
q_{min} &= 2.9 \\
q_{min} &= 3.0
\end{align*}$
Eigenmode Structure ($n = 1$)

Alfvén resonance

Higher freq.

Lower freq.

TAEs  Double TAE  RSAE
Excitation by Energetic Particles ($q_{\text{min}} = 2.6$)

- Without EP

- With EP

  $3 \times 10^{16}$ m$^{-3}$
  360 keV
  0.5 m

- With EP

  $1 \times 10^{17}$ m$^{-3}$
  360 keV
  0.5 m
Summary

- **Various kinds of Alfvén eigenmodes** have possibilities to be excited by energetic ions in toroidal plasmas.

- The study of linear stability requires **global kinetic analysis**, because the mode structure is sensitive to the \( q \) profile and the damping is sensitive to the parallel wave electric field.

- The existence of **RSAE** in the reversed magnetic shear configuration explains the large-scale frequency increase in the reverse magnetic configuration observed in JT-60U and JET.

- The calculated **threshold of fast ion pressure** is consistent with experimental conditions.

- **Remaining problems**
  - Coupling with drift waves in the low frequency range
  - Nonlinear analysis to estimate the loss of energetic ions
Alfvén Eigenmode in a Cylindrical Plasma (II)

- **GAE : Global Alfvén Eigenmode**
  \[ \omega \sim k_\parallel v_{A\text{min}} \]
  - Shear Alfvén wave can propagate.
  - No Alfvén resonance
  - Weak damping, easily excited

- **Effect of poloidal Magnetic Field**
  - Toroidal mode number : \( n \)
  - Poloidal mode number : \( m \)
  - Safety factor : \( q = rB_\phi/RB_\theta \)
  - Wave number parallel to the long field line : \( k_\parallel = \frac{m + nq}{qR} \)

- If \( k_\parallel v_A \) has local minimum, **GAE** may exist. \( \omega \sim (k_\parallel v_A)_{\text{min}} \)
Structure of TASK code system

• Integrated code for the analysis of toroidal plasmas